Education and Training under Labor Market Frictions

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Abstract

I consider a frictional labor market model with ex-post wage bargaining where a worker invests in education ex-ante and a firm trains the employee. It is assumed that a fraction of the training cost is subsidized by the government and there exists complementarity between education and training. Although the worker and the firm face hold-up problems, the conditions under which efficiency is achieved can be derived. The conditions require that the government’s cost share to training should be equal to the worker’s bargaining share and also require that the elasticity of the production function with respect to education should be equal to the worker’s “effective” bargaining power. These conditions tell us what the government should do to gain efficiency. The model is extended to include education subsidies.

1 Introduction

Many economists agree that human capital is a crucial factor in economic performance or growth. Human capital is accumulated mainly by education in school or training in a firm. So, it is important to consider the conditions under which the amounts of education and training are efficient. This paper derives such conditions and gives policy implications.

The model is based on a frictional labor market where it takes time for an agent to meet his or her partner. In this paper, I call investment behavior by a worker before entering the labor market “education”. Schooling is a typical example. Although in reality, the government subsidizes education, to get the clear results I exclude education subsidies from the basic model. They are included in Section 6. Next let me talk about training. The seminal work by Becker (1962) says that in the competitive labor market, if training is “firm-specific” in the sense that it is effective only in the current firm, a firm trains

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1For simplicity, I use “he” for a worker, “she” for a firm.
her employee and pays the cost of training. Moreover, recently Acemoglu and Pischke (1998, 1999a, 1999b) argue that under labor market frictions, she trains him and incurs the cost even if training is “general” in the sense that it is effective in all firms. Also in reality, the government provides financial assistance for firms to increase their training activities.\textsuperscript{23} So, it is natural to assume that after a firm meets a worker, she trains him and the cost is incurred by both her and the government. The government is assumed to pay a constant share of the training cost.

About education and training, complementarity is an important characteristic and assumed in this paper. Altonzi and Spletzer (1991) and Bartel and Sicherman (1998) find positive relationships between previous education and training incidence using the National Longitudinal Survey. Acemoglu and Pischke (1998) report similar results in the data for Germany. Given these empirical facts, Heckman (2000) emphasizes the significance of complementarity.

A more correct view of ability (or rather abilities) is that they are created in a variety of learning situations and that ability in turn fosters further learning. More able people acquire more skills; more skilled people become more able. Dynamic complementarity characterizes skill and ability formation and our economic models have to be modified to account for this. (p. 6 of Heckman (2000))

My model is, as far as I know, the first to incorporate such dynamic complementarity under search frictions. This comes from the timing of investments: before entering the labor market a worker invests in education and after he meets a firm, she chooses the amount of training positively replying to the amount of education.

The remaining feature of the model which is worth mentioning here is that wages are determined by ex-post Nash bargaining. Under search frictions, there is room for rent-sharing because it takes time for both a worker and a firm to find a partner, which implies that destruction of the match is costly. Thus, each agent has a bargaining power over the split of the surplus the match produces.

Now, I explain intuitions about why efficiency is achieved under some conditions. First, let’s consider the worker’s problem. He invests in education before entering the labor market. Under this situation, when he invests, he has to pay all the cost of education because at that moment, he has not yet formed the employment relationship with a firm, which means that he has no partner to share the cost. Given ex-post wage bargaining, this causes a hold-up problem. That is, even though he incurs all the cost, he cannot get all the output. Naturally he does not have the right incentive to invest.\textsuperscript{4} On the other hand, if he invests more, he can get larger amounts of training after matching because due to complementarity, a firm chooses the amount of training positively replying to the amount of education. Large amounts of training leads to much output.

\textsuperscript{2}For example, see Holzer et al. (1993).

\textsuperscript{3}Acemoglu and Pischke (1999a) support training subsidies because they mitigate a hold-up problem a firm faces.

