Sectoral Shocks, Mismatch and Monetary Policy*

Tim Bian† and Pedro Gete‡

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Abstract

This paper studies monetary policy in a two sector economy facing sector specific shocks, search frictions and mismatch between unemployed workers and vacant jobs. We compare the performance of interest rate rules that target the aggregate economy versus interest rate rules that are sector specific. We show that in an economy hit with sector specific shocks and labor mismatch there are considerable differences from taking into account, or not, sectorial considerations in the Taylor rule. For example, for inflation there is a tradeoff between the size of the deviations from steady state versus the speed of recovery of the shock. These results confirm concerns recently expressed by policy makers as Kocherlakota (2010).

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†Email: yb33@georgetown.edu

‡Email: pg252@georgetown.edu
1 Introduction

Do aggregate or sectoral shocks drive cyclical unemployment? This was a popular question in the 1980s as economists discussed if oil shocks and the shift from manufacturing to services could explain the observed high rates of unemployment (see for example Lilien 1982 or Hamilton 1988). Following the work of Abraham and Katz (1986), the question seemed resolved in favor of aggregate shocks. Consequently, all models that have integrated unemployment in the standard New Keynesian (NK) model of monetary policy assume aggregate shocks as the cause of unemployment fluctuations. However, the fact that the last three recessions have been "Jobless Recoveries", together with the impressive housing boom and bust of the last decade, and the increasing shift of employment away from manufacturing, have brought back the question to the academic and policy arena.

Moreover, in housing boom/bust countries as Spain, the U.K. or the U.S. it is quite popular the view that sectorial shocks came associated with a mismatch in skills or geographical location between unemployed workers and vacant jobs. The support comes from three facts: 1) most of the job losses in the recent recession were in housing related sectors and in manufacturing while the newly created jobs were in health care or education. 2) The bust in housing prices prevents households with high mortgages from selling their homes and moving to locations with lower unemployment. 3) The Beveridge curve (the observed negative relationship between the unemployment rate and the vacancy rate) has shifted in a way that is consistent with mismatch.

In this paper we analyze the monetary policy implications of sectorial shocks when there is mismatch between workers and vacancies. To do so we set up a two sector NK model where unemployment arises from matching frictions à la Mortensen and Pissarides (1994) with exogenous job separations. There is monopolistic competition and adjustment cost on pricing à la Rotemberg (1982) that give raise to nonneutral effects of monetary policy. We start with the simplifying assumption that mismatch is extreme, i.e. workers unemployed in one sector cannot work in other sector. This assumption rules out the labor reallocation channel. We focus on

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1 The standard NK model does not feature unemployment, only hours worked per worker vary over the business cycle. However, several papers have incorporated unemployment into the NK framework: Blanchard and Gali (2010), Chéron and Langot (2000), Christoffel et al. (2009), Faia (2008 and 2009), Gertler et al. (2008), Gertler and Trigari (2009), Krause and Lubik (2007), Krause et al. (2008), Ravenna and Walsh (2008 and 2010), Sala et al. (2008), Thomas (2008), Trigari (2009) or Walsh (2005).


3 For example Elsby et al. (2010), Kocherlakota (2010), Phelps (2008), Sahin et al (2010)

4 quote Phelan-Trejos (2000)

5 For simplicity we do not impose wage rigidities. Hence our model suffers the volatility puzzle pointed out by Shimer (2005) about the search and matching model.
the effects of monetary policy in the economy when it tries to counteract sector specific shocks via countercyclical demand management policies.

First we perform a comparison across different monetary policy rules. We compare the performance of interest rate rules that target the aggregate economy versus interest rate rules that are sector specific. We show that in an economy hit with sector specific shocks and labor mismatch there are considerable differences from taking into account, or not, sectorial considerations in the Taylor rule. For example, for inflation there is a trade-off between the size of the deviations from steady state versus the speed of recovery of the shock. These results confirm concerns recently expressed by policy makers as Kocherlakota (2010).

The paper proceeds as follows. Sections 2 describes the model. Section 3 shows the equilibrium conditions. Section 4 explains the calibration. Section 5 discusses the results and Section 6 concludes.

2 Model

The model is based on New Keynesian model with search and matching frictions studied by Faia (2008) and Krause and Lubik (2007), which we augmented to include two sectors and mismatch friction. In the economy, there is one representative household, which is composed of two kinds of workers: type-A workers and type-B workers; There are two industry sectors, A and B, producing different final goods separately.

2.1 Labor Markets

In each sector, there is a labor market with search and matching frictions that prevent some job-seekers from finding a job and some vacancies from being filled. New workers must be hired from the unemployment pool and posting a vacancy involves a fixed cost. Wages are determined through Nash bargaining. Each worker can be employed or unemployed. We assume that unemployed workers do not receive any income.

The household sends her unemployed workers to search in one sector or another. We denote by \( L^i \) the quantity of labor force for type-\( i \) workers, by \( N^i_{j,t} \) the type-\( i \) workers employed during

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6 In these papers the price setting and employment decisions are embedded within the same firm.
7 Shimer (2005) and Hall (2005) noticed that in a matching model a’ la Mortensen and Pissarides wages are too volatile since little adjustment takes place along the employment margin. They also noticed that the introduction of real wage rigidity helps to resolve some of the puzzling features of the standard matching model.
period $t$ in sector $j$; by $U^i_{j, t}$, the type-$i$ workers who are unemployed and make decision at period $t$ on which sector $j$ to apply to next period. Thus,

\[
U^i_{A, t} + U^i_{B, t} = U^i_t = L^i - (N^i_{A, t} + N^i_{B, t})
\]

(1)

\[
u^i_t = \frac{U^i_t}{L^i}
\]

(2)

\[N_{j, t} = N^A_{j, t} + N^B_{j, t}
\]

(3)

where $u^i_t$ is the unemployment rate for type-$i$ workers, and $N_{j, t}$ is the total employment in sector $j$. Here $i = \{A, B\}$ and $j = \{A, B\}$.

