Tying in Two-Sided Markets

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Abstract

Tying is widespread in two-sided markets, though it is not obvious why this should be the case. This paper offers an explanation. We extend the standard Hotelling model by allowing a duopoly to serve two distinct groups of consumers who generate externalities upon each other. We find that a quantity spillover across the two sides induces a fundamental change of strategic effects: In the presence of a large externality, price competition could lead to prices being strategic substitutes rather than strategic complements as in traditional markets. Consequently, tying works as a commitment to behave aggressively and will unambiguously hurt rivals but could be self-benefiting. Therefore, tying is adopted no matter whether a firm’s aim is to deter or to accommodate rivals. Our analysis also shows that, in a duopoly, firms may engage in "prisoners’ dilemma" tying. From a social planner’s point of view, tying may be desirable.

Keywords: Two-sided markets, Strategic complement, Strategic substitute, Tying

JEL Classifications: L13, L14, L41

1 Introduction

This paper examines the reason why tying is widespread in two-sided markets and its impact on social welfare. Two-sided markets refer to those in which firms operate as platforms that allow interactions between two distinct groups of customers who need each other. The defining characteristic of these markets is inter-group network externalities: It is more valuable for consumers on one side (or less valuable in presence of negative

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externalities) as more consumers get on the other side. Consequently, the pricing strategy on one side of an intermediary must account for the spillover effect on the other side. Two-sided platforms arise in many economically significant industries. For instance, credit cards provide a convenient method of transaction between consumers and merchants; computer operating systems court both users and application developers; portals, TV channels, newspaper and magazines bring advertisements from producers to potential consumers; auction sites, shopping malls and real estates attempt to match sellers and buyers, etc.¹

It is worth noting that tying is deployed in a wide variety of two-sided markets. One form of tying corresponds to the practice of bundling together two complementary goods. Consider the most famous example: Windows Media Player (WMP) is sold together with Windows. Choi (2007) analyzes this example and argues that the impact of tying on social welfare depends crucially on whether operating system users are exclusive on one media player or not. However, another form, illustrated by selling two completely independent goods together, arises even more frequently. For example, web portals offer a large bundle of services for free, such as email, web photo album, chatting tool, blog and personal website, which could as well be sold separately. Another familiar example is that CD or make-up is often offered together with magazines. This second kind of tying has so far received rare attentions. A major objective of our paper is to fill this gap.

Before analyzing tying of independent goods in the two-sided context, let’s recall the conventional wisdom points. Whinston (1990) argues that tying acts as a commitment to lower the opportunity cost of selling the competitive good. In the pricing game that follows, the firm practicing tying will price more aggressively and as a result both firms suffer losses in the tying regime. Hence, tying will not be adopted unless it enables a firm to drive its rivals out of the market. It is our interest to investigate whether this textbook result still holds in two-sided markets. Furthermore, according to Whinston (1990), tying could be welfare-reducing. It is especially true when there is little market expansion in response to lower prices. This result justifies many economic policies that discourage tying on the grounds of "anti-competition" or "welfare-reducing". However, the logic in one-sided markets may not work in two-sided markets (see Evans (2003), Wright (2003)). The welfare consequence of allowing tying in two-sided markets is therefore another issue that we will address here.

Although our model can be applied to a large range of two-sided industries, for convenient explanation, we will in follows relate our model to the example of magazine. The tying strategy is studied in the context of Hotelling model, where two horizontally

¹More examples can be found in Armstrong (2006), Evans (2003) and Rochet-Tirole (2003a).
different magazines compete for single-homing readers. We characterize two-sided by allowing duopoly to serve another group of consumers — advertisers — whose valuation of a magazine increases in its readership. The advertisers are allowed to be multi-homing so as to reap the maximal network effect. In the basic model, we assume that readers are indifferent of the size of advertisements. We find that the externalities generated by readers on advertisers may induce fundamentally strategic changes to duopoly: contrary to the case in one-sided markets, prices could be strategic substitutes when inter-group externalities are large enough. This is due to the fact that each platform sets price of its magazine by taking into account the effects on the advertisers’ side.

To start, we allow only one magazine to tie its issues with monopolized products, CD, and analyze the impact on the competition equilibrium and social welfare. We show that, in the presence of large externalities, tying can be a profitable strategy for the firm. Consequently, in this case, tying is adopted no matter whether the firm aims at accommodating or deterring its rival, which justifies widespread tying in two-sided markets.

We further show that tying can be welfare-enhancing. This is due to the presence of larger network effect in the regime of asymmetric platforms. The analysis thus has important implications for antitrust cases and provides a caution in applying the traditional theory in two-sided markets.

We then go on to test robustness of the basic model. We consider the possibility of two-direction externalities: Readers could be ad-lovers or ad-haters. We find that, so long as readers don’t dislike advertisements too much, the results in basic model still hold. By extending the model, the result applies to most two-sided industries instead of only to those characterized by one-direction externalities. We finally extend the analysis to the case in which both platforms can deploy tying. It turns out that the two platforms may be involved in a "prisoners' dilemma" in tying in the two-sided market rather than sticking to the "no tying" equilibrium as in one-sided markets.

A number of papers have analyzed tying in two-sided markets.

Rochet and Tirole (2003b) provide an economic analysis of the tying initiated by payment card associations such as Visa and MasterCard in which merchants who accept their credit cards were forced to also accept their debit cards. They show that in absence of tying, the interchange fee between merchants’ and cardholders’ banks on the debit cards is too low and tends to be too high on the credit cards as compared to the social optimum. Tying is shown to be a mechanism to rebalance the interchange fee structure and to raise social welfare.

