Optimal Trading and Tax Option Value of Defaultable Bonds with Asymmetric Capital Gain Taxes

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Abstract

Current U.S. tax laws provide investors an incentive to time the sales of their bonds to minimize tax liability. This gives rise to a tax timing option that affects bond value. In reality, corporate bond investors' tax-timing strategy is complicated by risk of default. Existing term structure models have ignored the effect of the tax-timing option and how much corporate bond value is due to the tax-timing option is unknown. In this paper we assess the effects of taxes and stochastic interest rates on the timing option value and equilibrium price of corporate bonds by considering discount and premium amortization, multiple trading dates, transaction costs, and changes in the level and volatility of interest rates. We find that the value of the tax-timing option can account for a substantial proportion of corporate bond price even when interest rate volatility is low. Ignoring the timing option value results in overestimation of credit spread, and underestimation of default probability and the marginal investor's income tax rate. These estimation biases generally increase with bond maturity and credit risk.

1. Introduction

In the traditional finance literature, taxes are considered as an important determinant of yield spreads (see Graham, 2003). Corporate bond returns are subject to both state and federal taxes. As a consequence, these bonds must yield sufficient returns to compensate investors' tax liability. Corporate bonds also expose investors to default risk, which has important tax ramifications. As default risk increases, bond yield rises and investors' tax liability increases because more taxes are paid as a result of higher interest income. An increase in default risk also raises the probability of capital loss. In the event of default, investors receive a tax rebate, which depends on their holding period and the recovery rate of the bond.

Measuring tax effects on defaultable bond pricing would be a relatively straightforward task, had investors followed a simple investment strategy such as buy-and-hold. However, in reality, tax considerations often affect bondholders' investment strategies. Under current U.S. tax laws, gains and losses on bond investments are not taxed until investors sell their holdings. Also, tax treatments are asymmetric: interest income and short-term capital gains are subject to higher tax rates than long-term capital gains. These differential tax treatments affect investors' trading strategies. Consider first that there is no default risk. Investors may realize short-term capital loss as soon as possible and postpone capital gains to reduce the tax burden. Alternatively, investors may change the holding period status from long- to short-term by selling the bond and repurchasing it back in order to realize future capital losses short-term. Still another possibility is that they may sell the bond to realize a capital gain and to increase the basis so that they can deduct the amortized premium from future interest income. Considering default risk further complicates the investor's trading strategy. Default risk affects the sensitivity of bond price to interest rates and coupon, and more generally, it causes uncertainty of future returns. The interactions among taxes, default and interest rate volatility result in a complicated environment, making it difficult to prescribe a universal optimal investment policy for corporate bonds.

In this paper we examine the equilibrium price and the timing option value of corporate bonds in the presence of default risk and asymmetric taxation. In particular, we explore the effects of both the level and volatility of interest rates, premium (or discount) amortization rules, multiple trading dates, transaction costs, and changes in tax rates on investors' trading strategy and bond prices. We find that default risk affects not only the relative pricing of defaultable and default-free bonds but also the tax-timing option value of corporate bonds.[1] While forced liquidation due to default is expected to reduce bond price, the effect of default on tax-timing option value is more complicated. The effect of default risk on the timing option value is greater for long-term bonds than for short-term bonds largely because the tax-timing option is a compound option. Since the timing option value of short-term bond is low in proportion to price, the effect of default risk is small. By contrast, default risk reduces the effective maturity of long-term bonds, making the compound nature of the tax-timing option much less valuable. On the other hand, asymmetric tax treatment of long-term and short-term gains increases the tax-timing option value, especially for long-term bonds. Taken together, we find that the tax-timing option retains considerable value even after accounting for the default effect.

[1] This suggests that it is important to consider the effect of default on the timing option value especially when default risk is high in a recession period.

Ignoring this option value thus leads to biased estimation of yield spreads, the marginal investor's tax rate, and the implied default probability.