We assume exogenous job separations at rate $\delta$, and that workers laid off at period $t$ cannot start looking for a job until next period. When unemployed workers look for jobs, there is a mismatch friction if a certain type workers search in the sector requiring different skills: $\tau$ fraction of them cannot find a job. This can be interpreted as if they have to devote a number of hours to learn the skill required in the other sector. Thus, if firms post vacancies $V_{j, t}$ in sector $j$’s labor market, the amount of new hires in that sector is given by the following Cobb Douglas matching functions

\[
m(U^A_{A, t-1} + (1 - \tau)U^B_{A, t-1}, V_{A, t}) = m_A(U^A_{A, t-1} + (1 - \tau)U^B_{A, t-1})^\xi (V_{A, t})^{1-\xi}
\]

(4)

\[
m((1 - \tau)U^A_{B, t-1} + U^B_{B, t-1}, V_{B, t}) = m_B((1 - \tau)U^A_{B, t-1} + U^B_{B, t-1})^\xi (V_{B, t})^{1-\xi}
\]

(5)

Among the newly matched workers in sector $j$ we assume there is a same probability they have the skill $A$ or $B$. In other words, the fraction of newly matched workers that are of type $A$ is just the fraction of type $A$ unemployed workers looking for a job in that sector. Also assuming that workers hired at period $t$ can produce immediately, the law of motion for employment of type-$A$ workers are

\[
N^A_{A, t} = (1 - \delta)N^A_{A, t-1} + \frac{U^A_{A, t-1}}{U^A_{A, t-1} + (1 - \tau)U^B_{A, t-1}}m_A(U^A_{A, t-1} + (1 - \tau)U^B_{A, t-1}, V_{A, t})
\]

(6)

\[
N^A_{B, t} = (1 - \delta)N^A_{B, t-1} + \frac{(1 - \tau)U^A_{B, t-1}}{(1 - \tau)U^A_{B, t-1} + U^B_{B, t-1}}m_B((1 - \tau)U^A_{B, t-1} + U^B_{B, t-1}, V_{B, t})
\]

(7)
We define the market tightness for each sector as

\[
\begin{align*}
\theta_{A,t} &= \frac{V_{A,t}}{U_{A,t-1}^A + (1 - \tau)U_{A,t-1}^B} \\
\theta_{B,t} &= \frac{V_{B,t}}{1 - \tau)U_{B,t-1}^A + U_{B,t-1}^B}
\end{align*}
\] (8) (9)

The probability that an unemployed worker is hired in each sector is

\[
\begin{align*}
p(\theta_{A,t}) &= \frac{m(U_{A,t-1}^A + (1 - \tau)U_{A,t-1}^B, V_{A,t})}{U_{A,t-1}^A + (1 - \tau)U_{A,t-1}^B} \\
p(\theta_{B,t}) &= \frac{m((1 - \tau)U_{B,t-1}^B + U_{B,t-1}^B, V_{B,t})}{(1 - \tau)U_{B,t-1}^A + U_{B,t-1}^B}
\end{align*}
\] (10) (11)

and the probability that a vacancy is filled in each sector is

\[
\begin{align*}
q(\theta_{A,t}) &= \frac{m(U_{A,t-1}^A + (1 - \tau)U_{A,t-1}^B, V_{A,t})}{V_{A,t}} \\
q(\theta_{B,t}) &= \frac{m((1 - \tau)U_{B,t-1}^B + U_{B,t-1}^B, V_{B,t})}{V_{B,t}}
\end{align*}
\] (12) (13)

Using these new variables we can rewrite (6) and (7) as

\[
\begin{align*}
N_{A,t}^A &= (1 - \delta)N_{A,t-1}^A + p(\theta_{A,t})U_{A,t-1}^A \\
N_{A,t}^B &= (1 - \delta)N_{A,t-1}^B + p(\theta_{A,t})(1 - \tau)U_{A,t-1}^B
\end{align*}
\] (14) (15)

Samely, the law of motion for employment of type-\(B\) workers are

\[
\begin{align*}
N_{A,t}^B &= (1 - \delta)N_{A,t-1}^B + p(\theta_{A,t})(1 - \tau)U_{A,t-1}^B \\
N_{B,t}^B &= (1 - \delta)N_{B,t-1}^B + p(\theta_{B,t})U_{B,t-1}^B
\end{align*}
\] (16) (17)

And the total employment in each sector evolves as

\[
\begin{align*}
N_{A,t} &= (1 - \delta)N_{A,t-1} + q(\theta_{A,t})V_{A,t} \\
N_{B,t} &= (1 - \delta)N_{B,t-1} + q(\theta_{B,t})V_{B,t}
\end{align*}
\] (18) (19)
2.2 Household

The household is composed of type-A and type-B workers. We follow the literature that incorporates unemployment into New Keynesian models and assume that workers are able to insure against idiosyncratic consumption risk.\(^8\)

The household has CRRA time additively separable preferences, with discount factor \(\beta\) and intertemporal elasticity of substitution \(\sigma\)

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \quad (20)
\]

\[
u(C_t) = \frac{(C_t)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \quad (21)
\]

