Consumption Smoothing and Portfolio Rebalancing:
The effects of adjustment costs*

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Abstract

This paper studies the dynamics of portfolio rebalancing and consumption smoothing in the presence of non-convex portfolio adjustment costs. The goal is to understand the household response to income and return shocks. The model includes the choice of two assets: one riskless without adjustment costs and a second risky asset with adjustment costs. With these multiple assets, a household can buffer some income fluctuations through bond holdings and engage in costly portfolio rebalancing less frequently. We estimate both preference parameters and the adjustment costs. The estimates are used for evaluating consumption smoothing and portfolio adjustment in the face of income and return shocks.

1 Introduction

This research studies household saving and portfolio choice in the presence of portfolio adjustment costs. We ask: how does the household response to income and return shocks when portfolio adjustment is costly?

The household optimization we study is enriched by the presence of multiple assets. This allows us to look at consumption smoothing and portfolio rebalancing separately. That is, the household could rebalance its portfolio, holding consumption fixed, or adjust consumption through a wide variety of asset trades. The costs of asset trading along with the preference parameters have not been estimated in a setting with multiple assets. These parameters are important for understanding the response of households to shocks to income and asset returns.

The estimation of preference parameters is traditionally based on a Euler equation approach. Results from this approach, such as Hall (1988), conclude that the household response to interest rate movement is near zero.

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However in the presence of adjustment costs, the standard Euler equation linking the marginal utility of consumption across periods does not always hold. We show that adjustment costs significantly change the estimates of preference parameters.

In terms of responses to income shocks, our model has the distinctive features of the buffer-stock saving model (Deaton (1991), Carroll (1992), Carroll (1997)). Households are impatient and accumulate wealth out of pre-cautionary motives. Consumption changes in response to income shocks, but is less volatile than income; while liquid wealth exhibit high volatility. However, in the traditional buffer-stock saving model, there exists only one asset – a risk-free bond. This makes it impossible for the traditional model to study the impacts of income shock on consumption smoothing and asset rebalancing. The extension to multiple asset model turns out to be difficult, as demonstrated in Heaton and Lucas (1997), because the return to risky asset is so high that the calibrated model generates no bondholding.

We consider the implications of an opportunity cost of asset trading and model it as a fraction of labor income. With this specification, trades of stocks and bonds will have an adverse effect on labor income. We estimate this opportunity cost, along with two other key parameters – coefficient of relative risk aversion and rate of time preference – via simulated method of moments.

Simulation of the model produces a number of interesting results. We do generate a positive demand for bonds. In an otherwise similar model without opportunity cost of stock trading, Heaton and Lucas (1997) find it extremely difficult to explain the observed bond holding in the data.

Households hold bond for two purposes, both are related to the infrequent adjustment of stock account due to the adjustment cost. First, when a household refrains from stock adjustment, bondholding is adjusted to accommodate changes to the state variables of the household. Secondly, households make large scale changes to their stock account once adjustment occurs. In this case, bond holdings are used to complement the stock adjustment. As a result, when a household reduces its stockholding, bondholding is often increased.

We also find that the response to shocks is nonlinear. For small income shocks, the household will respond with variations in bond holdings, without incurring a costly stock adjustment. But, for large enough income shocks, stock holdings and bond holdings are jointly adjusted. In response to return shocks, the portfolio rebalancing effect is evident.

2 Model

We study the intertemporal consumption and portfolio choice of a household. The household’s portfolio consists of two assets. The first is an asset which is riskless and has relatively low trading costs, such as money and bonds. The second asset is stocks which have a higher return on average, are riskier and more costly to trade. Throughout we refer to the first asset as bonds and the second as stocks.

\footnote{The model extends the household optimization problem studied in Bonaparte and Cooper (2010) by considering multiple assets.}
The agent faces both income risk and variations in the return on stocks. Thus the household has a desire to both smooth consumption and adjust its portfolio in response to shocks.

The key aspect of the model comes from the costs of adjusting the household’s portfolio. If we assume that trades in bond are costless, then the agent can smooth consumption through this asset. But in some states, the agents will choose to adjust its holdings of stocks as well. This will generally lead to both portfolio adjustment as well as a change in the household consumption.

We look at an infinite horizon stochastic optimization problem. Let $v(y, A_{-1}, R_{-1})$ be the value of the household’s problem. The state vector includes current household income $y$, the existing portfolio of $A_{-1}$ and the return vector from the previous period was $R_{-1}$. Here $A_{-1} = (A_{b_{-1}}, A_{s_{-1}})$ is the vector of asset holdings from the previous period, where $A_{b_{-1}}$ and $A_{s_{-1}}$ are the holding of bond and stock respectively. The return vector for $R_{-1} = (R_{b_{-1}}, R_{s_{-1}})$ provides information about returns over the next period. Total financial wealth equals $\sum_{i=b,s} R_{i_{-1}} A_{i_{-1}}$.