The closest model to ours is presented by Farhi and Hagiu (2007). They show that the
The possibility of subsidization of one side in a two-sided market can lead to fundamentally new strategic configurations in oligopoly. They present the conditions under which a cost-reducing investment by intermediaries may be a successful entry accommodation strategy and at the same time may also raise the profits of its rival, which will never happen in one-sided markets. As pointed out by Fudenberg and Tirole (1984) and Bulow Geanakoplos and Klemperer (1985), strategic effects in one-sided markets are determined by two factors: Whether actions in the competition game are strategic complements or substitutes, and whether cost-decreasing investments decrease or increase rival’s profits. However, in two-sided markets, it turns to be much more complicated because there are four prices corresponding to four supply levels that need to be considered, as compared to only two variables in traditional markets. In fact, many factors can induce fundamental changes of strategic effects: price is not necessarily strategic complement in competition, the effects of cost-reducing investments on four prices are ambiguous and platforms may earn negative margin on one side. Farhi and Hagiu (2007) emphasize the last factor and show that it is enough to make strategic effect totally different whereas, in our basic model, the first factor plays an important role: in presence of large externalities, price could be strategic substitute. As a result, tying appears to be self-serving rather than self-harming, which justifies widespread tying in two-sided markets.

As we have mentioned earlier, Choi (2007) analyzes the welfare consequence of tying two complementary goods in a model of competition between two-sided platforms, where one or both sides can multi-home. In his model, tying simply allows one of the platforms to reach all consumers by bundling the platform product in question with another product that all consumers need (the motivating example is the tying of Windows Media Player to the Windows Operating System, which every PC user needs). The impact of tying on social welfare depends on whether consumers can multi-home or not, but in all cases, tying unambiguously hurts the rival platform.

Amelio and Jullien (2007) consider a setting in which two-sided platforms would like to set prices below zero on one side of the market in order to solve the demand coordination problem, but are constrained to set non-negative prices. Tying can then serve as a mechanism to introduce implicit subsidies on one side of the market in order to solve the aforementioned coordination failure. As a result, tying can raise participation on both sides and can benefit consumers in the case of a monopoly platform. In a duopoly context tying also has a strategic effect on competition. But contrary to the monopoly case, tying may not be ex-post and/or ex-ante optimal for a contested platform. Moreover, the competing platforms benefit from tying if the equilibrium implicit subsidy is large enough. We also obtain this result in the present study, although as a particular case of a
broader setting. In our paper, we assume that marginal cost of magazine is large enough to avoid negative pricing and then exclude the possibility that the tied goods act as a subsidy to readers.

The remainder of the paper is organized as follows. In Section 2 we set up a two-sided duopolistic framework. Section 3 explores the competition equilibrium without tying. We analyze the effects of tying on market outcome in Section 4. Section 5 derives the welfare analysis and policy implication. In Section 6, we extend the analysis by setting that there are two-direction externalities, the advertisers are single-homing and both firms are allowed to implement tying. Concluding remarks follows.

2 Basic Model

Platforms: Magazines

We suppose that there are two magazines indexed by $i = A, B$. They compete for market share within readers (side 1) and advertisers (side 2). Let $q_i$ and $p_i$ denote the prices charged readers and advertisers respectively. Production in magazine market involves no fixed cost but incurs an expenditure of $c$ per magazine. The cost of serving advertisers side $d$ is neglected. The number of readers and advertisers who participate in platform $i$ are denoted by $n_i$ and $m_i$. We consider a situation in which at least one side is characterized by exclusive intermediation. More specifically, we assume that readers engage in single-homing while advertisers can participate on multiple platforms in order to reap maximal network benefits.

Using a standard Hotelling model: readers are heterogeneous and the number is normalized to one. They locate uniformly on a line with length equal to 1. The unit transportation cost on each side is assumed to be $t$. Platform $A$ and $B$ lie respectively on $x = 0$ and $x = 1$.

Without tying, the best reaction function is:

$$R_i(q_j) = \frac{4t - \alpha^2}{8t - \alpha^2} q_j + \frac{4t(t + d + \frac{2a - \alpha^2}{8t - \alpha^2})}{8t - \alpha^2} = \gamma q_j + \delta + \frac{2ta - d}{8t - \alpha^2}.$$

With tying, the best response system will be:

$$R_A(q_B) = \gamma q_B + \delta + \frac{2a - d}{8t - \alpha^2} - \frac{4t - \alpha^2}{8t - \alpha^2},$$

$$R_B(q_A) = \gamma q_A + \delta + \frac{2ta - d}{8t - \alpha^2}. $$
**Side 1: Readers**

The readers are single-homing, that is, they purchase at most one magazine. For time being, suppose that the reader side is indifferent of the size of advertisements. It can be justified on the ground that the readers come mainly for the "content". The intrinsic values of "content" of two magazines are symmetric, equivalent to \( v \), which is assumed to be large enough that the market is totally covered. The reader locating at point \( x \) derives utility of \( v - q_A - tx \) from magazine A while the net benefit of purchasing B is given by \( v - q_B - t(1 - x) \). Then we can identify the demand function of each magazine

\[
n_i = \frac{1}{2} + \frac{q_j - q_i}{2t} \quad (i = A, B).
\]

**Side 2: Advertisers**

The characteristic of two-sided market is captured by the assumption that the advertisers’ willingness to pay for one magazine depends positively on its readship. More precisely, each advertiser gains additional utility of \( \alpha > 0 \) from each reader who reads the magazine. The net benefit of \( x \)-advertiser on platform \( i \) is given by \( U^i_2(x) = \alpha n_i - p_i - x \). Suppose that advertisers are allowed to be multi-homing. It implies that the decision of participating relies only on the utility enjoyed on the platform, independent of that derived on the other one, which captures the fact that two magazines are not direct competitors on advertiser side and each has monopoly power. A \( x \)-type advertiser will participate on platform \( i \) if \( U^i_2(x) \geq 0 \). We assume that \( x \) is uniformly distributed in \([0, 1]\). The size of advertisers is normalized to 1. Thus, the number of advertisers on platform \( i \) is given by:

\[
m_i = \alpha n_i - p_i \quad (i = A, B),
\]

**Tying Good**

To analyze the effects of tying on competition equilibrium in two-sided markets, I assume that intermediary A is also a monopolist in CD market. The production cost of CD is normalized to 0. Readers each desire at most one unit of CD. All of them have identical reservation value of \( s > 0 \) for CD. Platform A can sell magazine and CD on a stand alone base or in a package.