Within each period she consumes goods from the two sectors according to CES preferences

\[
C_t = [\lambda_{A,t}(c_{A,t})^{\frac{1-\varepsilon}{1-\sigma}} + \lambda_{B,t}(c_{B,t})^{\frac{1-\varepsilon}{1-\sigma}}]^{\frac{1}{1-\varepsilon}} \quad (22)
\]

where \(\varepsilon\) is the intratemporal elasticity of substitution, and \(\lambda_{j,t}\) is a demand preference for good \(j\). We assume that demand preference \(\lambda_{A,t}\) follow AR(1) process

\[
\log \left( \frac{\lambda_{A,t}}{\lambda_A} \right) = \rho \log \left( \frac{\lambda_{A,t-1}}{\lambda_A} \right) + \varepsilon_{A,t}^\lambda \quad (23)
\]

\[
\varepsilon_{A,t}^\lambda \sim N(0, (\sigma_A^\lambda)^2) \quad (24)
\]

where \(\varepsilon_{A,t}^\lambda\) is a shock specific to sector \(A\).

The household’s problem is to decide consumption of each type of good, her savings in nominal risk free bonds \((B_t)\) subject to her budget constraint, and the number of unemployed workers to look for jobs in each sector \((U_{A,t}^A, U_{B,t}^A, U_{A,t}^B, U_{B,t}^B)\) subject to law of motion for employment. If we denote by \(p_{A,t}\) the price in dollars of one good \(A\), and by \(p_{B,t}\) the price of one good \(B\), then preference (22) implies that the price \((P_t)\) of one unit of aggregate consumption \((C_t)\) is

\[
P_t = [(\lambda_{A,t})^\varepsilon (p_{A,t})^{1-\varepsilon} + (\lambda_{B,t})^\varepsilon (p_{B,t})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (25)
\]

\(^8\)Andolfatto (1996) models an institutional structure that allows workers to insure themselves against earning uncertainty and unemployment. Basically the wage earnings have to be interpreted as net of insurance costs.
and the inflation rates are

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

(26)

$$\pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}}, \quad j = A, B$$

(27)

Moreover, household’s demand for goods $j$ satisfies

$$c_{j,t} = (\lambda_{j,t})^x \left( \frac{p_{j,t}}{P_t} \right)^{-x} C_t$$

(28)

The household’s budget constraint in units of aggregate consumption is

$$\frac{B_t}{P_t} + C_t \leq R_{t-1} \frac{B_{t-1}}{P_t} + w_{A,t}(N_{A,t}^A + N_{A,t}^B) + w_{B,t}(N_{B,t}^A + N_{B,t}^B) + \Theta_{A,t} + \Theta_{B,t}$$

(29)

where $\Theta_{j,t}$ are the profits of the firm in sector $j$, $w_{j,t}$ are wages and $R_{t-1}$ is the nominal interest rate between period $t-1$ and $t$.

### 2.3 Firms

In each sector $j$ there is a representative firm that takes wages as given and behaves as a monopolist in the good that it produces. It chooses prices $(p_{j,t})$, employment $(N_{j,t})$ and vacancies $(V_{j,t})$ to maximize the discounted value of its profits

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ \frac{p_{j,t}}{P_t} y_{j,t} - w_{j,t} N_{j,t} - \frac{p_{j,t}}{P_t} \kappa V_{j,t} - \frac{p_{j,t}}{P_t} \cdot \frac{\psi}{2} \left( \frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 y_{j,t} \right]$$

(30)

where $\frac{\psi}{2} \left( \frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 y_{j,t}$ is the cost of adjusting prices and $\psi$ is the sluggishness in the price adjustment process. The variable $\Lambda_{0,t}$ is the household’s stochastic discount factor to be defined in next section. The parameter $\kappa$ is the cost of posting a job vacancy.

If the search process is successful, the firm produces according to a technology that is linear in labor

$$y_{j,t} = z_{j,t} N_{j,t}$$

(31)

where $z_j = 1$ is a productivity scale parameter for sector $j$.

The firm’s problem in sector $j$ is to maximize (30) subject to law of motion (18) (or equation
(19)), (31), and the equation that internalizes the pricing power of the firm

\[ y_{j,t} = \left( \lambda_{j,t} \right)^{\xi} \left( \frac{p_{j,t}}{P_t} \right)^{-\xi} C_t \]  

(32)

### 2.4 Wage determination

Wages are obtained through Nash Bargaining, i.e., in sector \( j \) the wages solve

\[ \max_{w_{j,t}} (J_{j,t})^{1-\eta}(W_{j,t} - T_t)^\eta \]  

(33)

where \( \eta \) is the worker’s bargaining power. In general form, \( J_{j,t} \) is the value of an additional worker for the firm in sector \( j \). \( W_{j,t} \) is the value of an additional worker being employed in sector \( j \). \( T_t \) is the value of an additional worker being unemployed. The next equations define these values in detail.

For the firm in sector \( j \):

\[ J_{j,t} = \mu_{j,t}z_{j,t} - w_{j,t} + E_t \Lambda_{t,t+1} \left\{ (1 - \delta)(J_{j,t+1}) \right\} \]  

(34)

where \( \mu_{j,t} \) denotes the Lagrange Multiplier associated with constraint (32). Free entry implies that the value of an additional vacancy for the firm must be zero.