Our analysis draws on several important papers. In a seminal paper, Constantinides and Ingersoll (1984) examine the optimal trading strategies of taxable default-free bonds under asymmetric taxation. We generalize their analysis to incorporate the effects of default risk, recovery rates, and amortization of premium (or discount) in a setting with multiple trading dates and time-varying level and volatility of interest rates. Dammon, Dunn and Spatt (1989) show that the tax-timing option value depends critically on the pattern of realized returns and the tax treatment of unrealized gains at the end of the investment horizon. They find that the value of realizing long-term gains to re-establish short-term status (so-called the restarting option) is much smaller than predicted by Constantinides (1984). Dammon and Spatt (1996) further show that under asymmetric capital gain taxes, investors realize all capital gains when the ratio of price to the basis is below some critical cutoff level. In the short-term region, investors defer all gains but depending on the time remaining, they may defer small losses to avoid restarting the short-term holding period prematurely. While both studies deal with the equity trading strategy, we focus on the trading strategy of corporate bonds, and examine the impact of ignoring the tax-timing option value on bond pricing and empirical estimation of default risk and implicit marginal income tax rates. Liu and Wu (2004) examine the effects of taxes on investors' trading strategy and the pricing of corporate bonds. They derive a closed-form solution for the pricing and the tax-timing option value of defaultable consol bonds under the assumption of symmetric taxation and no amortization of premium or discount. In contrast, we examine the effects of personal taxes on the pricing and timing option values of finite-maturity corporate bonds under a much more general setting that accounts for the effects of amortization, asymmetric taxation and stochastic interest rates under different tax regimes.

The issue of tax-timing option value is relevant for bond valuation and estimation of credit spread and default probability. Most term structure models have ignored personal tax effects.[2] A main reason is that a rigorous treatment of personal tax produces extremely complex results. Among a few studies that address the effects of personal taxes, the assumption of buy-and-hold trading strategy is typically imposed in order to obtain a tractable model (see Yawitz, Moloney and Ederington, 1985; Green, 1993; Green and Odegaard, 1997; Elton, Gruber, Agrawal and Mann, 2001). The buy-and-hold model ignores tax-timing option value. If taxes are important and the tax-timing option value is not trivial, existing term structure models will be misspecified. In this paper, we find that bond values are significantly affected by personal taxes and the tax-timing option value is often sizable under asymmetry taxation. Ignoring the timing option value can lead to a serious bias in estimating spreads, default probability and marginal investor's income tax rate. Our finding suggests that the buy-and-hold model should be used with care in empirical application. One way is to adjust the observed bond price for the tax-timing option value before fitting the buy-and-hold model to estimate default probability, credit spreads and implicit income tax rates from bond data.

The remainder of this paper is organized as follows. In Section 2, we briefly review important tax provisions for corporate bond investments. In Section 3, we set up

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[2]These include both the structural models (e.g., Merton, 1974; Black and Cox, 1976; Longstaff and Schwartz, 1995; Leland, 1994; Leland and Toft, 1996; Collins-Dufresne and Goldstein, 2001; Collins-Dufresne, Goldstein and Martin, 2001) and the reduced-from models (e.g., Jarrow and Turnbull, 1995; Jarrow, Lando and Turnbull, 1997; Duffie and Singleton, 1997, 1999; Duffee, 1999).

the model to examine the effects of personal taxes and amortization on the pricing of defaultable bonds under alternative trading strategies. Here, we describe the investor's optimization problem and the dynamic programming procedure to obtain equilibrium bond price and the tax-timing option value. We then conduct numerical analysis in Section 4, and explore the implications of omitting taxes for empirical estimation of default probability and marginal tax rates in Section 5. Finally, we summarize our main findings and conclude our paper in Section 6.

2. The Tax Environment

In this section, we briefly review major provisions in the current tax code with respect to the treatment of corporate bond returns. Coupon payments on corporate bonds are taxed at the ordinary income tax rate while capital gains (or losses) are subject to the capital gains tax rate. The ordinary income tax rate varies with investors' tax brackets and capital gains tax rates depend on the holding period.