We consider a two-stage game. In stage one, firm A determines whether or not to tie magazine and CD. The decision can be observed by firm B. In stage two, two platforms pick prices of two sides simultaneously and price competition takes place.

To solve for the equilibrium of this model, we proceed by backward induction.
We first derive the market outcome in absence of tying. The analysis will be used as a benchmark to investigate the effects of tying in two-sided market.

If platform $A$ determines to sell magazine and CD separately, it can extract the entire consumers’ surplus and earn $s$ in the CD market. On the other hand, the profit from magazine market depends on the selling of magazines and on the size of advertising demand. Two platforms compete in prices to attract consumers on each side. Platform $i$’s objective function is given by:

$$\max_{p_i, q_i} \pi_i = (q_i - c)n_i + p_im_i$$

$$= (q_i - c)n_i + p_i(\alpha n_i - p_i).$$

As we mentioned in the previous section, each platform operates as a monopoly on advertiser side. The first order condition with respect with $p_i$ yields the monopoly price\(^3\):\(^3\)

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow p_i = \frac{1}{2} \alpha n_i,$$

with corresponding profit equal to $\frac{\alpha^2}{4} n_i^2$. The advertising price $p_i$, independent of $p_j$, varies positively with the readership. Intuitively, the advertisers place ads on the magazine for reaching prospective consumers. The higher the level of exposure is, the higher they are willing to pay for the ads and the higher the platform can earn from advertising.

The total profit in the magazine market can be written as a function of the readership: $\pi_i(n_i) = (q_i - c)n_i + \frac{\alpha^2}{4} n_i^2$. We will focus on competition strategy on reader side. The first order condition with respect with $q_i$ is given by\(^4\):

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\alpha^2}{4} 2n_i \frac{\partial n_i}{\partial q_i} + (q_i - c) \frac{\partial n_i}{\partial q_i} + n_i = 0.$$  \(^{(4)}\)

We find that the last two terms are regular in the Hotelling model: for maximizing the profit, firms should set a price by equalizing the marginal revenue from readers to the corresponding marginal cost. It is worth noticing the first term: a small increase in the magazine price not only influences profit on reader side but also makes it less attractive for advertisers. The fact that fewer potential consumers are exposed to the advertisement discourages the demand of advertisers and resultanty decreases the profit.

\(^3\)For example, suppose that the demand of market is $D(p) = a - p$ and the marginal cost $0$. As a monopoly, he will set the monopoly price $p^m = \frac{a}{2}$. The difference is that the willingness to pay relies on $n_i$ in our setting.

\(^4\)By assuming $t \geq \frac{\alpha^2}{8}$ (in assumption 2), the second order condition is satisfied.
This additional negative effect on advertiser side makes the platforms tend to set a lower price for readers relative to one-sided markets. From (3) and (4), the best response function of platform $i$ can be written as:

$$R_i(q_j) = \gamma q_j + \delta,$$

where $\gamma = \frac{4t - \alpha^2}{8t - \alpha^2}$ and $\delta = \frac{4t(t + \alpha^2 - \frac{\alpha^2}{t})}{8t - \alpha^2}$. From the system of best reaction functions, we derive a symmetric equilibrium:

$$q_i = t + c - \frac{\alpha^2}{4}, \quad (i = A, B).$$

The price on reader side can be interpreted as the standard Hotelling price $t + c$ adjusted by $\frac{\alpha^2}{4}$ representing its effect on profit of advertiser side resulting from inter-group externalities. In equilibrium, two magazines split the magazine market and charge the same price $p_i = \frac{1}{4} \alpha$ for the advertisers. The total profit of two firms are respectively:

$$\pi_A = \frac{t}{2} - \frac{\alpha^2}{16} + s, \quad \pi_B = \frac{t}{2} - \frac{\alpha^2}{16}.$$  \hspace{1cm} (5)

We need some assumptions to ensure the existence of a unique and stable equilibrium. First, for avoiding adverse selection and opportunistic behavior of agents that platforms could face by offering a direct monetary transfer to consumers, we suppose that the marginal cost of magazine is large enough that the equilibrium prices are non-negative. By doing this, we also differ our model from Amelio and Jullien (2007) in which tied goods acts as a subsidy on one side to solve the coordination problem.

**Assumption 1:** $c \geq \frac{\alpha^2}{4} - t$.

Since both platforms will employ more aggressive strategy in presence of externality across two groups of consumers, they earn less profit relative to one-sided case. In equation (5), we find that the larger benefits generated by readers on advertisers, the less the readers are supposed to pay for the magazine. When the externality is too large or the market competition is too vigorous, the platforms earn negative profits. In order to ensure that they are active, we make the second assumption. With this assumption, we also ensure that the profit function is concave.

**Assumption 2:** $t \geq \frac{\alpha^2}{8}$.

From these two assumptions, we can obtain that $\gamma < \frac{1}{2}$ and $\delta > 0$ as long as the industry exhibits inter-group externality. Compared with $\gamma = \frac{1}{2}$ in standard Hotelling model, the presence of advertising biases the firms towards adopting more aggressive strategy. For ruling out unstable equilibrium, we further need:

**Assumption 3:** $t > \frac{\alpha^2}{6}$. 

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It implies that $\frac{1}{2} > \gamma > -1$ and hence there will be a unique and stable equilibrium. We will see in following that when $\frac{1}{\alpha}$ is large, the reaction curve of each platform is very similar to that in one-sided markets. However, when $\frac{1}{\alpha^2}$ is small, it will be dramatically different.

3.1 Strategic Complements: $t \geq \frac{\alpha^2}{4} \Rightarrow \gamma \in [0, \frac{1}{2})$

The prices charged for readers are strategic complements: the price of one magazine increases in the price of its rival though the best reaction curve becomes less steeper. In the following figure, the point curve describes the best response function in one-sided market while the solid one represents that in two-sided market\(^5\).