\textsuperscript{4}The logic is same as Acemoglu and Shimer (1999).
and high wages. In this sense, he has the incentive to invest more. These two effects are offset under some parameter relationships. Next, let’s consider the firm’s problem. When she trains a worker, owing to training subsidies, she only has to pay a fraction of the training cost. About output, she can get only a fraction of it due to ex-post wage bargaining. Thus, given education, if the firm’s cost share is equal to her bargaining share, then she does not face any hold-up problems. In other words, she has the right incentive to train a worker.

Given the intuitions stated above, what are the conditions under which efficiency is achieved like? As will be clear later, they are composed of two equations. The first equation requires that the government’s cost share to training should be equal to the worker’s bargaining share. If this is satisfied, it follows that the firm’s cost share is equal to her bargaining share. This is consistent with the intuitions given above. The second equation is more complicated. It requires that the elasticity of the production function with respect to education should be equal to the worker’s “effective” bargaining power. The details about this and policy implications will be discussed in the following sections.

At this moment, I briefly review the related literature. I restrict attention to the search and matching models. Acemoglu and Shimer (1999) consider the situation where a firm invests in physical capital before entering the labor market. They show that the amount of investment is always sub-optimally low. Jansen (2010) extends this model to include competition among applicants. Acemoglu (1996) and Masters (1998) explore the situations where both a firm and a worker invest before entering the labor market. They show that there exist multiple equilibria and efficiency is never achieved. Acemoglu (1997a, b) also get the similar results. Masters (2011) extends the model by Acemoglu (1996) or Masters (1998) to the competitive search model and Navarro (2011) also modifies it so that there exist high and low productivity sectors. There are a few papers which deal with training. Moen and Rosén (2004) consider the conditions which guarantee efficiency in the competitive search model with training and endogenous turnover. Tripier (2011) investigates the effect of intrafirm bargaining on the efficiency of training and hiring. There are many papers which deal with education. Moen (1999) puts an assumption that a worker’s job-finding rate depends on his education level. Moen (1998) examines contingent loans which finance the education cost and Kaas and Zink (2011) deal with the similar situation in the competitive search model. Charlot and Decreuse (2005, 2010) and Charlot et al. (2005) consider the multi-sectors models. Coles and Masters (2000) assume that skills of an unemployed worker decline. Sato and Sugiuira (2003) investigate the effect of education subsidies and unemployment benefits on efficiency.

The plan of this paper is as follows: Section 2 describes the environment of the economy. Section 3 derives the equilibrium allocation. Section 4 derives the constrained efficient allocation. Section 5 derives the conditions which ensure efficiency and discusses these conditions and policy implications. Section 6 extends the model to include education subsidies. Section 7 concludes.
2 The environment

I follow the standard search and matching model. The model is set in continuous time. The economy consists of continuums of workers and firms, and the measure of workers is 1 and the measures of firms are exogenously given to satisfy the steady-state conditions. Workers exit the market at a constant, exogenous rate \( \delta \), and are replaced by a flow of new workers entering the labor market, so the measure of workers is constant. Let \( u \) denote the measures of unemployed workers. Each firm hires one worker at most. Thus a firm is either searching for an unemployed worker, i.e., vacant or having one worker and producing, i.e., filled. On the other hand, a worker is either searching for a vacant firm, i.e., unemployed or being hired by a firm, i.e., employed. Once a match is produced, it is subject to a separation shock at a rate \( s \). When this happens, an employed worker becomes unemployed. As stated in footnote 6, I assume that entry and exit of firms are exogenous, i.e., at rates \( \delta \) or \( s \), filled firms exit the market and are replaced by a flow of new firms entering the labor market. This means that the measures of firms are also constant. Both workers and firms are identical and risk-neutral. For simplicity, I assume that an unemployed worker receives a flow payoff of zero and the flow cost of posting a vacancy is also zero. All agents discount the future at the common rate \( r \).

The labor market is characterized by search frictions, which means that it takes time for each agent to find a partner. Thus, it is natural to assume that an unemployed worker meets a vacant firm at a Poisson arrival rate \( w \) and a vacant firm meets an unemployed worker at a Poisson arrival rate \( f \). Since the measures of workers and firms are exogenously given and I consider only the steady-state, the rates \( w, f \) are constant and exogenous.

