Abstract

In this paper, based on Diebold and Li (2003)’s term structure model, we derive a short interest rate model with two latent macro variables by taking into account monetary policies, namely, the McCallum Rule and the Taylor Rule. Given the nature of the model, we execute a two-step Kalman filter so as to not only estimate the inflation gap and the output gap but also forecast the future short interest rate. The model’s performance is superior to the random walk model and the Diebold and Li’s model.

JEL Codes: E43, E44, E47, E52.

Keywords: Term Structure, McCallum Rule, Taylor Rule, Short-term interest rate
The Term Structure, Monetary Policy Rules and Short-Term Interest Rate Forecasting: A Dynamic Latent Factor Model

1. Introduction

In this paper, the primary goal is to incorporate the central bank’s (CB) reaction functions into a term structure model, which is introduced by Diebold and Li (2003). By doing so, we are able to obtain a better forecasting ability on the future short-term rates (the short-term interest rate) due to the full recognition of information considered by the CB when the monetary policy is made. It is worth noting that we are considering the CB’s reaction functions responding not only to the variance of several major macroeconomic indicators (the Taylor Rule) but also to the changes in the financial market, namely, the bond spread and the short-term rate (the McCallum Rule).

This paper makes contributions in two dimensions. First, the mix of term structure model and the monetary policies enables us to generate a future-short-rate equation which differentiates from the most commonly used latent term structure model in several aspects. This model is capable of incorporating more information utilized when the expectation of future short rate is formed. Furthermore, unlike other well accepted latent factor models, the two latent factors considered in this model exhibit considerable macroeconomic implications. More specifically, they represent the deviation from the realized inflation rate to the CB’s inflation target and the realized output to the potential output, respectively.

Second, given the nature of the state-space equation that this model spins, we use a two-step Kalman filter method to estimate the model. Thus, with the consideration of all information available to us, we are able to achieve a better forecasting result.

For the sake of comparison, we also estimate the Diebold and Li (2003) term structure model as well as a random walk term structure model at the very end of this paper. Based on these two models, the in-sample one-month-ahead forecasts are performed so as to evaluate the forecast power of our model. In favor of our model, the comparison tells an
interesting story.

The reminder of this paper is organized as follows. In section 2, we review the current literature associated with the term structure studying. In section 3, we briefly discuss the theoretical model used in this paper. Section 4 and section 5 introduce the data source that we will use and our empirical strategy, respectively. In section 6, we present our results and section 7 concludes.

2. Reviewed Literatures

It is widely accepted that the shape of the yield curve embodies information on changes in major macroeconomic variables and in monetary policy rules.\(^1\) Thus, modeling the shape of the yield curve might offer a better understanding of the correlations between the variations of macroeconomic variables and the monetary policy manipulations. In this regard, most researchers claim that the shape of the yield curve is determined by three unobservable factors, which could be portrayed as level, slope and curvature factors, respectively. First, the level factor governs the shift of the whole yield curve. It has been found that changes in inflation level have considerable effects on the shift of the yield curve. Second, the slope factor determines the slope of the yield curve and it is believed to be closely related to the spread between the long-term yield and the short-term yield. Third, the curvature factor is used to picture the changes in the yields on the bonds with medium term. The correlation between the curvature factor and the major macroeconomic indicators has not been found yet.

The most popular term structure model is so-called affine model (Nelson and Siegel (1987), Duffine and Kan (1996), Dai and Singleton (2000)). The affine model is essentially a latent factor model. It requires a considerably small dimension of factors in the model so that the modeling process is easier to handle. Another attractive feature of it is that the stochastic volatility of the term premia could be easily considered and modeled. Hence, it could be used to test the validity of the “Expectation Hypothesis” by assuming that the term premia is time-varying instead. However, the criticisms have been raised

\(^1\) See Poole (2005), Theoret et al. (2004) and Bernanke (2006).
regarding two major pitfalls of the affine model. First, the affine model does poorly in the out-sample forecasting. Second, the affine model merely delivers insights on the economic inferences of those latent factors.

In response to the first criticism, Diebold and Li (2003) revise the Nelson-Siegel (1987) model. The new model is also a latent variable model in which three time-varying parameters may be interpreted as factors corresponding to the level, slope and curvature of the yield curve. Nonetheless, the Diebold and Li’s model is more straightforward and has a better fit when forecasting the yield curve. Especially, it shows much more satisfactory results to affine model in the out-sample forecasting. In response to the second criticism, Diebold et al. (2005) argue that the parameters in Diebold and Li (2003) are closely related to macro variables.