When two magazines are significantly different or readers create large externalities on advertise side, platforms still employ complement strategy. The platform will set a higher price in response to an increase in rival’s price and verse vice. When one magazine becomes more expensive, the other one will be more popular, which will play two roles: on one hand, it is more profitable on reader side to increase the price; on the other hand, a higher price induces more losses of advertising revenue. When the former effect dominates, firms will follow their competitor’s pricing strategy as derived in the standard Hotelling model. As we mentioned in benchmark, platforms will adopt more aggressive strategy in two-sided markets: since increasing price on reader side not only influences the profit of selling magazine but also reduces the profit from advertisers, platforms have less incentive to increase prices in two-sided market.

In this case, $q_i = c + t - \frac{\alpha^2}{4} \in (c, c + t)$. Although the equilibrium price is lower than one-sided markets, firms still set the price of magazine above marginal cost and earn money on both sides.

\(^5\)In fact, it is possible that $\delta > \frac{1}{2}(t + c)$ when $c > t$. However, $\gamma < \frac{1}{2}$ for sure.
3.2 Strategic Substitutes: \( t \in (\alpha^2/6, \alpha^2/4) \Rightarrow \gamma \in (-1, 0) \)

When the quantity spillover is quite large or the competition is quite vigorous, strategic effect changes radically. We show the best response curves in the above right figure. The key point is that, in this case, two prices are strategic substitutes: the platforms will sell magazines at a even lower price to hold larger market share when the rival behaves less aggressively.

**Proposition 1** When \( t \geq \alpha^2/4 \), magazine prices are strategic complements. When \( t \in (\alpha^2/6, \alpha^2/4) \), the prices charged on readers turns out to be strategic substitutes.

As we all know, when rival raises price on reader side, platform \( i \) has larger market share, that’s, higher \( n_i \). There are two effects stemming from higher \( n_i \): first, platform faces less competition pressure on reader side, tending to charge more for readers; second, since advertisers’ willingness to pay increases with \( n_i \), platform can earn more money from advertisers by attracting an additional reader and thus has incentives to cut down the price of magazine. These two effects conflict. When \( t > \alpha^2/4 \), the first effect dominates and platforms will adopt similar strategy as in one-sided market. When \( \alpha^2/6 < t < \alpha^2/4 \), the second dominates. The negative influence on advertiser side of increasing price is so significant that the best response to an increase in rival’s price is to decrease price further and then more money can be earned from sale of advertisements.

Note that, in this case, \( q_i = c + t - \alpha^2/4 < c \). The quantity spillover is so large that in equilibrium the platforms will set a price below cost to attract readers. As a result, two firms both lose money on reader side and recoup on advertiser side. This situation is very likely to occur in reality: in a lot of two-sided industries, for instance, the medias, the shopping malls and the date clubs, platforms act as a loss-leader on one side and make money on the other side.
4 Platform Competition with Tying

As is well-known, in one-sided context, tying of two independent goods will hurt rival and is self-harming as well (Whinston 90). Therefore, firms will not deploy tying if it is impossible to drive rival out of the market. Clearly, without fixed cost as in our model, if \( \alpha = 0 \), platform A will never sell magazine and CD as a bundling. The question is addressed here: will it make a difference when \( \alpha > 0 \)?

If platform A decides to tie two products, it sets price \( \tilde{q}_A \) for the tying products. The effective price of magazine A is \( q_A = \tilde{q}_A - s \). Reader locating on \( x \) will determine to buy magazine A with CD or only B by comparing \( v + s - \tilde{q}_A - tx \) with \( v - q_B - t(1 - x) \). Thus, the realized demands of A and B are respectively:

\[
\begin{align*}
    n_A &= \frac{1}{2} + \frac{q_B - (\tilde{q}_A - s)}{2t}, \\
    n_B &= \frac{1}{2} + \frac{(\tilde{q}_A - s) - q_B}{2t}.
\end{align*}
\]

(6) (7)

The tying firm’ profit maximization problem can be written as:

\[
\max_{p_A,q_A} \pi_A = (\tilde{q}_A - c)n_A + p_Am_A = (q_A + s - c)n_A + p_Am_A
\]

The F.O.C. with respect with \( p_A \) is analogous to no tying case, the price determined by platform A on advertiser side depends on the number of reader side: \( p_A = \frac{1}{2} \alpha n_A \). Replacing \( p_A \), the F.O.C. with respect with \( q_A \) yields:

\[
\frac{\partial \pi_A}{\partial q_A} = n_A + \left( -\frac{1}{2t} \right) (q_A + s - c) + \left( -\frac{1}{2t} \right) \frac{\alpha^2}{4} 2n_A = 0.
\]

It turns to be more costly to increase price in tying case. It is due to the fact that, under tying, in order to make profitable sales of CD, it must also make sales of magazine. Not surprising, in the pricing competition that follows, platform A will behave more aggressively in an effort to steal sales away from firm B. Tying arrangement has the same strategic effect as a reduction of the marginal cost on reader side. On the other hand, platform B’s reaction function doesn’t change. We derive the system of best response functions:

\[
\begin{align*}
    R_A(q_B) &= \gamma q_B + \delta - \frac{4t}{8t^2 - \alpha^2} s, \\
    R_B(q_A) &= \gamma q_A + \delta.
\end{align*}
\]

By solving the system, we obtain that, after tying, A’s price levied on advertisers increases while it sets a lower price for readers. Consequently, firm A has larger market share on both sides. \( p_B \) undoubtedly reduces whereas the change of \( q_B \) in response to tying is
ambiguous, depending on whether prices are strategic complements, precisely, whether \( t > \frac{a^2}{4} \). Recall the effects of tying in a one-sided markets: both firms cut down prices, the tying firm gets larger market share and both suffer loss from tying. We will see that these results may be different in two-sided markets. As in previous section, we will discuss this issue in two cases.