Before entering the labor market, a new worker invests in education just once and instantaneously. Let \( e \) denote the education level and \( d(e) \) its cost. \( d(e) \) is assumed to satisfy the following conditions: \( d'(e) > 0, d''(e) > 0 \) for all \( e > 0 \) and \( d(0) = 0, d'(0) = 0 \). Education is assumed to be “general” in the sense that it is effective in all firms and assumed not to depreciate. After meeting a worker, a firm trains him. In this model, for simplicity, I assume that training is done just once and instantaneously, not continuously. In other words, training for a new employee is assumed. Let \( t \) denote the training level and \( c(t) \) its cost. I specify \( c(t) \) as follows: \( c(t) = C \cdot t \) where \( C > 0 \) is constant. To simplify the analysis, training is assumed to be “firm-specific” in the sense

\(^5\)For example, see Pissarides (2000).

\(^6\)Usually it is assumed that the measures of firms are endogenously determined through entry and exit. However, the focus of this paper is on the incentives to invest in education and training, not on the incentives to enter and exit the market. Therefore, I assume that entry and exit of firms are exogenous and the measures of firms are exogenously given to satisfy the steady-state conditions. About how the measures of firms are given, see footnote 12. Also see Moen (1998, 1999).

\(^7\)Suppose the cost of education is composed of two parts. The first is monetary payment such as tuition and the second is disutility from education. Then, the latter seems to make the cost function convex.

\(^8\)For example, \( t \) can be considered as hours spent in training and \( C \) as opportunity costs caused by not producing during the training.
that it is effective only at the current employer. Suppose a worker with the education level $e$ meets a firm and she gives him the training level $t$, then the match is assumed to produce the output flow $f(e,t)$ with a price normalized to one. Since there exists complementarity between education and training, I specify $f(e,t)$ as follows: $f(e,t) = e^a \cdot t^{1-a}$ where $0 < a < 1$.

3 The equilibrium allocation

3.1 The asset value equations

This subsection derives the asset value equations, or Hamilton-Jacobi-Bellman (HJB) equations in the steady-state and also considers Nash bargaining.

First, I derive HJB equations for a worker. Let $U(e)$ denote the asset value of an unemployed worker with education $e$ and $E(e,t)$ denote that of an employed worker with education $e$ and training $t$. Note that a firm chooses the training level replying to the education level a worker chooses, so when I would like to emphasize this, I write down the training level as $t(e)$ rather than $t$. Let $(e^*, t^*)$ be the allocation at the equilibrium. Since an unemployed worker receives a flow payoff of zero, meets a firm at the rate $w$ and exits the market at the rate $\delta$, the HJB equation is as follows:

$$rU(e) = \lambda_w \{E[e, t^*(e)] - U(e)\} - \delta U(e)$$

The left hand side is the flow cost of holding the asset of “unemployment”, while the right hand side represents the flow value. This equation uses the fact that at the equilibrium, all firms choose the training level $t^*(e)$ because they are identical. The income after leaving the market is normalized to zero. Similarly, since an employed worker gets the wage flow, exits at $\delta$ and separates at $s$, the following holds:

$$rE[e, t(e)] = w[e, t(e)] - \delta E[e, t(e)] + s\{U(e) - E[e, t(e)]\}$$

$w(e,t)$ is the wage flow a worker with education $e$ and training $t$ gains.