However, Diebold and Li (2003) find that their model’s ability of forecasting deteriorates when they forecast the 1-month-ahead yield curve. They believe that the larger RMSE in the 1-month-ahead forecast is attributable to the pricing errors caused by the illiquidity. The accusation is reasonable. But lack of the CB’s monetary policy information may also leave Diebold and Li’s model open to critics. If the “Expectation Hypothesis” is somehow correct in revealing the relationship between the yield and the expected short-term rate, a change in the CB’s policy will definitely alter the shape of the yield curve.

Therefore, in the interest of answering this question, it is natural to exploit the relationships between macroeconomic variables and the latent factors considered in term structure model. A large body of literature on this subject has been developed. Based on the revised Nelson-Siegel model, Diebold et al. (2005) incorporate three macroeconomic variables (manufacturing capacity utilization, federal funds rate and annual price inflation) and find strong evidence of macroeconomic influences on the shape of the yield curve. Bekaert et al. (2005) construct a structural macro model with an affine term structure model to estimate the inflation gap and the output gap. The inclusion of the yield curve encourages them to claim that the use of term structure can provide more monetary authority’s information to the macro model. Thus, the estimation might be more reliable.
Bikbov and Chernov (2005) decompose the term structure model into a model resembling the Taylor rule. By adding more macro variables, they can perform a better forecasting of future short rates. Dewachter and Lyrio (2003) relate the affine model to three macro variables and use Kalman filter to estimate the long-term yield. With the help of the Extended Kalman Filter, Theoret et al. (2004) estimate a two-factor affine model and capture the unobserved stochastic volatility.

In this paper, based on Diebold and Li (2003)’s term structure model, the CB’s reaction functions responding not only to the variances in the macroeconomic variables but also to changes in financial factors (short-term rate and bond spread) will be taken into account. As a result, we will have a short-rate equation which is associated with two macroeconomic variables. In addition, with help of the Diebold and Li model, the McCallum rule and the Taylor rule, a state-space equation system can be formed. Therefore, the forecasting of the future short rate is able to be conducted by the Kalman filter.

3. Theoretical Model

The essential idea of this paper is to combine a term structure model (Diebold and Li’s model) with two monetary policy rules in order to obtain an equation of short rate. The first rule we consider here is called the McCallum Rule. It models the CB’s reaction function in the financial market. Specifically, it shows the willingness of the CB to adjust the short rates in response to the changes of the bonds’ spreads and the desire to smooth the interest rates over time. The second rule is called the Taylor Rule, in which the presence of the macroeconomic variables exhibits the reactions of CB to the variations of Macro factors. The combination of Diebold and Li’s model with these two monetary policy rules eventually will help us to generate a state-space model. The use of the two-step Kalman filter will allow us to forecast the future short rate with more information taken into account.

Diebold and Li (2003)’s revised Nelson-Siegel model

The basic model that Diebold and Li have laid out is in the following fashion:
Where, $\beta_1$ denotes level factor, $\beta_2$ stands for slope factor and $\beta_3$ is the curvature factor.

We also assume three latent variables are following an AR(1) transition process.

$$\beta_t = C \beta_{t-1} + \epsilon_t$$

Following Gallmeyer, Hollifield and Zin (2005), we convert the Diebold and Li’s model into an equation of short rate. First, given equation (1), we can introduce bond spread into our model

$$r_t^{(s)} = \beta_{1t} + \beta_{2t} \left( \frac{1-e^{\lambda t}}{\lambda t} \right) + \beta_{3t} \left( \frac{1-e^{-\lambda t}}{\lambda t} - e^{-\lambda t} \right)$$

(1)

Let $M$ denote $(s(s's)^{-1}s')^{-1}(sc(s's)^{-1}s')_{n\times n}$, $a$ stands for the coefficient of the error term and $spread$ represents all bond spreads. In order to generate an equation relating the short-rate at $t$ to the short-rate at $t-1$, equation (2.2) can be partitioned as:

$$
\begin{bmatrix}
  r_t \\
  r_t^2 - r_t \\
  r_t^3 - r_t \\
  \vdots \\
  r_t^n - r_t \\
\end{bmatrix}
= (s(s's)^{-1}s')^{-1}(sc(s's)^{-1}s')_{n\times n}
\begin{bmatrix}
  r_{t-1} \\
  r_{t-1}^2 - r_{t-1} \\
  r_{t-1}^3 - r_{t-1} \\
  \vdots \\
  r_{t-1}^n - r_{t-1} \\
\end{bmatrix}
+ (s(s's)^{-1}s')^{-1}s \epsilon_t
$$