4.1 Strategic Complements

When \( t > \frac{a^2}{4} \), the prices are still strategic complements, i.e. \( \frac{\partial q_i}{\partial q_j} = \gamma > 0 \). It is easy to show that tying shifts firm A’s reaction curve leftward. The change on the equilibrium can be seen in the figure.

A decrease in effective price of magazine A and in the price of magazine B follows tying. In equilibrium, both platforms cut down the price of magazines. The impact of tying on magazine B’s profit can be identified by calculating the differentiation of \( \pi_B \) with respect to the value of CD. Using Envelope Theory, the only influence is through strategic effect.

\[
\frac{d\pi_B}{ds} = \frac{\partial \pi_B}{\partial s} + \frac{\partial \pi_B}{\partial q_B} \cdot \frac{dq_B^*}{ds} + \frac{\partial \pi_B}{\partial q_A} \cdot \frac{dq_A^*}{ds} = (\frac{\alpha^2}{2} n_B^* + q_B^* - c) \cdot \frac{1}{2t} \cdot \left[ - \frac{8t - \alpha^2}{2(6t - \alpha^2)} \right] < 0.
\]

The higher the value of tied good is, the less profit platform B will make. As compared to no tying case, which can be viewed as equivalent to \( s = 0 \), platform B suffers loss from tying. This outcome is similar to that in one-sided market because A charges unambiguously lower price for readers and steals readers from B. Thereby, after tying, platform B serves less consumers at a lower price on both sides. Although tying makes firm A "tough", platform B survives with a positive profit due to the fact that producing a magazine doesn’t involve fixed cost.
Note that, direct effect and strategic effect comprise the influences of tying on \( A \)’s profit:

\[
\frac{d\pi_A}{ds} = \underbrace{\frac{\partial \pi_A}{\partial s}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \pi_A}{\partial q_B} \frac{dq_B}{ds}}_{\text{Strategic Effect}} = n_A^* + n^*_A \cdot \left[ -\frac{4t - \alpha^2}{2(6t - \alpha^2)} \right] < 1.
\]

Direct effect captures the impact on CD market. Under no tying, \( A \)’s profit increases in value of CD at a rate of 1, the realized size of purchasers of CD. However, only some rather than all the readers will purchase CD after tying. Thus, \( \frac{\partial \pi_A}{\partial s} < 1 \) points out that \( A \) loses money on CD market. The effect on the magazine market is represented by strategic effect: tying forces rival to be more aggressive and then the magazine market is less profitable. When competition on magazine market is not very fierce, \( A \) could only steal some rather than many readers from \( B \). When the quantity spillover from readers to advertisers is not significant, benefit of larger market shares on both side is unable to compensate losses resulting from price-cutting. Therefore, firm \( A \) also earns less compared to independent pricing case. All these conclusions are summarized in the following proposition.

**Proposition 2** When \( t > \frac{\alpha^2}{4} \), both platforms earn less in tying regime.

Thus, in this case, since it is impossible to drive platform \( B \) out of the market, firm \( A \) would never commit to implement tying. The result is very similar to what is obtained in one-sided markets. Here, "not tying" is an accommodating strategy, which is both self-benefit and "soft".

### 4.2 Strategic Substitutes

When inter-group externality is very large compared with transportation cost, the best response functions of both firms are strategic substitutes, i.e. \( \frac{\alpha^2}{6} < t < \frac{\alpha^2}{4} \) implies \(-1 < \gamma < 0\). In this case, platform \( A \) will adopt even more aggressive strategy for making more sales of monopolized good while platform \( B \) reduces its price. If the externality across two groups is very large or the competition is very vigorous, \( B \)’s profit from advertiser side decreases dramatically in response to tying so it becomes much less costly to increase the price of advertisement. Most important, this effect offsets the tendency to decrease price caused by large competition pressure from lower \( q_A \). The equilibrium is illustrated in the following figure: the effective price of magazine \( A \) decreases whereas that of magazine \( B \) increases. In this setting, tying becomes a creditable commitment to set a low price which will force the rival increases its price.
Let’s analyze the effects of tying on magazine market. As in case 1, tying is a "tough" strategy.

\[
\frac{d\pi_B}{ds} = \frac{\partial \pi_B}{\partial q_A} \frac{dq_A^*}{ds} < 0
\]

No matter whether the prices are strategic complements or substitutes, \( A \) decreases \( q_A \) and resultantly platform \( B \)'s profit will decrease in tying regime. However, the effect on platform \( A \)'s profit may be different. When \( A \) acts more aggressively, \( B \) becomes less aggressively, i.e. \( q_B^* \) augments. The possibility of opposite reaction of two firms leads to radically different effect on \( A \)'a profit.

\[
\frac{d\pi_A}{ds} = \frac{\partial \pi_A}{\partial s} + \frac{\partial \pi_A}{\partial q_B} \frac{dq_B^*}{ds} = \left( \frac{1}{2} + \frac{s}{6t - \alpha^2} \right) \left[ 1 - \frac{4t - \alpha^2}{2(6t - \alpha^2)} \right]
\]

The effect on CD market doesn’t change: since less consumers on side 2 will purchase CD, losing money on this market is unavoidable. However, platform \( A \) will earn more money on the magazine market, which is impossible in one-sided context. The net effect on two markets is ambiguous.

Furthermore, we find that the bundling price is \( q_A^* = q_A + s = t + c - \frac{\alpha^2}{4} + \frac{4t - \alpha^2}{2(6t - \alpha^2)} s \).

A surprising result is derived: When \( \frac{\alpha^2}{6} < t < \frac{\alpha^2}{4} \), platform \( A \) will sell the bundling at a even lower price than separate selling and suffer more losses on reader side while \( B \) reduces its losses on reader side due to both smaller market share and less losses on each reader. Although \( B \) "derives" more from readers, it loses much more on advertiser side owing to the dramatical diminution in realized demand of readers. On the other hand, \( A \)'s losses on reader side will be compensated on advertiser side.