The value of the household’s problem is given by the maximum over the options of adjusting, $v^a(y, A_{-1}, R_{-1})$, or not, $v^n(y, A_{-1}, R_{-1})$, is

$$v(y, A_{-1}, R_{-1}) = \max \{v^a(y, A_{-1}, R_{-1}), v^n(y, A_{-1}, R_{-1})\}$$

for all $(y, A_{-1}, R_{-1})$. Here adjustment refers to variations in the holdings of stock since adjustment of bondholding is not costly.

If the household adjusts its holdings of both assets, the value $v^a(y, A_{-1}, R_{-1})$ is given by

$$v^a(y, A_{-1}, R_{-1}) = \max_{A_{b_{-1}} \geq A_{b_{-1}}, A_{s_{-1}} \geq 0} u(c) + \beta E_{R, y'|R_{-1}, y} v(y', A, R).$$

The consumption level is given by

$$c = \sum_{i=b,s} R_{i_{-1}} A_{i_{-1}} + y \times \psi - \sum_{i=b,s} A_i - C(s_{-1}, s).$$

Here $A_i$ is the purchase of shares of asset $i$. There are two costs of adjustment in this problem. The first, represented by the function $C(\cdot)$, captures the direct cost of portfolio adjustment through payments to an intermediary. The second cost of adjustment, parameterized by $\psi$ in (3), is meant to capture the time cost of gathering information and making an optimal choice. This cost occurs whenever a household make additions or reductions to stock accounts. We assume the reinvestment of dividends are automatic and do not incur any cost.

If there is no adjustment of stock holdings, the value of the problem is:

$$v^n(y, A_{-1}, R_{-1}) = \max_{A_{b_{-1}} \geq A_{b_{-1}}} u(c) + \beta E_{R, y'|R_{-1}, y} v(y', A, R).$$

To the extent that portfolio shares and asset market participation have life-cycle patterns, a life-cycle considerations may be of additional interest.

Thus we study trading costs not as in the related literature on rational inattention, eg Abel, Eberly, and Panageas (2007) and Alvarez, Guiso, and Lippi (2010), costs of becoming informed about the current state.
where

\[ c = R_{t-1}^b A^b_{t-1} + y - A^b. \] (5)

When there is no adjustment of the risky asset, the household optimally adjusts its bondholding. This is seen in (5), where consumption includes the interest earnings on bond as well as the purchases of bond. The evolution of shares reflects the adjustment of bond as well as no adjustment in portfolio holdings. Since we assume that dividends on stock are reinvested without the payment of any adjustment cost, \( A = (A^b, A^s \times R^s_{t-1}) \) when there is no adjustment, as in (4).

Notice that the choice of next period’s bondholding is bounded by \( A^b \) while the holdings of stocks must be non-negative. \(^4\) If \( A^b < 0 \), the household is able to borrow. We allow for borrowing in the model and discuss the estimation of \( A^b \) below.

This problem includes both consumption smoothing and portfolio adjustment. It allows the household to partially smooth consumption through adjustment in their holdings of bonds. In addition, households can adjust their portfolio composition at a cost. We return to a discussion of the properties of these policy functions after estimating our model.

### 3 Exogenous Processes and Financial Trading Costs

The Appendix provides a detailed discussion of the data used in our study. We take from the data the stochastic processes for income and returns as well as the moments used for our estimation. In addition, we estimate direct trading costs for stocks.

**Income and Returns** The income process is estimated directly from the Panel Study of Income Dynamics (PSID). The persistence of income shocks is estimated at 0.842 with a standard deviation of the innovation to an AR(1) process of 0.29.

The stock return comes from Shiller\(^5\)\(^\text{http://www.econ.yale.edu/~shiller/data.htm}\). This return includes both dividends and capital gains. The estimated serial correlation of annual return not significantly different from zero. This parameterization of stock return process implies an average annual return of 6.33\% with a standard deviation of 15.5\%. The non-stochastic return on bonds was set at 1.02.

**Trading Costs** We allow the direct costs of trading stocks vary depending on whether stock is bought or sold.

\[ C^b(A^s_{t-1}, A^s) = \nu_0^b + \nu_1^b (A^s - A^s_{t-1}) + \nu_2^b (A^s - A^s_{t-1})^2 \] (6)

if the household buys stock so that \( A^s > A^s_{t-1} \). If instead the household sells stock, then

\[ C^s(A^s_{t-1}, A^s) = \nu_0^s + \nu_1^s (A^s_{t-1} - A^s) + \nu_2^s (A^s - A^s_{t-1})^2. \] (7)

\(^4\)As we do not have durable goods in the model, collateralized borrowing is not considered.
We use the estimates for these parameters provided by Bonaparte and Cooper (2010) who use monthly household account data to estimate these transactions costs for the buying and selling of common stocks. Their estimates are summarized in Table 1. As noted by Bonaparte and Cooper (2010), only the fixed costs of trade are economically significant. Moreover, as these costs were estimated from the 1991-96 time period which preceded the PSID data, the actual fixed costs of traders may be less than that estimated here. We return to that point below.