(2.2)
\[
\begin{pmatrix}
    r_t \\
    \text{spread}_{(n-1)\text{d}}
\end{pmatrix}_{\text{nod}} = 
\begin{pmatrix}
    M_{11} & M_{12} \\
    M_{21} & M_{22}
\end{pmatrix} 
\begin{pmatrix}
    r_{t-1} \\
    \text{spread}_{(n-1)\text{d}}
\end{pmatrix}_{\text{nod}} + \begin{pmatrix}
    a_t \\
    a_{2t}
\end{pmatrix} + \varepsilon_t
\]
(2.3)

From equation (2.3), the short-rate at time \( t \) can be solved with respect to its previous value and all bond spreads at time \( t \).

\[
r_t = (M_{11} - M_{12}M_{22}^{-1}M_{21})r_{t-1} + M_{12}M_{22}^{-1}\text{spread}_t + (a_t - M_{12}M_{22}^{-1}a_{2t})\varepsilon_t
\]
(3)

McCallum Rule

Next, we consider the CB’s reaction rule in the financial market. The McCallum rule suggests that the CB’s reaction will target on the last period short rate and also intends to smooth the spread between the long-term yield and the short-term rate. Consistent with Gallmeyer et al. (2005) and Razzak (2001), the McCallum Rule can be written as:

\[
r_t = \mu_r r_{t-1} + \left( \begin{array}{cccc}
    \mu_{f1} & \mu_{f2} & \cdots & \mu_{f(n-1)} \\
    \end{array} \right) \begin{pmatrix}
    r_t^{(2)} - r_t \\
    r_t^{(3)} - r_t \\
    \vdots \\
    r_t^{(n)} - r_t
\end{pmatrix} + w_t
\]
(4)

where, \( \mu_r \) and \( \left( \begin{array}{cccc}
    \mu_{f1} & \mu_{f2} & \cdots & \mu_{f(n-1)} \\
    \end{array} \right) \) are the policy rules of the CB in the financial market, \( w_t \) is assumed to have a normal distribution. Using notations in equation (3), equation (4) can be written as:

\[
r_t = \mu_r r_{t-1} + \left( \begin{array}{cccc}
    \mu_{f1} & \mu_{f2} & \cdots & \mu_{f(n-1)} \\
    \end{array} \right) \text{spread}_{(n-1)\text{d}} + w_t
\]
(4.1)

Combination of the McCallum Rule and the Diebold and Li’s Term Structure Model

Combining equation (3) and equation (4) will give a relationship between the short rate and its lagged value.
\[ r_t = H_i(\mu_r, \mu_f, \lambda, n)r_{t-1} + \gamma_t \]  

(5)

Where,

\[ A_{t+1} = (1 - M_{11}M_{22}^{-1}(\mu'_f, \mu_f)^{-1}\mu'_f, \mu_f)^{-1}(M_{11} - M_{12}M_{22}^{-1}M_{21} - M_{12}M_{22}^{-1}(\mu'_f, \mu_f)^{-1}\mu'_f, \mu_f)_{t+1} \]

\[ \gamma_{t+1} = (1 - M_{11}M_{22}^{-1}(\mu'_f, \mu_f)^{-1}\mu'_f, \mu_f)^{-1}b_t((a_t - M_{12}M_{22}^{-1}a_2)e_t - M_{12}M_{22}^{-1}(\mu'_f, \mu_f)^{-1}\mu'_f, \mu_f)_{t+1} \]

Equation (5) could be considered as a term structure model with a policy-motivated restriction. With incorporation of this monetary policy restriction, the Diebold and Li (2003) model may contain more information on the fluctuations of the short rate. In other words, this equation is supposed to have a better fit in the empirical study, particularly, in forecasting the short rate.

The Taylor Rule, The McCallum Rule and The Term Structure

However, it is obvious that with the exception of the Taylor rule, the information retained in our model is far from enough. Literally, lack of macroeconomic variables is the major pitfall of the term structure model. The CB will adjust the short-term rate with respect to the changes of macroeconomic variables. Consequently, the shape of the yield curve will be influenced by the changes in macroeconomic variables and in monetary policy manipulations. Therefore, it is critical to point out a way to incorporate related macroeconomic variables into our model.