Next, I derive HJB equations for a firm. Let $V$ denote the asset value of a vacancy and $J(e,t)$ denote that of a filled vacancy hiring a worker with education $e$ and training $t$. Since the flow cost of posting a vacancy is zero and a vacant firm meets a worker at $\lambda_f$, the HJB equation is as follows:

$$rV = \lambda_f \{J[e^*, t^*(e^*)] - V\}$$

This equation also uses the fact that at the equilibrium, all workers choose education $e^*$ and all firms choose training $t^*(e^*)$. In the same way, since a filled firm produces the output flow $f(e,t)$, pays the wage flow and exits the market by the exogenous shocks, the following holds:

$$rJ[e, t(e)] = f[e, t(e)] - w[e, t(e)] - (\delta + s)J[e, t(e)]$$
Rearranging these four equations, we get the four equations below:

\[ U(e) = \frac{\lambda_w}{r + \delta + s + \lambda_w} \cdot \frac{w[e, t^*(e)]}{r + \delta} \]  
\[ E[e, t(e)] = \frac{w[e, t(e)]}{r + \delta + s} + \frac{s}{r + \delta + s} U(e) \]  
\[ V = \frac{\lambda_f}{r + \lambda_f} J[e^*, t^*(e^*)] \]  
\[ J[e, t(e)] = \frac{f[e, t(e)] - w[e, t(e)]}{r + \delta + s} \]  

(Note that \( E[e, t^*(e)] = \frac{r + \delta + \lambda_w}{r + \delta + s + \lambda_w} \cdot \frac{w[e, t^*(e)]}{r + \delta} \).)

Finally, let’s consider Nash bargaining. As I stated before, wages are determined by Nash bargaining in this model. Let \( \beta \) denotes the worker’s bargaining power (0 < \( \beta < 1 \)). It follows that 1 - \( \beta \) is the firm’s bargaining power. Nash bargaining rule implies:

\[ (1 - \beta)\{E[e, t(e)] - U(e)\} = \beta\{J[e, t(e)] - V\} \]

Substituting Equations (1) to (4) into the above equation, we get the wage:

\[ w[e, t(e)] = \beta f[e, t(e)] + (1 - \beta) \frac{\lambda_w}{r + \delta + s + \lambda_w} w[e, t^*(e)] - \beta \frac{(r + \delta + s)\lambda_f}{r + \lambda_f} J[e^*, t^*(e^*)] \]

3.2 Characterization of the equilibrium allocation

This subsection derives the equations which characterize the equilibrium allocation. Since the structure of the model can be considered as a sequential-move game with perfect and complete information where a worker is a first-mover and a firm is a second-mover, I use the subgame-perfect equilibrium as the equilibrium concept. Here, I restrict attention to the equilibrium path, that is, the equilibrium allocation or outcome. I solve backward, i.e., first solve the decision problem of a firm and next solve that of a worker.

After a firm meets a worker with education \( e \), she trains him. The government pays a constant share of the training cost, so let \( \mu_t \) denote the government’s cost share (0 < \( \mu_t < 1 \)).\(^9\) It follows that a firm pays \((1 - \mu_t)c(t)\) when she chooses training \( t \). Hence, the firm’s decision problem is given as follows:

\[ \max_t J(e, t) - (1 - \mu_t)c(t) \]  

\(^9\)The implicit assumption is imposed that the government levies lump-sum tax to finance training subsidies.
Before considering this, I put an assumption that at the equilibrium, a firm is better off hiring and training a worker, i.e., the following inequality holds:

\[ J[e^*, t^*(e^*)] - (1 - \mu_t)c[t^*(e^*)] \geq V \]

From \( c(t) = Ct \) and Equation (3), this inequality can be written as:

\[ C \leq \frac{r}{r + \lambda_f} \cdot J[e^*, t^*(e^*)] \]

Thus I restrict parameter value \( C \) so that it satisfies the above inequality. Now I go back to the firm’s decision problem. Using Equations (4) and (5), Equation (6) is written as:

\[
\max_t 1 - \frac{\beta}{r + \delta + s} f(e, t) - \frac{\beta}{r + \delta + s + \lambda_w} w[e, t^*(e)] + \frac{\beta \lambda_f}{r + \lambda_f} J[e^*, t^*(e^*)] - (1 - \mu_t)c(t)
\]