**Proposition 3** When \( \frac{\alpha^2}{6} < t < \frac{\alpha^2}{4} \), platform \( B \) will suffer losses while platform \( A \) may benefit from tying.
When \( \frac{d\pi_A}{ds} \geq 1 \), platform A will tie magazine and CD even if deterring rival is impossible. Thereby, in two-sided context, platform A may choose tying which will never occur in absence of externality. This result justifies why tying is more widespread in two-sided markets.

We will identify in next subsection, the conditions in which the extra benefits on magazine market offset the losses on CD market.

### 4.3 Tying or not?

Since production of magazine doesn’t incur fixed cost at all, it is impossible to deter rival through tying. Then the decision of whether tying or not is driven by maximizing \( \pi_A \).

Fudenburg and Tirole (1984) define the following strategies when firm A aims at accommodating rival:

<table>
<thead>
<tr>
<th>Accomodation</th>
<th>Tough</th>
<th>Soft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic complements</td>
<td>Puppy Dog</td>
<td>Fat Cat</td>
</tr>
<tr>
<td>Strategic substitutes</td>
<td>Top Dog</td>
<td>Lean and Hungry</td>
</tr>
</tbody>
</table>

In one-sided market, since prices are strategic complements and tying has a negative effect of rival’s profit, tying will unambiguously hurt itself. Obviously, firm A will act as "Puppy Dog": Tying will never be adopted.

From the previous subsection, we know that when \( t > \frac{a^2}{4} \), firm A is still "Puppy Dog". However, when \( \frac{a^2}{6} < t < \frac{a^2}{4} \), price becomes strategic substitute and then A may benefit from tying in magazine market. Then, in two-sided markets, firm A may be "Top Dog". Although it continues to earn less on CD market relative to separate selling, it may make more money on magazine market, in particular, on advertiser side. The gain of firm A by tying two independent goods is:

\[
\Delta \pi_A = \frac{s}{4(6t - \alpha^2)^2} [8 - \frac{6t - \alpha^2}{8t - \alpha^2}(16t - 3\alpha^2)].
\]

Firm A will choose tying if and only if extra profit on magazine market can offset losses on CD market. Precisely, \( \Delta \pi_A > 0 \) when \( s > \hat{s} \equiv (16t - 3\alpha^2)\frac{6t - \alpha^2}{8t - \alpha^2} \).

**Proposition 4**

1. When \( \frac{a^2}{6} < t \leq \frac{3a^2}{16} \), platform A will choose tying for all \( s > 0 \).
2. When \( t \in (\frac{3a^2}{16}, \frac{5a^2}{24}) \), firm A will choose tying if the value of CD is large enough \( s > \hat{s} \).
3. When \( t \geq \frac{5a^2}{24} \), tying is never adopted.

**Proof.** Since the market is covered and reader side is not allowed to expand in this model, \( n_A^* = \frac{1}{2} + \frac{a}{6t - \alpha^2} \leq 1 \). If firm A sells the magazine with a CD of value \( s \geq \hat{s} = \frac{6t - \alpha^2}{2} \),
it holds the entire magazine market. In other words, all the tying goods of value $s > \bar{s}$ have the same strategic effect on magazine market. If tying is not a profitable strategy as $s = \bar{s}$, the tying firm will earn the same profit on magazine market by tying a good of $s > \bar{s}$. (In the following figure, we could see that as $s > \bar{s}$, A’s profit increases in $s$ at rate equal to 1, which represents the profit on CD market increases in $s$ at rate equal to 1 while that on magazine market is constant as $s \geq \bar{s}$.)

Therefore, $\bar{s}$ and $\bar{s}$ are two critical points. If and only if $\bar{s} < \bar{s}$ and $s \geq \bar{s}$, tying is a self-benefiting strategy. Straightforward, $\bar{s} < \bar{s}$ if and only if $t < \frac{5a^2}{24}$.

- When $\frac{3a^2}{16} < t \leq \frac{5a^2}{16}$, $\bar{s} < 0$. $\Delta \pi_A > 0$ as long as $s > 0 > \bar{s}$.
- When $\frac{3a^2}{16} < t < \frac{a^2}{4}$, $\bar{s} > 0$. $\Delta \pi_A > 0$ if $s > \bar{s} > 0$.
- When $t \geq \frac{5a^2}{24}$, tying is definitely self-harming.

In the figure, we could find A’s profit curves respectively in three cases which we mentioned in proposition.

In case of $\frac{1}{\alpha^2} \in (\frac{1}{5}, \frac{3}{16})$, strategy of tying dominates: A always sells magazine and CD together. Large externality or small transport cost implies that a little advantage on reader side will lead to dramatical change of market share and then induce more profit on advertiser side. The additional benefit on advertiser side is large enough to compensate the losses on CD market and reader side. When $\frac{1}{\alpha^2} \in (\frac{3}{16}, \frac{1}{4})$, there is a tradeoff between losses on CD market and extra benefit on magazine market. Larger $s$ implies that less consumers give up purchasing CD. Then, the loss on CD market is easier to compensate. Platform A can finds a critical point $\bar{s}$, above which it should practice tying. However $s > \bar{s}$ could be infeasible due to the fixed market size.
5 Welfare Analysis

In this section, we compare the market outcomes with tying and without trying to provide a welfare analysis.

Without tying, each magazine holds half of the market. The social welfare of magazine market is composed of advertisers’ surplus, readers’ surplus and two platforms’ profits.

\[
W_M = \int_0^{n_A^*} (an_A^* - x) dx + \int_0^{n_B^*} [an_B^* - x] dx
\]

\[+
\int_0^{n_A^*} (v - tx - d) dx + \int_{1-n_B^*}^1 [v - t(1 - x) - c] dx
\]

\[=
v - c - \frac{t}{4} + \frac{3}{16} \alpha^2
\]

where \(W_i(i = 1, 2)\) is social welfare of side \(i\). Social welfare on CD market is \(W_{CD} = s\). The total social welfare is equal to \(W = v - c - \frac{t}{4} + \frac{3}{16} \alpha^2 + s\).