The manner of Taylor Rule sheds light on our path. Complying with Bikbov and Chernov (2005), the Taylor Rule is given as,

\[ r_t = r_{t-1} + K(\Delta p_t - \Delta \bar{p}_t) + F\bar{y}_t + h_t \]  

(6)

where, \( r_{t-1} \) refers to the short rate in the previous period. The second term is interpreted as the derivations from current inflation rate to the optimal inflation rate (CB target rate). The third term is considered as the gap between current output and potential output. With
the appearance of the Taylor Rule, we can write a state-space model by considering equation (5). First, we can write equation (6) in a matrix form.

\[ r_t = r_{t-1} + (K, F) \left( \Delta p_t - \Delta \tilde{p}_t \right) + h_t \]  

(7)

Where, \( h_t \) is an error term which is assumed to be normal distribution. Then, we use equation (7) to derive expressions of \( r_t \) and \( r_{t-1} \), respectively. Substitute them into equation (5). Thus, we have an equation highlighting the correlation between value of macroeconomic variables at \( t \) and their values at time \( t-1 \). (We will use notation \( A \) to denote coefficient vector \( (K, F) \) in equation (7).)

\[ r_{t-1} + A \left( \frac{\Delta p_t - \Delta \tilde{p}_t}{\gamma_t} \right) + h_t = H_1 (r_{t-2} + A \left( \frac{\Delta p_{t-1} - \Delta \tilde{p}_{t-1}}{\gamma_{t-1}} \right) + h_{t-1} + \gamma_t \]  

(7.1)

After collecting and combining terms, we have

\[ \begin{pmatrix} \Delta p_t - \Delta \tilde{p}_t \\ \gamma_t \end{pmatrix} = B (\mu_r, \mu_f, K, F, \lambda, n) \begin{pmatrix} \Delta p_{t-1} - \Delta \tilde{p}_{t-1} \\ \gamma_{t-1} \end{pmatrix} + g_t \]  

(8)

Where, \( B_{2x2} = (A' A)^{-1} A' H_1 A_{1x2} \)

\[ g_t = (A' A)^{-1} A' (H_1 r_{t-2} - r_{t-1} - h_t + H_1 h_{t-1} + \gamma_t) \]

Therefore, the state-space model can be written as

Observation Equation:

\[ r_t = r_{t-1} + (K, F) \left( \Delta p_t - \Delta \tilde{p}_t \right) + h_t \]  

(7)

State Equation:

\[ \begin{pmatrix} \Delta p_t - \Delta \tilde{p}_t \\ \gamma_t \end{pmatrix} = B (\mu_r, \mu_f, K, F, \lambda, n) \begin{pmatrix} \Delta p_{t-1} - \Delta \tilde{p}_{t-1} \\ \gamma_{t-1} \end{pmatrix} + g_t \]  

(8)
As shown above, the combination of the term structure and the monetary policy rules forms a state-space system. This system consists of two parts, namely, the observation equation and the state equation. The observation equation represents a linear measurement equation relating the observed yields to the state vectors (latent variables). The state equation shows a VAR(1) transition equation summarizing the dynamics of the vector of the latent variables. Our special interest lies in the coefficients of the state equation. These coefficients include plenty information on the term structure (the factor loadings of Diebold and Li’s model are kept in the state equation’s coefficients.) and the monetary policy rule in the financial market. Given all the coefficients of this equation system, we can forecast the future short-rate by taking macroeconomic variables as unobservable factors. It is also true, that as byproducts, we are able to estimate the inflation gap and the output gap.

4. Data Description
The dataset is from Bliss (1997). It includes the unsmoothed Fama-Bliss approximation of the zero coupon bonds’ yields with maturities 3, 6, 9, 12, 15, 18,…, 360 months from 1978:1 to 2003:12. (Essentially, it covers bonds with 26 different maturities.)

5. Empirical Practice
We use a two-step Kalman filter\(^2\) to estimate the state-space model. First, we use the Kalman filter method to estimate the coefficients of the McCallum Rule (equation (4)) by assuming those coefficients are time-varying. Then we plug those time-varying coefficients into equation (8) (the state-equation). All other variables in the coefficient set are given explicit values according to previous studies.\(^3\) Thus, the macroeconomic variables in the model might be treated as latent factors (unobservable) and the loadings of those factors are known to us.