Note that given \( f(e, t) = e^\alpha t^{1-\alpha} \) and \( c(t) = Ct \), the objective function is strictly concave in \( t \). Therefore the first order condition is the necessary and sufficient conditions for maximization. By the first order condition given \( e \), we can get the best response function of a firm, i.e., \( t^* = t^*(e) \):

\[
1 - \frac{\beta}{r + \delta + s} f_t(e, t^*) = (1 - \mu_t)c'(t^*)
\]

Using the specification that \( f(e, t) = e^\alpha t^{1-\alpha} \) and \( c(t) = Ct \), the closed-form of the best response function can be achieved:

\[
t^* = \left[ \frac{(1 - \beta)(1 - \alpha)}{(r + \delta + s)(1 - \mu_t)C} \right]^{1/\alpha} e
\]

Finally, the following equation is one of the necessary and sufficient conditions for the equilibrium.

\[
1 - \frac{\beta}{1 - \mu_t} f_t(e^*, t^*) = (r + \delta + s)c'(t^*)
\]

Next let me solve the decision problem of a worker.\(^{10}\) Before entering the labor market, he invests in education. So, his decision problem is given as follows:

\[
\max_{e} U(e) - d(e)
\]

Equation (5) implies:

\[
w[e, t^*(e)] = \beta \frac{r + \delta + s + \lambda_w}{r + \delta + s + \beta \lambda_w} f[e, t^*(e)] - \beta \frac{r + \delta + s + \lambda_w}{r + \delta + s + \beta \lambda_w} \left( \frac{(r + \delta + s)\lambda_f}{r + \lambda_f} J[e^*, t^*(e^*)] \right)
\]

\(^{10}\)From Equations (1) and (2), \( E[e^*, t^*(e^*)] \geq U(e^*) \) always holds at the equilibrium. So, a worker is always better off being hired by a firm.
Using this equation and Equation (1), Equation (9) can be written as:

\[
\max_{\epsilon} \frac{\beta \lambda_w}{r + \delta + \lambda_w} f[\epsilon, t^*(\epsilon)] - \frac{\beta}{r + \delta + \lambda_w} \lambda_f f[\epsilon^*, t^*(\epsilon)] - d(\epsilon)
\]

Note that from the specification \( f(e, t) = e^{\alpha}t^{1-\alpha} \) and Equation (7), \( f[\epsilon, t^*(\epsilon)] = e^{\alpha}[t^*(\epsilon)]^{1-\alpha} = [\epsilon^{1-\alpha}]^{\alpha} \cdot \epsilon \). Therefore, the objective function is strictly concave in \( \epsilon \) because \( d(e) \) is strictly convex. So, the other necessary and sufficient conditions for the equilibrium is as follows:

\[
\frac{\beta \lambda_w}{r + \delta + \lambda_w} \{f_e[e^*, t^*(\epsilon)] + f_t[e^*, t^*(\epsilon)] \frac{dt^*(\epsilon)}{d\epsilon}\} = (r + \delta)d'(\epsilon^*)
\]

Using Equation (7) and the specification that \( f(e, t) = e^{\alpha}t^{1-\alpha} \), the above equation is written as:

\[
\frac{\beta \lambda_w}{r + \delta + \lambda_w} \left[ \alpha(e^*)^{\alpha-1}(t^*)^{1-\alpha} + (1-\alpha)(e^*)^\alpha(t^*)^{-\alpha} \right] \left[ \frac{(1-\beta)(1-\alpha)}{(r + \delta + s)(1 - \mu_t)C} \right]^{1/\alpha} = (r + \delta)d'(\epsilon^*)
\]

Using the fact that at the equilibrium, \( \left[ \frac{(1-\beta)(1-\alpha)}{(r + \delta + s)(1 - \mu_t)C} \right]^{1/\alpha} = (e^*)^{-1}(t^*) \) holds, this equation can be simplified to:

\[
\frac{\beta \lambda_w}{r + \delta + \lambda_w} (e^*)^{\alpha-1}(t^*)^{1-\alpha} = (r + \delta)d'(\epsilon^*)
\]

We have characterized the equilibrium allocation.