If the bond is purchased at par and held to maturity, only coupon interest is taxed at the ordinary income tax rate. If a bond is issued below (or above) par, the difference between the price and par is termed Original Issue Discount (or premium). An investor may ignore the discount (or premium) amortization rule if it is less than one-fourth of 1% of the stated redemption price at maturity multiplied by the number of full years from the date of original issue to maturity. This is known as "de minimis" OID. At the end of maturity, this discount (or premium) will be recognized as a capital gain (or loss). If a bond is issued substantially above the par value,[3] an investor has two choices. The investor could treat the difference as capital tax losses and claim a tax rebate at maturity.

Alternatively, the investor could amortize the premium annually until the maturity date. For bonds issued since September 27, 1985, the bond premium must be amortized by using a constant yield method on the basis of the bond's yield to maturity. For bonds issued prior to September 27, 1985, the premium can be amortized by any reasonable method, including straight-line amortization. The amount amortized in a tax year will be a deduction against interest income and the basis of the bond is reduced accordingly. If the bond is sold before maturity, the difference between the sell price and basis is treated as a capital gain (loss).

On the other hand, if a bond is issued substantially below the par value (greater than "de minimis"), the discount is treated as ordinary interest income. IRS allows an investor to pay ordinary income tax on the discount when the bond matures or sold. If the bond is held to maturity, the total amount of the discount is taxed as ordinary income at maturity. If the bond is sold before maturity, the difference between the proceeds from sale and the basis is a gain or loss.[4] If there is a loss, the loss is treated as a capital loss. If there is a gain and the gain is greater than the accrued market discount (or the amortized portion of the discount), the difference between the proceeds from sale and the amortized basis is taxed at the capital gain rate, and the accrued market discount portion is taxed as ordinary income. On the other hand, if the gain is less than or equal to the accrued market discount but greater than zero, the entire gain is taxed as ordinary income (see Sec. 1276 of Internal Revenue Code, 2002).

The situation becomes slightly more complicated when there is a possibility of default, which is equivalent to a forced liquidation. Assume that the investor receives a

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[3] That is, the premium is greater than "de minimis" amount.
[4] In our model, we assume no other purchase costs.

residual value δ upon default before maturity. If δ is less than the basis, the loss is treated as the capital loss. If δ is greater than the basis, we need to separate the total gain into ordinary income or capital gains in the same way as described above for bond sale before maturity. It is quite unusual to have a gain at default, and in fact the historical average of residual (or recovery) value received by bond investors is well below the purchase price or face value (Altman and Kishore, 1998). As a consequence, the loss due to default generally results in a tax rebate. Conversely, when there is no default before maturity, the tax rule described above applies.

Capital gains or losses are classified as long-term or short-term according to the investor's holding period. The current tax law requires at least one-year holding period to obtain a long-term capital gain status.[5] Short-term capital gains are taxed at an ordinary income tax rate. The maximum long-term capital gains tax rate for individuals has recently been reduced from 20% to 15% recently.[6] For corporations, the long-term capital gains tax is the same as the ordinary income tax rate.

When an investor has both long-term and short-term capital gains and losses, these gains or losses are netted separately. Short-term and long-term capital losses, including loss carryovers from previous tax years are deducted from short-term and long-term gains, respectively. All capital gains and losses incurred are then totaled. A net short-term loss can offset net long-term capital gains, or a net long-term loss can offset net short-term gains. A maximum amount of $3,000 capital losses can be deducted from

[5] The current law has a lower rate for assets held more than five years (effective now for taxpayers in the lowest regular tax bracket).
[6] The new tax bill was passed May 23, 2003 and the new tax rates were retroactive to January 1, 2003. For lower income individuals, the top capital gains rate is reduced to 5% from 10%, effectively May 6, 2003. For low-income taxpayers, the capital gains tax is phased out in 2007. The new law also reduces the top dividend tax rate from 38.5% to 15% retroactive to January 1, 2003. However, both capital gains and

the ordinary income for noncorporate taxpayers. Capital losses over $3,000 can be carried forward but separated into short-term and long-term categories. Noncorporate taxpayers may carry over a net capital loss indefinitely until the loss is exhausted. A firm can use losses for a tax year only to offset capital gains in that year but not against ordinary income. A firm is allowed to carry back unused capital losses to the three preceding tax years and to carry over losses to the five following tax years.