Table 1: Estimated Trading Costs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Buying</th>
<th>Selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant $\nu_0^i$</td>
<td>56.10</td>
<td>61.44</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>Linear $\nu_1^i$</td>
<td>0.0012</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(1.63e-06)</td>
<td>(1.93e-06)</td>
</tr>
<tr>
<td>Quadratic $\nu_2^i$</td>
<td>$-1.01e^{-10}$</td>
<td>$-1.28e^{-10}$</td>
</tr>
<tr>
<td></td>
<td>(2.88e-13)</td>
<td>(9.26e-13)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.251</td>
<td>0.359</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>1,746,403</td>
<td>1,329,394</td>
</tr>
</tbody>
</table>

4 Estimation

We estimate the preference parameters, ($\gamma, \beta$), along with the trading cost, $\psi$, using simulated method of moments. We also use the estimate trading costs from Table 1 as well as the stochastic processes for income and returns in this estimation. Finally, we discuss the procedure used to determine the borrowing limit, $A^b$.

4.1 Moments

We match four moments. The data appendix provides detailed information about the sources and computation of these moments. These moments are for stockholders only which is consistent with our model.

The first moment is the adjustment rate for shareholders of 47% biannually. This moment is particularly informative about the opportunity of stock trading, one of the key elements in our model. We calculate it from five waves of the PSID survey, 1999-2007. The survey asked stockholders whether they bought or sold any non-IRA stocks since the previous interview. Within our sample, the percentage of stockholders that had positive answers ranged between 53% and 68% over the sample. This represents the gross adjustment rate. For these gross-adjustors, PSID further asked whether they put money into stocks, or take money out of them, or put about as much in as they took out. About 16.2% of the gross-adjustors reportedly put about as much in as they took out. In net, 47% of the stockholders either put money into stock or take money out of them from 1999-2007. This net adjustment is consistent with the definition of stock adjustment in our
4.1 Moments

Bonaparte and Cooper (2010) calculate an adjustment rate of 71% from Survey of Consumer Finance. The survey asks stockholders who own a brokerage account how many times they bought or sold stock over the past year. In Bonaparte and Cooper (2010), a stockholder is said to make stock adjustment if she bought/sold stocks at least once. Therefore the 71% is annual gross adjustment rate. It is higher than the 47% biannual gross adjustment rate in PSID. It should be noted that 47% biannual adjustment rate is not to be converted into 23.5% annual adjustment rate, unless totally different households did stock adjustment in year two relative to year one. On the other hand, if the same set of households who make adjustment in year one also did so in year two, then the annual adjustment rate from PSID should also be 47%. Notice that SCF adjustment rate is for stockholders that own a brokerage account, while PSID adjustment is for all non-IRA stock holders. A higher adjustment rate in SCF implies that stockholders that own a brokerage account are more active traders.

The second moment is the portfolio composition of shareholders. We interpret bonds as the sum of deposits in transaction account and CDS, directly held bonds and saving bonds in SCF. The data counterpart of stocks in the model is the amount of money invested in stock (directly and indirectly) reported by SCF stockholders.

The fraction of stocks relative to the sum of stocks and bonds is 69%. This moment contains information on households’ degree of risk aversion ($\gamma$) and the opportunity cost of stock trading ($\psi$). The existing literature has found that an unrealistically high coefficient of relative risk aversion is required in order to induce households to hold bond due to high equity premium. We show that a modest $\gamma$ coupled with a small $\psi$ induces sizable bondholding, bringing the portfolio composition in the model closer to that in the data.

For the third moment, we follow the literature on intertemporal consumption and study the response of consumption growth to interest rate movements. This moment, termed the “EIS”, is computed from the consumption of shareholders, as in Vissing-Jorgensen (2002), with a point estimate of 0.2984.

The fourth moment is the median of the wealth to income ratio and is 2.127 in the data. This moment is important for estimating the discount factor $\beta$ and relative risk aversion $\gamma$. Intuitively a high $\beta$ is associated with high wealth-income ratio because households value future consumption. Similarly a high $\gamma$ leads to high buffer-stock saving, hence high wealth-income ratio.

To estimate the parameters, we solve

$$\min_{(\gamma,\beta,\psi)} (M^s - M^d)'W(M^s - M^d).$$

(8)

Here $M^d$ are the moments from the data and $M^s$ are the simulated values of those moments for a given set of parameters, $(\gamma,\beta,\psi)$. The weighting matrix, $W$, is the inverse of a matrix with the variances of the moments along the diagonal. Since our moments come from different data sources, we are unable to calculate

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5The stock in our model should be interpreted as the composite of individual stocks, such as S&P500.
covariances directly.

The simulated moments are obtained by solving the household’s dynamic optimization problem through value function iteration. The solution method is described in the Appendix. Through simulation, a panel with 500 households and 800 time periods is created. The simulated moments are calculated from this panel in exactly the same manner as they are calculated in the actual data. In particular, the adjustment rate is calculated bi-annually in the simulated data as it is in the PSID.