**Proposition 1** The steady-state subgame-perfect equilibrium allocation is summarized by a pair \( (e^*, t^*) \) which solves Equations (8) and (10).

### 4 The Efficient Allocation

An allocation is constrained efficient if it maximizes the net output of the economy subject to search restrictions. So, let’s consider a social planner’s problem. Although the true planner’s problem involves solving for an optimal transition path, a much simpler problem is to restrict attention to the case where the discount rate tends to zero. In the limit as \( r \to 0 \), the planner assigns no weight to the transitional period and the planner’s objective is to maximize the stationary total payoff.\(^\text{11}\) Given all agents are risk-neutral, the social planner

\[^{11}\text{From Masters (2011), “Using subjective discount rates that are different from zero complicates the welfare analysis without any qualitative effect on the results. For realistic parameter values even the quantitative effects are small. See Hosios (1990) and Pissarides (2000) for a discussion.” In the search literature, Charlot and Decreuse (2005, 2010), Charlot et al. (2005), Coles and Masters (2000) and Masters (2011) consider the case where the discount rate tends to zero.} \]
chooses education $e$ and training $t$ to maximize the aggregate welfare flow in the steady-state:

$$\max_{e,t} (1 - u)f(e, t) - \delta d(e) - (1 - u)(\delta + s)c(t)$$

The first term is the flow value of production times the measures of matches, the second term is the flow cost of education times the measure of workers, i.e., 1, and the third term is the flow cost of training times the measures of matches. This problem is subject to the steady-state condition about workers:

$$(\delta + s)(1 - u) = \lambda_w u (\leftrightarrow u = \frac{\delta + s}{\delta + s + \lambda_w})$$

The left hand side is the inflow to unemployment, while the right hand side is the outflow.\footnote{About the measures of firms, they are exogenously given to satisfy the steady-state conditions. Let $m$ denote the measures of firms. Then the steady-state condition about firms implies that $(\delta + s)(1 - u) = \lambda_f [m - (1 - u)]$. So, using the fact that $u = \frac{\delta + s}{\delta + s + \lambda_w}$ from the steady-state condition about workers, the measures of firms are exogenously given as $\frac{\lambda_w}{\delta + s + \lambda_w} \frac{\delta + s + \lambda_f}{\lambda_f}$.}

Substituting this into the objective function, we get the following:

$$\max_{e,t} \frac{\lambda_w}{\delta + s + \lambda_w} f(e, t) - \delta d(e) - \frac{\lambda_w}{\delta + s + \lambda_w} (\delta + s)c(t)$$

Given the specification that $f(e, t) = e^\alpha t^{1-\alpha}$ and $c(t) = Ct$, and the assumption that $d(e)$ is strictly convex, the objective function is strictly concave in $(e, t)$. Thus, the first order conditions are the necessary and sufficient conditions for the efficient allocation. Let $(e^E, t^E)$ be an efficient allocation. Then $(e^E, t^E)$ is characterized by the following two equations.

$$f_e(e^E, t^E) = (\delta + s)c'(t^E)$$

$$\frac{\lambda_w}{\delta + s + \lambda_w} f_e(e^E, t^E) = \delta d'(e^E)$$

Using the specification that $f(e, t) = e^\alpha t^{1-\alpha}$, Equation (12) is written as:

$$\frac{\lambda_w}{\delta + s + \lambda_w} \alpha (e^E)^{\alpha - 1} (t^E)^{1-\alpha} = \delta d'(e^E)$$

We get the following proposition.

\textbf{Proposition 2} \textit{The steady-state efficient allocation is characterized by a pair $(e^E, t^E)$ which solves Equations (11) and (13).}
The conditions and policy implications

The previous sections characterize the equilibrium and efficient allocations. This section derives the conditions where these allocations coincide and discusses their meaning and policy implications.