Thus, in the second step, the Kalman filter method can be employed again to estimate a

\(^2\) For details of Kalman filter method, see Hamilton (1994).
typical latent factor model (equation (7) and equation (8)).

The two-step Kalman filter may allow us to accomplish several objectives. For instance, we can use it to forecast t+s future short interest rate.

6. Empirical Result

In Table (1), we report the basic statistical descriptions on selected bonds’ yields. It can be seen that the bond yield is larger as its duration is longer. For instance, the mean value of 3-month bond yield is 6.55 and it is smaller than that of the bond which matures after 120 months. This evidence is consistent with the facts observed in the bond market. That is, due to the real interest risk and the inflation risk, the bonds with longer duration have to offer higher return to the investors.

Table (2) gives the estimation of the McCallum Rule’s time-varying coefficients (the first step Kalman Filter estimation). The estimation shows that the CB would adjust the short rate with respect to its lagged value within a range from 0.589 to 1.265. This finding demonstrates that the current interest rate is highly correlated with its lagged value. Moreover, it proves that the CB has incentive to smooth its monetary policy (or monetary policy inertia). Rudebush (2002) also documents the similar estimation results. Hereafter, the coefficients of the latent variables in the final state space equations (equation 8) can be calculated. In light of these known coefficients, we estimate the two latent variables in equation 8 (the inflation gap and the output gap) and the result is reported in Table (3). In Table (3), we see that both the inflation gap’s and the output gap’s mean values are negative. One explanation is that there are two recessions in our sample period. One of them is around 1990s after the burst of the IT bubble. The other one is the aftermath of 9-11.

Provided that the concern of this paper is to forecast future short interest rate, Table (5) essentially needs us to pay more attention. In Table (5), the one-month-ahead forecasting

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3 We choose $K=0.5$, $F=0.5$, $\lambda=-0.0602$, $n=3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 60, 72, 84, 96, 108, 120, 180, 240, 300, 360.$
error of future short interest rate from 1979:1 to 2002:12 is reported with mean 0.0254. The number may be satisfactory but it is hard to evaluate the qualification of our model if no comparison is made. Therefore, we estimate two well documented approaches, which have gained adequate attentions and also been taken as benchmark models in other relative literature.

These two models are random walk term structure model and the aforementioned Diebold and Li’s model, respectively.

1. Random Walk

The random walk model has a very natural perception on the dynamic of yield curve. That is the change of future short rate is unpredictable at all. Thus, in compliance with this assertion, the model is as follows

\[ \hat{r}_{t-1} = r_{t-1} \]  

(9)

Obviously, the underlying assumption is that there is no useful information but the short rate in the last period. The forecast result is reported in Table (6).

2. Diebold and Li’s model

\[ \hat{r}^{(a)}_t = \beta_{it} + \beta_{2i} \left( \frac{1-e^{\lambda n}}{\lambda n} \right) + \beta_{3i} \left( \frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \]  

(1)

We use the model with the exactly same form as equation (1) and equation (2). First of all, we calculate the coefficients of latent variables by assuming \( \lambda = -0.0609 \). For the sake of simplicity, we estimate equation (1) with OLS.

\[ \beta_{it} = \beta_{it-1} + \epsilon_{it} \]  

(2)
Then, use $\beta_n$ to regress on $\beta_{n-1}$. Thus C becomes one of known variables. Following equation (2), the forecast of three latent variables can be carried out. Furthermore, we can forecast the future short rate with the use of equation (1). Table (4) reports the estimated coefficients of equation (1). In addition, Table (7) summarizes the forecast results.

To some extent, the forecasting power of our model which embodies rich information of term structure and monetary policies is satisfactory (the forecast error of our model is reported in Table (5)) when we look at the forecast errors from all three models. The Diebold and Li’s model does the worst in the in-sample one-month-ahead forecast. This result reckons the aforementioned weakness of their model that might be caused by the lack of the real economic variables. In contrast, the random walk model generates much smaller forecast error. However, our model, at least within the time period we consider, has outperformed the other two models, indeed. For instance, the mean of error (0.0254) is lower than the alternative models (-0.0395, -1.05). Standard deviation and MSE are consistent as well. If we look at the autocorrelation coefficients of the forecast error, after 12 months, the autocorrelation has totally phased out in our model but not in the other two. Thus, we conclude that the monetary policies do have profound effects on the changes of the short rates and no doubt to mention, a model without consideration of monetary policy may end up with poor forecasting power on the future short rate.