For type-A and type-B workers employed in sector \( j \):

\[ W_{j,t}^A = w_{j,t} + E_t \Lambda_{t,t+1} \left\{ (1 - \delta)W_{j,t+1}^A + \delta T_{t+1}^A \right\} \]  

(35)

\[ W_{j,t}^B = w_{j,t} + E_t \Lambda_{t,t+1} \left\{ (1 - \delta)W_{j,t+1}^B + \delta T_{t+1}^B \right\} \]  

(36)

For unemployed workers, they make decision on which sector to search in,

\[ T_{t}^A = \max \left( \frac{E_t \Lambda_{t,t+1} \{ p(\theta_{A,t+1})W_{A,t+1}^A + [1 - p(\theta_{A,t+1}) + 1 - \delta)]T_{t+1}^A \}}{E_t \Lambda_{t,t+1} \{ p(\theta_{B,t+1})(1 - \tau)W_{B,t+1}^A + [1 - p(\theta_{B,t+1})(1 - \tau)]T_{t+1}^A \}} \right) \]  

(37)

\[ T_{t}^B = \max \left( \frac{E_t \Lambda_{t,t+1} \{ p(\theta_{B,t+1})(1 - \tau)W_{B,t+1}^A + [1 - p(\theta_{B,t+1})(1 - \tau)]T_{t+1}^B \}}{E_t \Lambda_{t,t+1} \{ p(\theta_{B,t+1})(1 - \tau)W_{B,t+1}^B + [1 - p(\theta_{B,t+1})]T_{t+1}^B \}} \right) \]  

(38)

Given our negative demand shock in sector \( A \), type-A workers will look for jobs in both sec-

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9We will assume that the firm’s bargaining power is equal to the elasticity of the matching function with respect to vacancies (\( \xi = \eta \)). This is Hosios (1990) condition for firms to internalize the congestions that they create in the labor market in a way that leads them to post the efficient number of vacancies.
tors (they are indifferent to either sector), while type-B workers only prefer to search in sector B. Then the above two equations (37) and (38) can be rewritten as

$$E_t \Lambda_{t,t+1} \{ p(\theta_{A,t+1})(W_{A,t+1}^A - T_{t+1}^A) \} = E_t \Lambda_{t,t+1} \{ p(\theta_{B,t+1})(1 - \tau)(W_{B,t+1}^A - T_{t+1}^A) \}$$ (39)

$$T_t^B = E_t \Lambda_{t,t+1} \{ p(\theta_{B,t+1}) W_{B,t+1}^B + [1 - p(\theta_{B,t+1})] T_{t+1}^B \}$$ (40)

So wages in sector A ($w_{A,t}$) is obtained through

$$\max_{w_{A,t}} (J_{A,t})^{1-n} (W_{A,t}^A - T_{t+1}^A)^n$$ (41)

which implies that in sector A, there are only type-A workers.

While wages in sector B ($w_{B,t}$) is obtained through

$$\max_{w_{B,t}} (J_{B,t})^{1-n} \left[ \frac{N_{B,t}^A}{N_{B,t}^A + N_{B,t}^B} (W_{B,t}^A - T_{t+1}^A) + \frac{N_{B,t}^B}{N_{B,t}^A + N_{B,t}^B} (W_{B,t}^B - T_{t+1}^B) \right]^n$$ (42)

which implies that in sector B, there are both type-A and type-B workers.

### 2.5 Monetary Policy Rules

Interest targeting rules are widely employed in New Keynesian models because such rules are thought to provide both a good positive description of, and a normative prescription for, monetary policy (Ang et al. 2007, Clarida et al. 2000, Taylor 1993, Taylor and Williams 2010, Woodford 2001). We investigate the performance of two alternative types of nominal interest rate rules:

$$R_t = R_{t-1}^{\phi_r} \left[ R^* \left( \frac{\Pi_{t-1}}{\Pi^*} \right)^{\phi_u} \left( \frac{1 - \bar{u}_{t-1}}{1 - \bar{u}^*} \right)^{\phi_u} \right]^{1-\phi_r}$$ (43)

1) Rules focused on aggregate variables:

$$\Pi_t = \Pi_t$$ (44)

$$\bar{u}_t = u_t \equiv \frac{\sum_{i=A,B} U^i_t}{\sum_{i=A,B} L^i}$$ (45)
2) Rules adjusted for sectorial shocks:

\[
\Pi_t = \{\pi_{A,t}, \pi_{B,t}\}
\]

\[
\bar{u}_t = \{u^A_t, u^B_t\}
\]

where \(u_t\) and \(u^i_t\) are respectively the aggregate and worker type specific unemployment rates. Variables with asterisk denote steady-state values. The parameter \(\phi_r\) is the degree of policy rate smoothing, \(\phi_{\Pi}\) and \(\phi_u\) are non-negative policy coefficients on aggregate or sectorial inflation and unemployment rates.

\section{Calibration}

We calibrate the model following Faia (2008). We set a period to be a quarter. There are three sets of parameters to choose:

- **Preferences**: We set the discount factor \(\beta = 0.99\), which implies an annual interest rate of 4\%. As in most of the real business cycle literature we assume \(\sigma = 0.5\), which under CRRA preferences implies a value for risk aversion of 2. Concerning the intratemporal elasticity of substitution between goods in two sectors \((\varepsilon)\), we assume \(\varepsilon = 1.17\). A number in the ballpark of those assigned to the elasticity of substitution between housing and other goods. At first we assume the same share of consumption in each good (the steady state \(\lambda_A = \lambda_B = \frac{1}{2}\)), but later give more weight to the consumption of good \(B\) \((\lambda_A = 0.4, \lambda_B = 0.6)\). We assume the persistence of the demand shock \(\rho = 0.95\).