The equilibrium allocation \((e^*, t^*)\) is characterized by Equations (8) and (10), which are reproduced here with \(r \to 0\):

\[
1 - \frac{\beta}{1 - \mu_t} f_t(e^*, t^*) = (\delta + s)c'(t^*) \tag{8}
\]

\[
\frac{\beta \lambda_w}{\delta + s + \beta \lambda_w}(e^*)^{\alpha - 1}(t^*)^{1-\alpha} = \delta d'(e^*) \tag{10}
\]

While, the efficient allocation \((e^E, t^E)\) is characterized by Equations (11) and (13), which are also reproduced:

\[
f_t(e^E, t^E) = (\delta + s)c'(t^E) \tag{11}
\]

\[
\frac{\lambda_w}{\delta + s + \lambda_w} \alpha(e^E)^{\alpha - 1}(t^E)^{1-\alpha} = \delta d'(e^E) \tag{13}
\]

By comparing Equation (8) with (11) and Equation (10) with (13), the necessary and sufficient conditions for efficiency is as follows:

\[
\frac{1 - \beta}{1 - \mu_t} = 1 \quad \text{and} \quad \frac{\beta}{\delta + s + \beta \lambda_w} = \frac{\alpha}{\delta + s + \lambda_w} \Rightarrow \mu_t = \beta \quad \text{and} \quad \alpha = \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta
\]

What is the meaning of these conditions? The first condition requires that the government’s cost share to training should be equal to the worker’s bargaining share. This means that the firm’s cost share, i.e., \((1 - \mu_t)\) is equal to her bargaining share, i.e., \((1 - \beta)\), which implies that a firm pays \((1 - \beta)\) fraction of the training cost and gets \((1 - \beta)\) fraction of the surplus. So, a hold-up problem is avoided. The second condition is more complicated. The left hand side is the elasticity of the production function with respect to education. Roughly speaking, it represents the importance of education in the economy. The right hand side can be thought of as the worker’s “effective” bargaining power. (Note that \(0 < \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta < 1\)). That’s because when a worker faces search frictions, his “effective” bargaining power should include the extent to which they are serious. To consider this in detail, let me do two thought experiments. First, suppose that a worker does not face any frictions, i.e., \(\lambda_w \to \infty\). Then, his “effective” bargaining power becomes one because even if the current match is destroyed, immediately he can find a new partner while it takes time for the current employer to find a new partner. In other words, destruction of the match is not costly at all for him and is costly for her. This is consistent with...
the fact that \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \to 1 \) as \( \lambda_w \to \infty \). Next, suppose that a worker faces so serious frictions that he can never meet a firm, i.e., \( \lambda_w \to 0 \). Then the match is never formed, which implies that an unemployed worker remains unemployed. This means that he has no superiority in searching to firms and his “effective” bargaining power becomes his usual bargaining power which is exogenously given. This is consistent with the fact that \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \to \beta \) as \( \lambda_w \to 0 \). In this sense, \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \) can be considered as the worker’s “effective” bargaining power.\(^\text{13}\) I go back to the meaning of the second condition. It requires that the importance of education in the economy should be equal to the worker’s “effective” bargaining power. Since a worker chooses the education level taking into account his “effective” bargaining power, if this condition is satisfied, it can be said that the strength of his incentive to invest is equal to the importance of education in the economy and the education level is properly chosen from the social point of view. I summarize what we have achieved as proposition.

**Proposition 3** The equilibrium allocation is efficient if and only if the following two conditions are satisfied.

1. The government’s cost share to training is equal to the worker’s bargaining power \( (\mu_t = \beta) \).

2. The elasticity of the production function with respect to education is equal to the worker’s “effective” bargaining power \( (\alpha = \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta) \).

These conditions tell us what the government should do to gain efficiency. From the first condition, the government’s cost share to training should be equal to the worker’s bargaining power. The policy implication from this condition is easy to grasp. The problem lies in the second condition. There is no reason why this condition is satisfied, i.e., it is rarely satisfied in reality. So, there is room for policy intervention. When \( \alpha > \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \) holds, the government should take a policy to make the right hand side (R.H.S.) larger. Given the R.H.S. is increasing in \( \lambda_w \), a policy which mitigates frictions a worker faces is justified.