Throughout this analysis, we focus on financial wealth, ignoring housing and other durables. Further, the portfolio share relates to financial assets alone. The consumption measure includes the service flow from housing. This is consistent with a model in which housing is rented by households who hold a portfolio of financial assets. This is also the outcome of a model in which the housing stock is held by households and is costlessly adjusted.

4.2 Results

The results are presented in Table 2. The first row in the table summarizes the data moments. The second row contains our results.

These results are for the case of $A^b = 0$ so that households are unable to borrow on their bond accounts. The restriction of $A^b = 0$ comes from estimating this lower bound along with the other parameters of the model. In the end, the best fit was with the tightest borrowing constraint of $A^b = 0$.

The simulated moments are all reasonably close to the actual data moments. The adjustment rate is met almost exactly. The other two moments are close as well. The weighting matrix gave most weight to the stock share moment but the model did not match that as well as the other moments.

In contrast to Heaton and Lucas (1997), our model generates positive demand for low return riskless bond. With reasonable parameter values, the share of bond in total financial wealth is about 26%. The major reason for the increased demand for bonds is the opportunity cost of stock trading.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Adj. Rate</th>
<th>Stock Share</th>
<th>EIS</th>
<th>Wealth/Income Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.467</td>
<td>0.6902</td>
<td>0.2984</td>
<td>2.127</td>
</tr>
<tr>
<td>std.</td>
<td>(0.0693)</td>
<td>(0.1200)</td>
<td>(0.0906)</td>
<td>(0.2557)</td>
</tr>
<tr>
<td>Est. Model</td>
<td>0.466</td>
<td>0.739</td>
<td>0.223</td>
<td>2.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Beta</td>
<td>0.575</td>
<td>0.625</td>
<td>0.294</td>
<td>14.245</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt. Income</td>
<td>0.395</td>
<td>0.848</td>
<td>0.109</td>
<td>0.417</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The parameter estimates indicate that a low discount factor of 0.7048 is necessary to match these moments. As noted earlier, the median ratio of wealth to income is particularly sensitive to the discount
factor.

The other two parameter estimates indicate a curvature estimate of around 6, which is about half of the inverse of the estimated EIS from the data. The point estimate of the adjustment costs is $\psi = 0.988$. With average income of about $72,000 in the data used to estimate the trading costs reported in Table 1, this part of the adjustment cost is about $850. Combined with the estimated fixed cost of financial trades, the combined fixed cost is around $905.

4.3 Robustness

The bottom two rows of Table 2 shows the results from two experiments. In the first one, $\beta$ is increased to a more traditional level of 0.90. As shown in the table, this generates a wealth/income ratio much higher than in the data. While the other moments are also affected by this large $\beta$, it is clear that matching the data wealth/income ratio requires more discounting than usual.

A second experiment replaces the income process estimated in the PSID with that assumed by Heaton and Lucas (1997). The point here is to see whether the income process assumed in that paper, which is less risky than the one we use, can still generate bond holdings. This is indeed the case through the stock share is much larger than in our baseline.

5 Responding to Shocks

Given these estimates, we turn to a main point of our study: how do households respond to income and return shocks? This is a traditional question of the consumption literature. Our answer differs because of the presence of portfolio adjustment costs.

We first present and discuss the policy functions of our estimated model. We then turn to simulation results to display the properties of these policy functions.

5.1 Policy Functions

Figure 1 shows the decisions rules (policy functions) for future stock and bond holdings as a function of current stockholding. Here, the measure of current stockholdings includes the realized return on stocks: in the notation of the model this is $A_{i}^{t-1} R_{i}^{t-1}$. The decision rules are shown for three different realizations of income. Current period’s bondholding is fixed at the average of the simulated panel (about 30% of mean income) and the return state is at its mean of 6.33%.

The decision rules are plotted together with the 45 degree line. Given the inaction in stock adjustment created by the non-convex adjustment costs, future stockholding is the same as current stock holding, indicated by decision rules which coincide with the 45 degree line.

The upper panel is for a household whose income realization is at low level, one standard deviation below the mean. The middle panel is for a household whose income realization is at the mean. The lower panel is
This figure shows the policy functions for three different income states. Current stock holdings (including current returns) are shown along the horizontal axis. Future stock and bond holdings are indicated along the vertical axis.
for a household whose realized income is one standard deviation above the mean.

As indicated in these policy functions, there is a sizable inaction region for stock adjustment for all three income states. It is only in the tails of the current stockholding that adjustment in this margin occurs. Over these regions of inaction in stock adjustment, the bond holdings are a decreasing function of the stock holdings. As the value of the stock account increases, households will consume at a higher level. This is financed by lower bond holdings.

When adjustment of the stock account does occur, the bond holdings move in the opposite direction. So, for example, high income households with very large stock holdings will reduce their stock accounts and increase bond holdings in order to finance higher consumption.