- **Technology**: As discussed in Krause and Lubik (2003), the choice for the value of price adjustment cost \(\psi\) ranges from 20 to 105. We choose an intermediate value \(\psi = 40\).

- **Labor Market**: We assume there are type-\(A\) and type-\(B\) workers with the same amount of labor force \(L^A = L^B = 1\). We assume the bargaining power of the workers to be \(\eta = 0.5\). To satisfy the Hosios condition we set the elasticity of the unemployed in the matching function to \(\xi = 0.5\). The exogenous job separation rate \(\delta\) is assumed 0.10. These last values are standard in the literature (Faia 2008). The scale parameters of the matching function \((m_A, m_B)\) and the cost of posting a vacancy \((\kappa)\) are calibrated, to match a steady-state unemployment rate of 10\% in each sector and a steady-state probability that a vacancy is filled \(\bar{q}(\theta) = 0.9\). For comparison purpose, we let the mismatch friction \(\tau\) to range from 0 to 1.

\(^{10}\)See Gete (2010) for references
4 Results

4.1 Mismatch Friction

This section examines the dynamic of variables in response to an increasing mismatch friction, both under steady-state conditions and facing a negative demand shock. We set parameters $\lambda_A = 0.4$ and $\lambda_B = 0.6$ (the household prefers consumption of good $B$), so that a fraction of type-$A$ workers look for jobs in sector $B$ (and incur mismatch frictions), while type-$B$ workers never look for jobs in sector $A$.

Figure 1 shows the change in selected variable values under steady-state condition, when the parameter for mismatch friction ($\tau$) ranges from 0 to the value at which there is no cross-sector job search. Facing an increasing $\tau$, unemployed type-$A$ workers are less willing to look for jobs in sector $B$, and fewer of them manage to enter sector $B$’s labor market so that the proportion of type-$A$ workers employed in sector $B$ keeps decreasing. In turn, unemployed type-$A$ workers are more willing to search in sector $A$, and more of them get employed. On the other hand, total number of unemployed workers (type-$A$ and type-$B$ combined) searching in sector $B$ decreases, then there are fewer employed workers in sector $B$. The production and consumption of good $A$ increases while the production and consumption of good $B$ decreases, causing aggregate consumption to decrease.

The aggregate unemployment rate displays an "inverse U" shape, which means the total number of unemployed first increases along with mismatch friction ($\tau$) until a maximum level, then starts to decrease with mismatch friction. This is mainly due to the number of unemployed type-$A$ workers who fail to reach sector $B$’s labor market ($\tau \cdot U_{AB}$, those who get unemployed due to mismatch). This number also displays an "inverse U" shape: first goes up along with an increasing $\tau$, later decreases due to a shrinking $U_{AB}$.

Figure 2 shows impulse responses of selected variables to a negative demand shock in sector $A$, under different mismatch frictions ($\tau = 0.1$ v.s. $\tau = 0.4$). Here monetary policy targets
aggregate variables ($\Pi_t$ and $u_t$). As shown, unemployment fluctuation is larger, and interest rate adjustment is larger, for higher value of $\tau$. This implies that when mismatch friction is large, facing a negative demand shock to sector $A$, there are more unemployment in type-$A$ workers when they decide to search for jobs in sector $B$. And the monetary authority has to adjust their interest rate more facing a higher unemployment.

Insert Figure 2 here

4.2 Monetary Policy

This section examines the impulse responses of several key variables to a negative demand shock in sector $A$, under different monetary policy rules. There are three monetary policy rules that respond to inflation and unemployment: 1) Targeting aggregate inflation ($\Pi_t$) and aggregate unemployment ($u_t$); 2) Targeting inflation in sector $A$ ($\pi_t^A$) and unemployment rate of type-$A$ workers ($u_t^A$); 3) Targeting inflation in sector $B$ ($\pi_t^B$) and unemployment rate of type-$B$ workers ($u_t^B$). All rules assume $\phi_\Pi = 1.5$, $\phi_u = 0.5$ and interest rate smoothing weight $\phi_r = 0.9$. Mismatch friction $\tau = 0.1$.

In Figure 3, after a negative demand shock in sector $A$, firms in sector $A$ lower good $A$’s price. They also cut off their production and vacancy posting, so the unemployment rate for type-$A$ workers increases. As consumers shift their relative demand to goods $B$, firms in sector $B$ increase their production, post more vacancies and hire more workers. The unemployment rate of type-$B$ worker decreases. For the whole economy, aggregate price decreases, which causes a negative change in inflation rate. This leads the central bank to lower the nominal interest rate.

Comparing three different monetary rules, we notice that policy rule 3 which focus on $\pi_t^B$ and $u_t^B$ causes the least deviation from steady state, while policy rule 2 which focus on $\pi_t^A$ and $u_t^A$ causes the largest deviation. The policy targeting aggregate variables falls in the middle.

Insert Figure 3 here

Figure 4 displays the Phillips curves corresponding to three monetary policy rules. We can conclude that the policy rule 3 which targeting $\pi_t^B$ and $u_t^B$ is better than the rest two, since with the same unemployment rate, this rule will cause the least inflation.