The total social welfare change due to tying can be expressed as:

\[
\Delta W = \Delta W_1 + \Delta W_2 + (n_A^* - 1) s
\]

\[\alpha^2 s^2 \left( \frac{3 \alpha^2 s^2}{(6t - \alpha^2)^2} \right) + \frac{s^2}{(6t - \alpha^2)^2} + (n_A^* - 1) s
\]

\[= s \cdot \frac{(20t - \alpha^2)^2}{(6t - \alpha^2)^2} [s - \frac{2(6t - \alpha^2)^2}{20t - \alpha^2}]
\]

Tying is welfare-enhancing if \(s \geq \hat{s} = \frac{2(6t - \alpha^2)^2}{20t - \alpha^2}\). There are three channels through which tying can affect social welfare. First, since there is no market expansion effect on single-homing side, the reader side worsens off due to the increasing transportation costs. Second, less readers will buy CD. These two negative effects can explain why tying is always welfare-reducing in one-sided market. However, there is the third effect in two-sided market. Tying induces larger network effects: One big network plus one small network are better than two medium ones.\(^6\) Positive effect on advertiser side and negative effect on reader side conflict. The net effects of tying on social welfare depends on three features: externality \(\alpha\), transportation cost \(t\) and value of tying good \(s\).

**Proposition 5** When \(t \geq \frac{1}{4} \alpha^2\), tying never improves social welfare. When \(\frac{1}{8} \alpha^2 < t < \frac{1}{4} \alpha^2\), tying is welfare-enhancing if and only if \(s \geq \hat{s} = \frac{2(6t - \alpha^2)^2}{20t - \alpha^2}\).\(^6\)

\[^6\frac{1}{2}(a^2 + b^2) \geq (\frac{a+b}{2})^2.\]
We find that, tying is welfare-enhancing if and only if the value of tied good is large enough. Since it is impossible to expand the market further when $n_A^* = 1$, the welfare outcome of tying a CD of value $s = \bar{s}$ is the same as with a CD of $s > \bar{s}$. Therefore, it is sufficient to compare $\hat{s}$ and $\bar{s}$.

As $t \geq \frac{1}{4} \alpha^2$, $\hat{s} \geq \bar{s}$. Tying is never welfare-enhancing. As transportation cost is very large relative to externality, the loss on reader side is unable to be covered even though $n_A = 1$. However, as $t \in (\frac{1}{6} \alpha^2, \frac{1}{4} \alpha^2)$, $\hat{s} < \bar{s}$. The intuition behind the second result is that the positive network effect will offset the negative effect if $s$ is large enough and the resultant $n_A$ is large enough. We can conclude that, in some circumstances, tying is desirable in two-sided market from view of social planner while it is always an welfare-harming action in one-sided context.

Recall that platform $A$ will tie magazine and CD only if $\frac{1}{6} \alpha^2 < t \leq \frac{5}{24} \alpha^2$. We will try to figure out the set in which platform $A$ will adopt a welfare-enhancing tying.

![Graph showing the two-dimensional figure of (s, t)](image)

We could conclude all these results in a two-dimension figure of $(s, \frac{1}{t^2})$. In the figure, the gray part represents the regime in which platform $A$ will tie two goods and tying is socially desirable. In the black part, tying is welfare-harming but profitable for platform $A$.

6 Extentions

6.1 Ad-lover Readers or Ad-hater Readers

We extend the basic model to a more general one: Advertisements also generate externalities on readers. The externalities may be positive or negative correspond to advertisement-lover or advertisement-hater readers. Furthermore, by extending the model, it can apply to most of the two-sided industries rather than some special industries which are characterized by one-direction externality.
As we got in section 3, when $\beta = 0$, prices are strategic substitutes if $t \in \left[\frac{\alpha^2}{6}, \frac{\alpha^2}{4}\right]$. In the figure above, we can see that there will be a set corresponding to different value of $\beta$ in which prices are strategic substitutes. When $\beta$ is too large or too small, we get a vide set. As $\beta$ is quite large, competitors are able to use strategy substitute but a stable equilibrium is not feasible.

**Proposition 6** When $\beta \in \left[-\frac{\alpha}{3}, \alpha\right]$, price competition is strategic complement as long as

$$t \in \left[\frac{1}{6}(\alpha^2 + 4\alpha \beta + \beta^2), \frac{1}{4}(\alpha^2 + 3\alpha \beta)\right].$$

We could prove that the main results in our model still hold as long as $\alpha > 0$, $\beta \in \left[-\frac{1}{3}\alpha, \alpha\right]$.

As $t \in \left[\frac{1}{6}(\alpha^2 + 4\alpha \beta + \beta^2), \frac{1}{4}(\alpha^2 + 3\alpha \beta)\right]$, in the magazine market, prices are strategic substitute. If firm A adopts tying, the best-response system will be:

$$\begin{align*}
R_A(q_B) &= \frac{\Omega q_B + F}{\Gamma} - \frac{16t^2 - 4t(5\alpha \beta + \beta^2) + \alpha \beta^2(5\alpha + 3\beta)}{\Gamma} s, \\
R_B(q_A) &= \frac{\Omega q_A + F}{\Gamma} + \frac{-\beta(4t - \alpha^2 - 3\alpha \beta)}{\Gamma} s.
\end{align*}$$

Notably, in presence of two-direction externalities, a commitment to tying not only affects its own reaction function but also that of rival.

In the same manner as in previous section, we could figure out the conditions in which platform A will pratice tying to maximize profit.

**Proposition 7** If magazine prices are strategic substitutes,

(1) When $t \in \left[\frac{1}{6}(\alpha^2 + 4\alpha \beta + \beta^2), \frac{1}{16}(3\alpha^2 + 10\alpha \beta + 3\beta^2)\right]$, firm A will choose tying for all $s > 0$. 