As indicated in Figure 1, the bond policy function is very non-linear even though the adjustment costs only apply to stocks. Of course, once the adjustment cost is paid for stocks, then the choice of bond holdings is quite different from the states of no adjustment in stocks.

5.2 Simulation Results

To illustrate the choice problem of agents, we study some simulation results. This enables us to see the response to households to income and interest rate shocks. These responses indicate how households use their portfolios to smooth consumption as well as portfolio rebalancing in response to income and return variations.

The figures which summarize the simulations show stockholding and bondholding as controls over time. Further, the stockholding series has diamonds to indicate periods of inaction.

5.2.1 Response to income shocks

Given the presence of the fixed adjustment costs, the response to income shocks is nonlinear. Accordingly we distinguish the response of the household to relatively small income shocks, Figure 2, from the response to larger income shocks, Figure 3.

As indicated in the top left panel of Figure 2, a household’s bond holdings increase with an income shock. From the bottom left panel, there is also an increase in consumption at the time of the shock. Since the income shock is relatively small, stock holding does not instantly change, as indicated by the top right panel. Instead, the stockholding builds through the reinvestment of dividends without any adjustment (as indicated by the diamond). After a few periods, the stock account has grown enough to warrant an adjustment. At the time of adjustment (periods 5, 10, etc.), the stock account falls and the bond account grows, as shown in the bottom right panel.

Thus these figures combine two effects. The first is the response to the temporary income shock. The

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6 This type of spillover is found in other optimization problems in which a non-convex adjustment cost applies to only a subset of the choice. For example, in a dynamic capital adjustment problem with no adjustment costs of labor, the nonlinearity in the capital decision rule will induce a nonlinearity in the labor decision rule.
The figure plots the dynamics of stockholding, bondholding and consumption in response to a small income shock. The stock return is fixed at the average level (6.33% per year). Diamonds mark inaction of stock account.

Figure 2: Small income shock.

second is the ongoing interaction between the stock and bond accounts. Due to the fixed cost, there is a natural dynamic of infrequent adjustment from the stock account to the bond account (portfolio rebalancing) along with the financing of consumption from the bond account (consumption smoothing). These dynamics are akin to those found in the old literature on money demand associated with Baumol and Tobin.

The response to a larger income shock is somewhat different. Looking at the top right panel of Figure 3, the larger income shock induces an immediate response of stock adjustment. The increase in the stock account implies that the bond account does not increase proportionately as much with the larger income shock, than it did with the smaller income shock. This is seen by the top right panel. Consumption responds to the income shock as before. Once the response to the income shock is over, the usual dynamic of portfolio rebalancing occurs again, as shown in the bottom right panel.

5.2.2 Response to Return Shocks

We now study the response of households to return shocks. This is illustrated in figure 4. For this simulation, income is held at its mean and initial stockholding and bondholding are both the averages of the simulated path of 500 households for 800 periods.

Households adjust their stockholding in response to the movements in stock returns. The adjustment
The figure plots the dynamics of stockholding, bondholding and consumption in response to a large income shock. The stock return is fixed at the average level (6.33% per year). Diamonds mark inaction of stock account.

Figure 3: Large income shock and asset rebalance

is basically a shift of wealth between stocks and bonds, while consumption is relatively unaffected. This response shows both consumption smoothing and portfolio rebalancing at work.

The initial high return shock in the figure leads to an immediate response in stockholding (there is no diamond in period 1 of the bottom right panel). The stock account is reduced and the bond account is increased. The consumption increase is financed out of the bond account. The stock account continues to build until the return falls and the bond holdings are replenished around period 10. Overall, stockholding is positively correlated with stock return while bondholding is negatively correlated with stock return. Further there is a negative correlation of -0.103 between stocks and bonds at the individual level. This is the portfolio rebalancing effect.

Figure 5 shows the aggregate implications of this experiment. There are two points to note. First, looking at the bottom right panel, at the aggregate level the correlation between stocks and bonds is clearly positive. In fact, it equals 0.94 in contrast to the negative correlation found at the individual level. The aggregate correlation is a different sign since the aggregation smooths over the lumpy portfolio adjustment at the individual level.

A similar result is reported for the correlation between hours and employment at the plant-level and in aggregate by Cooper, Haltiwanger, and Willis (2004).
Second, as seen in the bottom left panel, the correlation between consumption growth and the stock return is evident as well. This comovement underlies the regression results used as a moment in Table 2.

### 5.2.3 Regression Results

For further evidence on the structure of household decision rules, we study the state-dependence of bond and stock holdings as well as the adjustment decision through a regression structure. Though the regressions are only coarse approximations, they are illustrative of the forces at work. Our results are shown in Table 3.

The first five rows report the regressions of decisions regarding stockholding, bondholding and consumption on current state variables: income, stock return, stockholding and bondholding. These regressions were run at the household level from the same simulated panel used to calculate our moments. Each regression yields a very high R-squared, indicating highly linear decision rules. These are intensive margin decisions because we split households into adjustors and non-adjustors.