7. Conclusion

In this paper, based on Diebold and Li (2003)’s term structure model, we derive a short interest rate model with two latent macro variables by taking into account monetary policies, namely, the McCallum Rule and the Taylor Rule. Given the appearance of the model, we are allowed to execute a two-step Kalman Filter so as to not only estimate the inflation gap and the output gap but also be able to forecast the future short interest rate. In the end, our model has been proved to be superior to the random walk and the Diebold and Li’s model.
References:


Table 1. Data Description of Bond Yield  

<table>
<thead>
<tr>
<th>Maturity (n)</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Max</th>
<th>Min</th>
<th>$\rho(1)$</th>
<th>$\rho(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-month</td>
<td>6.55</td>
<td>3.07</td>
<td>15.92</td>
<td>1.16</td>
<td>0.97</td>
<td>0.66</td>
</tr>
<tr>
<td>12-month</td>
<td>6.93</td>
<td>3.12</td>
<td>16.30</td>
<td>1.14</td>
<td>0.97</td>
<td>0.68</td>
</tr>
<tr>
<td>120-month</td>
<td>7.72</td>
<td>2.76</td>
<td>15.53</td>
<td>2.33</td>
<td>0.98</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Note: $\rho(1)$ and $\rho(2)$ refer to autocorrelation coefficient.
Table 2. McCallum Rule time-varying coefficient estimation (first Kalman filter)
Equation (4)
(1979:7-2002:12)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Mean</th>
<th>Stdv</th>
<th>Medium</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>ur</td>
<td>0.98</td>
<td>0.103</td>
<td>0.982</td>
<td>1.265</td>
<td>0.589</td>
</tr>
<tr>
<td>uf(1)</td>
<td>4.065</td>
<td>0.906</td>
<td>4.262</td>
<td>6.51</td>
<td>0.042</td>
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<tr>
<td>uf(2)</td>
<td>-4.817</td>
<td>1.473</td>
<td>-5.182</td>
<td>1.153</td>
<td>-6.46</td>
</tr>
<tr>
<td>uf(3)</td>
<td>-0.785</td>
<td>0.423</td>
<td>-0.741</td>
<td>1.409</td>
<td>-2.095</td>
</tr>
<tr>
<td>uf(4)</td>
<td>1.997</td>
<td>0.793</td>
<td>2.028</td>
<td>3.13</td>
<td>-1.154</td>
</tr>
<tr>
<td>uf(5)</td>
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<td>0.552</td>
<td>1.059</td>
<td>2.405</td>
<td>-1.333</td>
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<tr>
<td>uf(6)</td>
<td>0.745</td>
<td>0.435</td>
<td>0.832</td>
<td>1.33</td>
<td>-1.255</td>
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<tr>
<td>uf(7)</td>
<td>-0.198</td>
<td>0.544</td>
<td>0.031</td>
<td>0.948</td>
<td>-1.548</td>
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<tr>
<td>uf(8)</td>
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<td>0.034</td>
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<tr>
<td>uf(9)</td>
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<td>-0.535</td>
<td>0.785</td>
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<tr>
<td>uf(10)</td>
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<td>-1.152</td>
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<td>uf(11)</td>
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<td>uf(12)</td>
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<td>uf(13)</td>
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<td>uf(14)</td>
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<td>-0.645</td>
<td>1.109</td>
<td>-1.691</td>
</tr>
<tr>
<td>uf(15)</td>
<td>1.589</td>
<td>0.694</td>
<td>1.822</td>
<td>3.051</td>
<td>-1.167</td>
</tr>
<tr>
<td>uf(16)</td>
<td>0.428</td>
<td>0.773</td>
<td>0.377</td>
<td>2.476</td>
<td>-2.134</td>
</tr>
<tr>
<td>uf(17)</td>
<td>0.238</td>
<td>0.478</td>
<td>0.214</td>
<td>1.503</td>
<td>-1.065</td>
</tr>
<tr>
<td>uf(18)</td>
<td>-0.23</td>
<td>0.631</td>
<td>-0.196</td>
<td>0.691</td>
<td>-2.03</td>
</tr>
<tr>
<td>uf(19)</td>
<td>-0.352</td>
<td>0.743</td>
<td>-0.098</td>
<td>0.727</td>
<td>-1.885</td>
</tr>
<tr>
<td>uf(20)</td>
<td>-0.585</td>
<td>0.74</td>
<td>-0.607</td>
<td>1.395</td>
<td>-2.071</td>
</tr>
<tr>
<td>uf(21)</td>
<td>-0.591</td>
<td>0.806</td>
<td>-0.325</td>
<td>1.966</td>
<td>-2.784</td>
</tr>
<tr>
<td>uf(22)</td>
<td>-0.27</td>
<td>0.527</td>
<td>-0.077</td>
<td>1.421</td>
<td>-1.277</td>
</tr>
<tr>
<td>uf(23)</td>
<td>-0.632</td>
<td>0.857</td>
<td>-0.368</td>
<td>0.973</td>
<td>-2.42</td>
</tr>
<tr>
<td>uf(24)</td>
<td>0.121</td>
<td>0.781</td>
<td>-0.229</td>
<td>1.61</td>
<td>-0.873</td>
</tr>
<tr>
<td>uf(25)</td>
<td>1.132</td>
<td>0.659</td>
<td>0.936</td>
<td>2.372</td>
<td>-1.657</td>
</tr>
</tbody>
</table>
Table 3. The State-Space Equation Estimation (Second Kalman filter)
Equation (8)