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When \( t \in \left[ \frac{1}{16}(3\alpha^2 + 10\alpha\beta + 3\beta^2), \frac{1}{21}(5\alpha^2 + 14\alpha\beta + 5\beta^2) \right] \), tying is implemented if \( s \geq \hat{s} \equiv \left[ 16t - (3\alpha^2 + 10\alpha\beta + 3\beta^2) \right] \frac{6t - (\alpha^2 + 4\alpha\beta + \beta^2)}{\left(6t - (\alpha^2 + 4\alpha\beta + \beta^2)\right)}.

What we obtain in the main part is a special case of these results.

### 6.2 Prisoner’s Dilemma

In this section, suppose platform \( B \) monopolizes a DVD market. For simplicity, we assume that DVD has the same value \( s \) as CD. We give a new two-stage game: In stage one, platform \( A \) and \( B \) decides to tie or not simultaneously. The decisions are common knowledge. Price competition will happen in stage two.

First, let’s look at the Nash-equilibrium in absence of externality.

<table>
<thead>
<tr>
<th>Platform B</th>
<th>Platform A</th>
<th>no tying</th>
<th>tying</th>
</tr>
</thead>
<tbody>
<tr>
<td>no tying</td>
<td>( \frac{\alpha}{2} + \frac{s}{3} )</td>
<td>( \frac{\alpha}{2} + \frac{s}{3} + \frac{2s}{15} )</td>
<td></td>
</tr>
<tr>
<td>tying</td>
<td>( \frac{\alpha}{2} + \frac{2s}{3} - \frac{s'}{15} )</td>
<td>( \frac{\alpha}{2} + \frac{s}{3} + \frac{2s}{15} - \frac{s'}{15} )</td>
<td></td>
</tr>
</tbody>
</table>

Given the rival chooses "no tying", no one will choose "tying" because "tying" decreases its own profit. Provided the rival ties the two goods, "no tying" is still a profitable strategy.\(^7\) Therefore, "no tying" is a dominant strategy. Clearly, (no tying, no tying) is a Nash-equilibrium.

Now we move to the case in presence of externality. We will skip the case in which firms act as in one-sided market. The most interesting case is that given rival chooses no tying, firm \( i \) will choose tying as long as \( s \geq 0 \), which will occur when \( \frac{\alpha^2}{6} < t < \frac{3}{16} \alpha^2 \).

\[
\begin{array}{c|c|c}
\alpha > 0 & \text{no tying} & \text{tying} \\
\hline
\text{no tying} & \frac{\alpha}{2} - \frac{\alpha^2}{16} + s, \quad \frac{\alpha}{2} - \frac{\alpha^2}{16} + s \quad \varphi(\chi + 2s)^2 \varphi(\chi - 2s)^2 + s, \\
\hline
\text{tying} & \varphi(\chi - 2s)^2 + s \varphi(\chi + 2s)^2 \quad \frac{\alpha}{2} - \frac{\alpha^2}{16} \quad \frac{\alpha}{2} - \frac{\alpha^2}{16} \\
\end{array}
\]

where \( \varphi = \frac{8t - \alpha^2}{16(6t - \alpha^2)} \) and \( \chi = 6t - \alpha^2 \).

In this case, provided that platform \( i \) sells two goods as a bundling, the best response of platform \( j \) is "not tying" if the value of tying good satisfies \( s > - (16t - 3\alpha^2) \frac{6t - \alpha^2}{8t - \alpha^2} \). Therefore, we could derive two Nash-equilibriums: (tying, not tying) and (not tying, tying). By contrast, when the externality is large and the value of tying good is not very large, "tying" is a dominant strategy.

\(^7\)Since \( n_A \leq 1, s \leq 3t \). Then \( \frac{\alpha}{2} + \frac{2s}{3} - \frac{s'}{15} > \frac{\alpha}{2} \).
Proposition 8 When $\frac{a^2}{6} < t < \frac{3}{16} \alpha^2$ and $s < -(16t - 3\alpha^2) \frac{6t - \alpha^2}{6t - \alpha^2}$, (tying, tying) will be a Nash-equilibrium.

If the benefit from DVD market is not very large, platform $B$ would like to sacrifice the monopolized market and involved in "prisoner’s dilemma" war of magazine market.

7 Conclusion

Traditional analysis of tying has focused on conventional markets. In such markets a general insight is that the firm can harm the rival by tying two independent goods, which will also reduce its own profit. Our analysis has shown that this is challenged in a two-sided market. We construct a simple model of two-sided markets, in which two magazines competes for readers as in the standard Hotelling model and, on the other hand, they serve the advertisers whose demand relies positively on the size of readers. When the externality generated by the readers on the advertisers is large enough, the prices set by the duopoly are strategic substitutes and then tying could be self-benefiting. As a result, tying will be adopted whether the firm aims at accommodating or deterring rival.

Our analysis then proceeds to examine the effects of tying on social welfare. Contrary to the conventional wisdom, tying could be welfare-enhancing in two-sided markets. It is due to the fact that, in presence of the network effect, optimal allocation of consumers on both sides should be asymmetric. In tying regime, the inefficiency in the market of magazine is mitigated. This result has important implications for competition policy in two-sided markets.

Our study has been started by a setting where only readers generate externalities on advertisers. Then we check the robustness of results in presence of two-direction externalities and the main results in this paper survive in more general setting. By doing this, we avoid incorporating any of the particularities of the media market into the model and therefore highlight the most common mechanisms of tying in two-sided markets. In the end, we extend the analysis by allowing both firms to tie the magazine with a monopolized good and argue that they may be involved in “prisoner’s dilemma”.

In our model, we assumed that the value of monopolized good is exogenous and we find that it plays an important role when the platform decides to tie or not. Interesting future work might relax this assumption and analyze a model in which the platform should determine the value (or the quality) of the monopolized goods preceding the game in the basic model.
References