Adjustors and non-adjustors have totally different decision rules. Bondholding is negatively correlated with income due to the substitution between stock and bond as shown in the impulse response figures – when stock adjustment occurs, it is accommodated by bond adjustment in the opposite direction. For non-adjustors, higher current income leads to more future bondholding. However stock return and current
5.2 Simulation Results

The figure plots the dynamics of aggregate stockholding, bondholding and consumption in response to shocks to stock return.

Figure 5: Return shocks and aggregate decisions

stockholding are negatively correlated with future bondholding. Due to wealth effect, high stock return and current stockholding induces more consumption which is financed by reduced bondholding. The marginal effect of state variables on consumption decisions are about the same for adjustors and non-adjustors, implying good consumption smoothing.

The last row reports the linear regression of the adjustment decision on the current state and size of income shock approximated by the absolute deviation of income from the mean. Higher income induces more asset holding for consumption smoothing purpose. Since we assume returns to stock are re-invested automatically in case of non-adjustment, a higher income decreases the likelihood of adjustment. Adjustment is more likely when the return on stocks or stockholding is high for portfolio rebalancing reasons. A larger income shocks increases the likelihood of adjustment, as in the impulse response functions.

5.2.4 Substitution Between Stock and Bond – Data Evidence

One of the most notable phenomena in the simulation results is the substitution between stock and bond holdings. When stock is not adjusted and dividends are re-invested, bondholding is reduced for consumption smoothing purposes. When stock adjustment occurs, asset rebalancing takes place and bond account is adjusted in the opposite direction of stock. We look into PSID data and find evidence supporting such
Table 3: Regression Results

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variables (state variables)</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision</td>
<td>$y$, $R^e$, $A^s$, $A^b$, $\text{abs}(y - \bar{y})$</td>
<td></td>
</tr>
<tr>
<td>stock adjustor</td>
<td>0.742 0.865 0.578 0.3770</td>
<td>0.978</td>
</tr>
<tr>
<td>bond adjustor</td>
<td>-0.135 0.685 0.374 0.5890</td>
<td>0.935</td>
</tr>
<tr>
<td></td>
<td>0.606 -0.239 -0.150 0.9200</td>
<td>0.994</td>
</tr>
<tr>
<td>non-adjustor</td>
<td>0.408 0.240 0.158 0.0960</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>0.394 0.239 0.150 0.1010</td>
<td>0.983</td>
</tr>
<tr>
<td>consum. adjustor</td>
<td>-0.347 0.243 0.103 -0.1430</td>
<td>0.157</td>
</tr>
<tr>
<td>non-adjustor</td>
<td>0.394 0.239 0.150 0.1010</td>
<td>0.983</td>
</tr>
<tr>
<td>adjust.</td>
<td>-0.347 0.243 0.103 -0.1430</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Table 4: Contemporaneous correlation

<table>
<thead>
<tr>
<th></th>
<th>Model data</th>
<th>data (subsample)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y$</td>
<td>$\Delta B$</td>
<td>$\Delta y$</td>
</tr>
<tr>
<td>$\Delta B$</td>
<td>0.036</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.601)</td>
</tr>
<tr>
<td>$\Delta S$</td>
<td>0.196</td>
<td>-0.435</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.019)</td>
</tr>
</tbody>
</table>

Contemporaneous correlation between stock and bond holding. $\Delta Y$, $\Delta B$ and $\Delta S$ refer to changes in un-predictable income, bondholding and stockholding respectively. Data (subsample) refer to the subsample that has 20% largest income shocks removed.

Based on the panel data in PSID 1997-2007, we run panel VAR on the changes in un-predicted income, stockholding and bondholding. Many of the coefficients on lagged terms are not significant. However, the contemporaneous correlation, reported in table 4, clearly shows the negative correlation in the data as in the model.

6 Conclusion

This study asked how households respond to income and return shocks when there are portfolio adjustment costs. We estimate the opportunity cost of stock trading, along with discount factor and curvature in preference via simulated method of moments.

We find that this response comes in two forms: portfolio rebalancing and consumption smoothing. The response is highly nonlinear. The stock account is adjusted in response to large shocks while the bond account buffers consumption from smaller shocks. Our simulation results reveal a strong negative correlation between stock and bond holding at household level. When households refrain from adjusting stock account to avoid negative correlation between stock and bond holdings. Data description is given in the appendix.

Based on the panel data in PSID 1997-2007, we run panel VAR on the changes in un-predicted income, stockholding and bondholding. Many of the coefficients on lagged terms are not significant. However, the contemporaneous correlation, reported in table 4, clearly shows the negative correlation in the data as in the model.
costs, stock account is increased due to the re-invested dividend, but bond account is run down to finance
costs. On the other hand, when stock adjustment occurs, bond account is replenished. We find data
evidence in PSID supporting such substitution.