<table>
<thead>
<tr>
<th>Latent variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Medium</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p_t - \bar{\Delta p}_t$</td>
<td>-0.055</td>
<td>0.3395</td>
<td>-0.0163</td>
<td>1.1159</td>
<td>-2.3562</td>
</tr>
<tr>
<td>$\bar{y}_t$</td>
<td>-0.0395</td>
<td>0.4378</td>
<td>0.0067</td>
<td>1.5092</td>
<td>-2.9794</td>
</tr>
</tbody>
</table>

Note: All numbers are in percentage.

Table 4. Descriptive Statistics of Estimated Latent Factors
(Diebold and Li’s model)
(1979:9-2002:12)

<table>
<thead>
<tr>
<th>Factor</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Max</th>
<th>Min</th>
<th>ρ(1)</th>
<th>ρ(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_{1t}$</td>
<td>8.38</td>
<td>2.66</td>
<td>16.15</td>
<td>3.34</td>
<td>0.96</td>
<td>0.75</td>
</tr>
<tr>
<td>$\hat{\beta}_{2t}$</td>
<td>-1.90</td>
<td>1.99</td>
<td>6.25</td>
<td>-5.55</td>
<td>0.90</td>
<td>0.36</td>
</tr>
<tr>
<td>$\hat{\beta}_{3t}$</td>
<td>-0.0095</td>
<td>1.73</td>
<td>5.46</td>
<td>-7.20</td>
<td>0.59</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Note: $\hat{\beta}_u$ denotes to the level factor, $\hat{\beta}_{2t}$ refers to the slope factor, $\hat{\beta}_{3t}$ refers to the curvature factor.
Table 5. In-Sample One-Month-Ahead Forecast error for the Future Short Rate
Equation (7)  

<table>
<thead>
<tr>
<th>Forecast Error</th>
<th>Mean</th>
<th>Stdv.</th>
<th>MSE</th>
<th>$\rho(1)$</th>
<th>$\rho(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0254</td>
<td>0.3698</td>
<td>0.185</td>
<td>0.170</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Note: Forecast error is insignificant from zero.

Table 6. In-Sample One-Month-Ahead Forecast error for the Future Short Rate
(the Random Walk)  

<table>
<thead>
<tr>
<th>Forecast Error</th>
<th>Mean</th>
<th>Stdv.</th>
<th>MSE</th>
<th>$\rho(1)$</th>
<th>$\rho(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.0395</td>
<td>0.4378</td>
<td>0.388</td>
<td>0.210</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Note: Forecast error is insignificant from zero.

Table 7. In-Sample One-Month-Ahead Forecast error for the Future Short Rate
(the Diebold and Li’s model with an AR(1) Factor Dynamic)  

<table>
<thead>
<tr>
<th>Forecast Error</th>
<th>Mean</th>
<th>Stdv.</th>
<th>MSE</th>
<th>$\rho(1)$</th>
<th>$\rho(12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.1015</td>
<td>0.1822</td>
<td>0.433</td>
<td>0.503</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Note: Forecast error is insignificant from zero.
Figure 1. Estimated Inflation Gap 

Figure 2. Estimated Output Gap 