Insert Figure 4 here
5 Conclusions

To be added
Tables and Figures

Table 1: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor: $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Intratemporal Elasticity of Substitution: $\varepsilon$</td>
<td>1.17</td>
</tr>
<tr>
<td>Intertemporal Elasticity of Substitution: $\sigma$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Demand for good A: $\lambda_A$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Demand for good B: $\lambda_B$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Labor force: $L^A = L^B$</td>
<td>1</td>
</tr>
<tr>
<td>Scale parameter for matching function: $m_A, m_A$</td>
<td>0.9</td>
</tr>
<tr>
<td>Elasticity of unemployed in matching function: $\xi$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Job separation rate: $\delta$</td>
<td>0.10</td>
</tr>
<tr>
<td>Vacancy posting cost: $\kappa$</td>
<td>0.128</td>
</tr>
<tr>
<td>Worker's bargaining power: $\eta$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Sluggishness in price adjustment process: $\psi$</td>
<td>40</td>
</tr>
<tr>
<td>Autoregressive coefficient demand shock: $\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>Mismatch friction: $\tau$</td>
<td>[0, 1]</td>
</tr>
</tbody>
</table>

Notes: This table shows the parameters used to numerically solve the model and construct the figures of Section 5. The parameter values are discussed in page 9.
Figure 1: Steady-state conditions with increasing mismatch friction $\tau$
Figure 2: Impulse responses to negative demand shock in sector A for different mismatch frictions
Figure 3: Impulse responses to negative demand shock in sector A for Taylor Rules. The solid, dashed, and dotted line correspond to Taylor rules that target aggregate variables, Sector A & type-A worker variables and Sector B & type-B worker variables, respectively.
Figure 4: Phillips Curve for different monetary policy rules.
References


The Economist: Aug 26th 2010, There is more to America’s stubbornly high unemployment rate than just weak demand.


7 Appendix

7.1 Equilibrium conditions

In the competitive equilibrium, household and firms optimize, the Central Bank follows the policy rule (43) and markets clear.

Household’s optimal consumption and savings are determined by (28), the Euler equation

\[ u'(C_t) = \beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\} \]  

(48)

and the budget constraints

\[ C_t = w_{A,t}(N_{A,t}^A + N_{A,t}^B) + w_{B,t}(N_{B,t}^A + N_{B,t}^B) + \Theta_{A,t} + \Theta_{B,t} \]  

(49)

From our CRRA assumption (21) we can define the household’s stochastic discount factor

\[ \Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \]  

(50)

Given our negative demand shock in sector A, the household’s optimal choices on the number of unemployed workers to apply to each sector are determined by the following three equations:

\[ mc^A_t = \beta E_t m_A(\theta_{A,t+1})^{1-\xi}[U'(C_{t+1})w_{A,t+1} - mc^A_{t+1}] \]  

(51)

\[ mc^A_t = \beta E_t m_B(\theta_{B,t+1})^{1-\xi}(1 - \tau) [U'(C_{t+1})w_{B,t+1} - mc^A_{t+1}] \]  

(52)

\[ mc^B_t = \beta E_t m_B(\theta_{B,t+1})^{1-\xi}[U'(C_{t+1})w_{B,t+1} - mc^B_{t+1}] \]  

(53)

\[ U_{U,t} = 0 \]  

(54)

where \( mc^i_t \) denotes the Lagrange Multiplier associated with labor force constraint (1), and is the utility value from an additional type-\( i \) worker.

The optimal choice of vacancies and prices for a firm in sector \( j \) is given by

\[ \frac{p_{j,t} - \frac{\kappa}{P_t q(\theta_{j,t})}}{P_t q(\theta_{j,t})} = \mu_{j,t} z_{j,t} - w_{j,t} + (1 - \delta)E_t\Lambda_{t,t+1} \left[ \frac{p_{j,t+1} - \frac{\kappa}{P_{t+1} q(\theta_{j,t+1})}}{P_{t+1} q(\theta_{j,t+1})} \right] \]  

(55)

\[ 1 - \varepsilon - (\lambda_{j,t})^{-\varepsilon} C_{t+1}^{-1}(\frac{p_{j,t+1}}{P_{t+1}})^\varepsilon V_{j,t} - \psi (\pi_{j,t} - 1) \pi_{j,t} - \frac{\psi}{2} (1 - \varepsilon) (\pi_{j,t} - 1)^2 \]  

\[ + \mu_{j,t} \varepsilon \left( \frac{p_{j,t}}{P_t} \right)^{-1} + E_t\Lambda_{t,t+1} \left[ \psi (\pi_{j,t+1} - 1) (\pi_{j,t+1})^{2-\varepsilon} (\Pi_t)^{\varepsilon-1} \left( \frac{\lambda_{j,t+1}}{\lambda_{j,t}} \right)^{\frac{\varepsilon}{\kappa}} \right] = 0 \]  

(56)
where we have used (28) to get rid of the sectoral consumption variables. Equation (55) implies that in equilibrium the cost of posting a vacancy must equate the discounted expected return of a filled vacancy.\footnote{As already noticed in Krause and Lubik (2007) in a new Keynesian model with matching frictions the marginal cost of firms is given by the marginal productivity of each single employee, and by the future value of current employees. Since posting vacancy is costly a successful match today is valuable as it reduces future search costs. The future value of current employees depends on the evolution of unemployment: if the number of searching workers increases, the probability of filling a future vacancy increases and the future value of current employees decreases.}