7 Appendix

7.1 Data Appendix

Exogenous processes We calculate stock return process from Robert Shiller’s online data of S&P500 for
the period 1947-2007. The return is defined as \( \frac{p_{t+1} + d_t}{p_t} \) where \( p_t \) is S&P500 index and \( d_t \) is the dividend. The return is then deflated by CPI. We run an AR(1) regression on stock return, and we find that the persistence is not statistically different from zero. So we represent stock return as an i.i.d. process. The mean and standard deviation of return are 6.33% and 15.5% in our sample period.

Income process is calculated from the Panel Study of Income Dynamics (PSID) for the period of 1968-
1997. Income is defined as salary plus other source of labor income such as commission, overtime, labor part of business income, labor part of farm income and others. Totally there are 78 household who have complete income information during the 30 years and satisfies the following criteria (i) non-SOE sample\(^8\) (ii) full information on age, gender, marital status, and education attainment (iii) stockholders. The survey does not provide stockholding information except in year 1984, 1989 and 1994 during the our sample periods. So we drop all the households that are not stockholder during the three waves of survey in order to meet criterion (iii). We take two steps to calculate the size of persistence of income shocks.

1. Pool all the observations together, and regress income on age, \( age^2 \), education attainment, gender and
marital status.

2. Take the residues from the above regression, and use the residues to run the AR(1) process.

\[
y_t = \rho y_{t-1} + \epsilon_t
\]

The persistence of income shock is estimated to be \( \rho = 0.84224 \), and shock size is \( \sigma_\epsilon = 0.29027 \).

Data moments We calculate data moments from 3 sources: Survey of Consumer Finance (SCF), Panel Study of Income Dynamics (PSID), and the consumption data complied and used in Vissing-Jorgensen (2002). We use multiple sources for two reasons. First, it is impossible to calculate all the moments in a single data. Secondly, each data source has its own strength and drawbacks, so we choose the best combination possible.

We calculate two moments from SCF: the median of stock share in total financial wealth, and median

\(^8\)Survey of Economic Opportunity (SEO) sample focuses on low-income families.
we first calculate the ratios at the household level, then calculate the median of the household level ratios. Although SCF provides a weight for each household, we use un-weighted median. This is because our sample includes stockholders only, while the SCF weights are meant to make the whole sample, including non-stockholders, nationally representative. The SCF is known for its excellent quality on household’s asset-holding levels and composition. In addition, it provides detailed information on U.S. labor force participation, liabilities, and other social demographic characteristics such as age, education and gender. The survey also collects information on total family earnings and wealth. The actual number of respondents in the survey is approximately 4,300 where for each observation there are another five imputed observations. Stock in SCF is defined as the sum of directly and indirectly held stock market investment. We define bond as the sum of investment in transaction accounts, CDs, directly-held bonds and saving bonds. Total financial wealth is defined as the sum of stock investment and bond investment, which is used in the calculation of both moments. Income reported in SCF is the total income from wage and salaries before deduction for taxes. Table 5 reports the moments for various years and the average over years.

We calculate the moment of stock adjustment rate from PSID, using the 1999-2007 sampling period. Starting from 1999, PSID surveys are conducted bi-annually, each wave provides wealth supplements which includes information about the respondents’ holding of non-IRA stocks. The survey asks three questions regarding stock adjustment: (i) whether the respondent buy stock since last survey, (ii) whether sell stock since last survey, (iii) whether buy more, sell more, or put as much in as took out. We define “gross adjustment rate” as the percentage of households who answered positively to either question (i) or (ii). The gross adjustors include the subset of households who report “put as much in as take out”. This subset of households did rebalance among different stocks, but didn’t use stock for consumption smoothing purpose. We excludes these households from gross adjustors, and define remaining as net adjustor. The adjustment rate consistent with our model definition is “net adjustment rate” – the percentage of households who are net adjustors. Table 6 shows the gross adjustment rate and net adjustment rates for the four intervals in our sample.

Response rate of aggregate consumption to stock return is calculated from data used in Vissing-Jorgensen (2002). The data is semi-annual, so we convert the consumption growth rate and return into annual ones by taking geometric means. As in Vissing-Jorgensen (2002), we run two-stage least square to obtain the response of consumption growth to stock return, using the growth rate of dividend as the instrument. Changes in family size are also controlled for.

Table 5: Moments from SCF

<table>
<thead>
<tr>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks share</td>
<td>0.479</td>
<td>0.571</td>
<td>0.691</td>
<td>0.782</td>
<td>0.749</td>
<td>0.773</td>
<td>0.690</td>
<td></td>
</tr>
<tr>
<td>Wealth/income</td>
<td>1.942</td>
<td>2.020</td>
<td>1.843</td>
<td>2.042</td>
<td>2.099</td>
<td>2.369</td>
<td>2.573</td>
<td>2.127</td>
</tr>
</tbody>
</table>
### Dynamics of Income and Asset-Holdings in PSID