\(^\text{13}\)Although it may seem strange because entry or exit of firms are exogenous, let me put assumptions that a vacant firm pays the positive flow cost of posting a vacancy and the asset value of a vacancy is zero, i.e., \( V = 0 \). Then, at the equilibrium, the wage is as follows (from Equation (5) with \( r \to 0 \)):

\[
w(e^*, t^*) = \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \cdot f(e^*, t^*)
\]

So, a worker can get \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \) times the output as wages. This fact also seems to justify the interpretation that \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \) can be considered as the worker’s “effective” bargaining power.
We know that from the above discussion, such policy strengthens his “effective” bargaining power. To construct public employment security offices is one example. On the other hand, when \( \alpha < \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \) holds, a policy which lowers the R.H.S. is needed. Since the R.H.S. is decreasing in \( s \), the government should take a policy which causes frequent labor turnover. If separation shocks happen more frequently, the worker’s “effective” bargaining power becomes weaker because the value of the match decreases. To deregulate employment protection is one example. Hence, by proper policy intervention stated above, the second condition is expected to be satisfied.

6 Extension

This section extends the model to include education subsidies and briefly discusses the conditions and policy implications.

I assume that the government subsidizes education as well as training. So, let \( \mu_e \) denote the government’s cost share to education \( (\mu_e < 1) \). It follows that a worker pays \((1 - \mu_e)d(e)\) when he chooses education \( e \). This extension changes only the worker’s decision problem, which is given as follows:

\[
\max_e U(e) - (1 - \mu_e)d(e)
\]

The analysis proceeds in the same way as in the previous sections. Here, I state only the results.

**Proposition 4** The equilibrium allocation with education subsidies is efficient if and only if the following two conditions are satisfied.

1. \( \mu_t = \beta \)

2. \( \mu_e = 1 - \frac{1}{\alpha} \cdot \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \)

The first condition is the same as that of the last section. This is straightforward because the extension gives no changes to the HJB equations for a firm and a worker, Nash bargaining rule, and the firm’s decision problem. Although it seems a little complicated, the meaning of the second condition is easy to understand. From the discussion in the last section, \( \alpha \) represents the importance of education in the economy and \( \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \) can be considered as the worker’s “effective” bargaining power (the strength of his incentive to invest). If \( \alpha > \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta \) holds, which implies that he has less incentive to invest compared to the social optimum, \( 0 < \mu_e < 1 \) holds from the second condition. This means that the government has to subsidize education to give
him more incentive to invest, i.e., to foster investment.\(^{14}\) On the other hand, if \(\alpha < \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta\) holds, which implies that he has more incentive to invest than the social optimum, it follows that \(\mu_e < 0\). In this case, the government has to impose education tax on a worker to reduce his incentive. Finally, if \(\alpha = \frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta\) holds (maybe just by chance), then efficiency is gained from Proposition 3 and subsidies to education are not needed, i.e., \(\mu_e = 0\). From the above discussion, it can be said that the government should set education subsidies to equalize the importance of education in the economy to the strength of the worker’s incentive to invest so that education is properly chosen from the social point of view.

7 Conclusion

This paper considered the frictional labor market model with ex-post wage bargaining where a worker invests in education and a firm trains him. I derived the conditions under which efficiency is achieved. These conditions tell us what the government should do to gain efficiency.

One possible future work is empirical estimation of the theory. Especially, it is interesting to see whether in reality, the conditions are satisfied or not.

References


\(^{14}\)Note that the last section says that the government should increase \(\lambda_w\) to make \(\frac{\delta + s + \lambda_w}{\delta + s + \beta \lambda_w} \beta\) larger when it is smaller than \(\alpha\), but in this section the government can control \(\mu_e\), so I consider how the second condition is satisfied by controlling \(\mu_e\).