The solution to the Nash Bargaining problem in sector $A$ (41) gives the wage

$$w_{A,t} = \eta(\mu_{A,t} z_{A,t} + \kappa E_t \Lambda_{t,t+1} \frac{P_{A,t+1}}{P_{t+1}} \theta_{A,t+1})$$  \hspace{1cm} (57)$$

and the solution to the Nash Bargaining problem in sector $B$ (42) determines the wage through following equations:

$$\eta \left( \frac{p_{B,t}}{P_t} \frac{\kappa}{m_B(\theta_{B,t}) - \xi} \right) = (1 - \eta) \left[ \alpha_t \cdot SA_t + (1 - \alpha_t) \cdot SB_t \right]$$  \hspace{1cm} (58)$$

$$\left[ \alpha_t \cdot SA_t + (1 - \alpha_t) \cdot SB_t \right] = w_{B,t} + \alpha_t \cdot E_t \Lambda_{t,t+1} \left\{ \left[ 1 - \delta - m_B(\theta_{B,t+1})^{1-\xi}(1 - \tau) \right] \right. SA_{t+1} \right\}$$

$$+ (1 - \alpha_t) \cdot E_t \Lambda_{t,t+1} \left\{ \left[ 1 - \delta - m_B(\theta_{B,t+1})^{1-\xi} \right] SB_{t+1} \right\}$$

$$E_t \Lambda_{t,t+1} \left( \eta \frac{p_{A,t+1}}{1 - \eta \frac{p_{A,t+1}}{P_{t+1}}} \kappa \theta_{A,t+1} \right) = E_t \Lambda_{t,t+1} \left( (1 - \tau)m_B(\theta_{B,t+1})^{1-\xi} SA_{t+1} \right)$$  \hspace{1cm} (59)$$

$$\alpha_t = \frac{N_{A,t}^A}{N_{A,t}^A + N_{B,t}^B}$$  \hspace{1cm} (60)$$

where $SA_t = W_{B,t}^A - T_t^A$ and $SB_t = W_{B,t}^B - T_t^B$ are the difference in value from an additional employed worker in sector $B$, compared to an additional unemployed worker.

Market Clearing in the sector $j$ is that production equals consumption plus the resources wasted in the search and matching process and the cost of adjusting prices

$$y_{j,t} = c_{j,t} + \kappa V_{j,t} + \frac{\psi}{2} \left( \frac{p_{j,t}}{p_{j,t-1}} - 1 \right) y_{j,t}$$  \hspace{1cm} (62)$$

Concerning the bond market we assume that they are in zero net supply.
7.2 System of Equations

7.2.1 Labor Markets

Labor force = employed + unemployed:

\[ L^A = U^A_{A,t} + N^A_{A,t} + N^B_{A,t} \]
\[ L^B = U^B_{A,t} + U^B_{B,t} + N^A_{B,t} + N^B_{B,t} \] (63) (64)

Market tightness:

\[ \theta_{A,t} = \frac{V_{A,t}}{U^A_{A,t-1} + (1 - \tau)U^B_{A,t-1}} \] (65)
\[ \theta_{B,t} = \frac{V_{B,t}}{(1 - \tau)U^A_{B,t-1} + U^B_{B,t-1}} \] (66)

Law of Motion for employment:

\[ N^A_{A,t} = (1 - \delta)N^A_{A,t-1} + m_A(\theta_{A,t})^{1-\xi}U^A_{A,t-1} \] (67)
\[ N^A_{B,t} = (1 - \delta)N^A_{B,t-1} + (1 - \tau)m_B(\theta_{B,t})^{1-\xi}U^A_{B,t-1} \] (68)
\[ N^B_{A,t} = (1 - \delta)N^B_{A,t-1} + (1 - \tau)m_A(\theta_{A,t})^{1-\xi}U^B_{A,t-1} \] (69)
\[ N^B_{B,t} = (1 - \delta)N^B_{B,t-1} + m_B(\theta_{B,t})^{1-\xi}U^B_{B,t-1} \] (70)

7.2.2 Household

Euler Equation:

\[ U'(C_t) = \beta R_tE_t \left\{ \frac{P_t}{P_{t+1}}U'(C_{t+1}) \right\} \] (71)

Unemployed workers’ choice on which sectors to apply to:

\[ mc^A_t = \beta E_t m_A(\theta_{A,t+1})^{1-\xi}[U'(C_{t+1})w_{A,t+1} - mc^A_{t+1}] \] (72)
\[ U^A_{B,t} = 0 \]
or
\[ mc^A_t = \beta E_t m_B(\theta_{B,t+1})^{1-\xi}(1 - \tau)[U'(C_{t+1})w_{B,t+1} - mc^A_{t+1}] \] (73)
\[ U^B_{A,t} = 0 \] (74)
Demands for good A and B:

\[ c_{A,t} = (\lambda_{A,t})^\varepsilon \left( \frac{p_{A,t}}{P_t} \right)^{-\varepsilon} C_t \tag{75} \]

\[ c_{B,t} = (\lambda_{B,t})^\varepsilon \left( \frac{p_{B,t}}{P_t} \right)^{-\varepsilon} C_t \tag{76} \]

Aggregate Price:

\[ P_t = [(\lambda_{A,t})^\varepsilon (p_{A,t})^{1-\varepsilon} + (\lambda_{B,t})^\varepsilon (p_{B,t})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{77} \]

Inflations:

\[ \Pi_t = \frac{P_t}{P_{t-1}} \tag{78} \]

\[ \pi_{A,t} = \frac{p_{A,t}}{p_{A,t-1}} \tag{79} \]

\[ \pi_{B,t} = \frac{p_{B,t}}{p_{B,t-1}} \tag{80} \]