We study the dynamics of income asset-holdings empirically based on PSID 1999-2007. The sample includes households that (i) have complete income information during the sampling period (ii) are stockholders (iii) have valid information on education attainment, age of head and sex of head in 1991 (iv) are aged 48 or less in 1991 so that they do not retire during the sampling period (v) have valid information on bond holding and stock holding. Totally there are 302 households in the sample. Starting from 1999, PSID provides wealth supplements bi-annually. Besides housing wealth, PSID has 3 types of wealth: (i) non-IRA stock (ii) money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, or treasury bills (checking and saving) (iii) other savings or assets, such as bond funds, cash value in a life insurance policy, a valuable collection for investment purposes, or rights in a trust or estate (bond and insurance). In addition wealth supplement provides information about debt that does not include home or vehicle loans, such as credit card charges, student loans, medical or legal bills, or on loans from relatives. We group wealth into two categories: stock and bond. Stock is simply the non-IRA stock. Bond is defined as “checking and saving + bond and insurance - debt”. Using residue income, stock and bond, we run panel VAR to study the dynamics among these variables.

### 7.2 Computation Appendix

The state space of the dynamic optimization problem in our model is \((y, A_{-1}, R_{-1})\). Here \(y\) is income of the current period, \(A_{-1} = (A^b_1, A^s_{-1})\) is the beginning-of-period asset holdings, and \(R_{-1} = (R^b_1, R^s_{-1})\) is the vector of bond return and stock return.

We discretize the AR(1) processes for labor income using the methodology proposed in Tauchen and Hussey (1991). As a result, income process is represented by a 5-state Markov chain. The i.i.d. stock return process is approximated by two return states, \(-9.083\%\) and \(-21.83\%\) with equal probability, implying a mean return of \(6.33\%\) and a standard deviation of \(15.5\%\). Bond return is fixed in the baseline case, and not kept track of in the programming problem. In the robustness check we allow bond return to be risky and persistent, and approximate the process by a two-state Markov chain.

The dynamics programming problem has 3 discrete state variables and 2 continuous state variables.
(stockholding and bondholding). There are 2 control variables – a household needs to decides how much stock and bond to hold in the beginning of each period. This is a high-dimensional programming problem which is computational intensive. Furthermore we need to solve the model many times to find parameter values that best match model moments with data moments. In order to solve the model with good precision within reasonable amount of time, We take the following strategy.

First, we combine stock return and stockholding into one state variable. This is feasible because stock return is i.i.d. That is, current stock return contains no information about next period’s return. Therefore we can simply treat the stockholding after realization of return as a state variable, rather than keep track of both stock return and stockholding before realization of return. Notice that stockholding as a control variable refers to the holding before realization of return. In simulation, we solve for stockholding as a control variable, then multiply it with return to make it a state variable for the next period.

Second, we use a mixture of grid search and spline interpolation to execute value function iteration. Specifically, we define two grids for the Cartesian space of stockholding and bondholding. The first one is a coarse grid with 25 points for stockholding and 20 points for bondholding, denoted $A_s^{\text{coarse}} \times A_b^{\text{coarse}}$. The second one is a fine grid with $400 \times 150$ grid points, denoted $A_s^{\text{fine}} \times A_b^{\text{fine}}$. We start with an initial guess of value function $v(D, A_s^{\text{fine}} \times A_b^{\text{fine}})$ where $D$ stands for the product of discrete state variables. Then we take the following steps to update the value function.

1. On the coarse grid, compute the value of the sub-optimal decision of not adjusting stockholding, denoted $v^n(D, A_s^{\text{coarse}} \times A_b^{\text{coarse}})$. The value of the sub-optimal decision of always adjusting stockholding, denoted $v^a(D, A_s^{\text{coarse}} \times A_b^{\text{coarse}})$, is also computed.

2. Use $v^n(D, A_s^{\text{coarse}} \times A_b^{\text{coarse}})$ and $v^a(D, A_s^{\text{coarse}} \times A_b^{\text{coarse}})$ to interpolate the values on fine grid, denoted $v^n(D, A_s^{\text{fine}} \times A_b^{\text{fine}})$ and $v^a(D, A_s^{\text{fine}} \times A_b^{\text{fine}})$. Due to the high non-linearity of the value functions, we using spline interpolation.

3. Compute the updated value function as $v(D, A_s^{\text{fine}} \times A_b^{\text{fine}}) = \max\{v^n(D, A_s^{\text{fine}} \times A_b^{\text{fine}}), v^a(D, A_s^{\text{fine}} \times A_b^{\text{fine}})\}$.

After the value function converges, we compute policy functions on the fine space, and use these policy functions to simulate artificial data. In the simulation, we draw random shocks to income and returns of 500 households for 800 periods, so the dimension of simulated data is $500 \times 800$.

Third, we design both coarse grid and fine grid carefully. The upper-bounds of asset holdings grid are important. Very high upper-bounds reduce the efficiency of the program, because a large area of the grid near the upper-bounds are never reached in simulation. On the other hand, when the upper-bounds are too low, households’ optimal decision rules are distorted. After many experiments, we find that the upper-bounds of 40 times of mean income for stockholding and 20 times of mean income for bondholding are efficient and non-binding. In addition, we put more points near the lower-bounds of asset holdings.
References