### 7.2.3 Firms

Optimal choice for vacancies:

\[ \frac{p_{A,t}}{P_t} \frac{\kappa}{m_A(\theta_{A,t})^{-\varepsilon}} = \mu_{A,t} z_{A,t} - w_{A,t} + (1 - \delta) E_t \Lambda_{t,t+1} \left[ \frac{p_{A,t+1}}{P_{t+1}} \frac{\kappa}{m_A(\theta_{A,t+1})^{-\varepsilon}} \right] \tag{81} \]

\[ \frac{p_{B,t}}{P_t} \frac{\kappa}{m_B(\theta_{B,t})^{-\varepsilon}} = \mu_{B,t} z_{B,t} - w_{B,t} + (1 - \delta) E_t \Lambda_{t,t+1} \left[ \frac{p_{B,t+1}}{P_{t+1}} \frac{\kappa}{m_B(\theta_{B,t+1})^{-\varepsilon}} \right] \tag{82} \]

Optimal choice for prices:

\[ 1 - \varepsilon - (\lambda_{A,t})^{-\varepsilon} \kappa (\frac{p_{A,t}}{P_t})^\varepsilon V_{A,t} (C_t)^{-1} - \psi (\pi_{A,t} - 1) \pi_{A,t} - \frac{\psi}{2} (1 - \varepsilon) (\pi_{A,t} - 1)^2 \]
\[ + \mu_{A,t}^\varepsilon \left( \frac{p_{A,t}}{P_t} \right)^{-1} + E_t \Lambda_{t,t+1} \left[ \psi (\pi_{A,t+1} - 1) (\pi_{A,t+1})^{2-\varepsilon} (\Pi_{t+1})^{\varepsilon-1} \left( \frac{\lambda_{A,t+1}}{\lambda_{A,t}} \right)^\varepsilon \frac{C_{t+1}}{C_t} \right] = 0 \tag{83} \]

\[ 1 - \varepsilon - (\lambda_{B,t})^{-\varepsilon} \kappa (\frac{p_{B,t}}{P_t})^\varepsilon V_{B,t} (C_t)^{-1} - \psi (\pi_{B,t} - 1) \pi_{B,t} - \frac{\psi}{2} (1 - \varepsilon) (\pi_{B,t} - 1)^2 \]
\[ + \mu_{B,t}^\varepsilon \left( \frac{p_{B,t}}{P_t} \right)^{-1} + E_t \Lambda_{t,t+1} \left[ \psi (\pi_{B,t+1} - 1) (\pi_{B,t+1})^{2-\varepsilon} (\Pi_{t+1})^{\varepsilon-1} \left( \frac{\lambda_{B,t+1}}{\lambda_{B,t}} \right)^\varepsilon \frac{C_{t+1}}{C_t} \right] = 0 \tag{84} \]

Production Technology:

\[ y_{A,t} = z_{A,t} (N_{A,t}^A + N_{A,t}) \tag{85} \]

\[ y_{B,t} = z_{B,t} (N_{B,t}^A + N_{B,t}) \tag{86} \]
7.2.4 Wage Determination

Wages in sector $A$:

$$w_{A,t} = \eta (\mu_{A,t} \tilde{z}_{A,t} + \kappa E_t \Lambda_{t,t+1} \frac{P_{A,t+1}}{P_{t+1}} \theta_{A,t+1})$$  \hspace{1cm} (87)

Wages in sector $B$ is determined by the following four equations:

$$\eta \left( \frac{p_{B,t}}{P_t} m_B (\theta_{B,t})^{-\xi} \right) = (1 - \eta) [\alpha_t \cdot SA_t + (1 - \alpha_t) \cdot SB_t]$$ \hspace{1cm} (88)

$$[\alpha_t \cdot SA_t + (1 - \alpha_t) \cdot SB_t] = w_{B,t}$$

$$\alpha_t \cdot E_t A_{t,t+1} \left\{ 1 - \delta - m_B (\theta_{B,t+1})^{1-\xi} (1 - \tau) \right\} SA_{t+1} + (1 - \alpha_t) \cdot E_t A_{t,t+1} \left\{ 1 - \delta - m_B (\theta_{B,t+1})^{1-\xi} \right\} SB_{t+1}$$ \hspace{1cm} (89)

$$E_t A_{t,t+1} \left( \frac{\eta p_{A,t+1}}{1 - \eta} \right) = E_t A_{t,t+1} \left( (1 - \tau) m_B (\theta_{B,t+1})^{1-\xi} S A_{t+1} \right)$$ \hspace{1cm} (90)

$$\alpha_t = \frac{N_{A,t}}{N_{B,t} + N_{A,t}}$$ \hspace{1cm} (91)

7.2.5 Interest Rules

$$R_t = R_{t-1}^\phi \left[ R^* \left( \frac{\bar{\Pi}_{t-1}}{\bar{\Pi}^*} \right) \phi_{u_t} \left( \frac{1 - \bar{u}_t}{1 - \bar{u}^*} \right)^{\phi_{u_t} - 1} \phi_{u_t} \right]$$ \hspace{1cm} (92)

7.2.6 Shock to demands

$$\log \left( \frac{\lambda_{A,t}}{\lambda_A} \right) = \rho \log \left( \frac{\lambda_{A,t-1}}{\lambda_A} \right) + \varepsilon_{A,t}$$ \hspace{1cm} (93)

7.2.7 Market clearing condition

$$y_{A,t} = c_{A,t} + \kappa V_{A,t} + \frac{\psi}{2} \left( \frac{P_{A,t}}{P_{A,t-1}} - 1 \right)^2 y_{A,t}$$ \hspace{1cm} (94)

$$y_{B,t} = c_{B,t} + \kappa V_{B,t} + \frac{\psi}{2} \left( \frac{P_{B,t}}{P_{B,t-1}} - 1 \right)^2 y_{B,t}$$ \hspace{1cm} (95)