The tax laws prohibit an investor from deducting a loss from the sale of securities in a wash sale. A wash sale occurs when an investor sells securities at a loss and within 30 days before or after the sale, (1) buys substantially identical securities; (2) acquires substantially identical securities in a fully taxable trade, or (3) acquires a contract or option to buy substantially identical securities. If the loss is disallowed because of the wash sale rules, the investor can add the loss to the cost of the new securities to form the new basis. This adjustment postpones the loss deduction until the disposition of the new securities.

In addition to federal taxes, corporate bond returns are subject to state and local taxes. State and local taxes vary by different states. In some states, there are no income or capital gain taxes. In others, the marginal state and local tax rate could be as high as 10%. State and local taxes are deducted from income for the purpose of federal taxes.

In view of these complicated tax rules, it is necessary to abstract from nuances of the tax code by focusing on the most important aspects of the tax treatments that have material effects on bond pricing to keep analytical results tractable. We assume that bond investors face an asymmetric capital gain tax $\tau_x(t,\hat{t})$, which depends on the time of

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dividend tax cuts are "sunset" provisions because they will expire December 31, 2008 unless future purchase, $\hat{t}$, and the time of sell, t. Short-term capital gains (or losses) with the holding period less than one year are subject to a higher tax rate than long-term capital gains. Specifically, at the selling time t,

$$\tau_x\left(t,\hat{t}\right)=\begin{cases}\tau & if \;\; t-\hat{t}<1\; year \\ \tau_l & if \;\; t-\hat{t}\geq 1\; year\end{cases} \tag{1}$$

where the subscripts $x = s$, l stand for short- and long-term, respectively, and $0 \leq \tau_l \leq \tau$. We assume that premiums and discounts are amortized linearly until maturity and the regular interest income is adjusted by these amortizations each period. The amortization method can be extended to other rules at a cost of complexity. We ignore the complication of the offset rule of gains and losses, and assume that all return realizations are taxed separately. In addition, we assume no capital loss deduction limitation and no restrictions on wash sales since the wash sale rule can be easily circumvented (see Green, 1993). If investors sell bonds before the maturity date, they will buy the same bonds back immediately. Given this tax setting, we now turn to derivation of the model.

3. The Model

The setup of the model is as follows. Corporate bonds are noncallable and have a par value equal to one, coupon rate c_t and maturity date T. $V(t, T)$ is the time t price of a corporate bond maturing at time $T > t$. Initially there are no transaction costs and later on we relax this assumption. Discounts and premiums are amortized and the basis adjusted each period accordingly. Bonds are defaultable and in the event of default a residual value is distributed to bondholders. The probability of default is λ and the recovery rate

Congresses extend them.

is equal to δ. We adopt the recovery of face value formulation, which specifies the residual value upon default as a fraction of the face value of the bond. Investors receive a tax rebate from the capital loss in the event of default.

3.1 Buy-and-Hold Strategy

Under the buy-and-hold strategy, an investor buys a bond and holds it to maturity or claims a residual value if default occurs before maturity. The value of the bond, $U(t,T;v(t,T),\hat{t})$, to the investor at time t is the expected present value of the bond $U(t+1,T,v(t,T),\hat{t})$ at the end of the period, $t+1$, and the after-tax coupon $(1-\tau)c_t$ in the event of no default, plus the expected present value of residual value and the tax rebate in the event of default at $t+1$. In addition, the bond value is affected by whether the bond is discount or premium.

For premium bond, the amortization of premium is deducted from the interest income in each period. Let $\hat{v}(\hat{t},T)$ be the basis established at the time when the trading takes place $\hat{t}$ (in buy-and-hold case, $\hat{t} = t_0$) and let $v(t,T)$ be the basis at time t. The amortization rate at current time t is

$$a(t) = \frac{1-\hat{v}(\hat{t},T)}{T-\hat{t}} \tag{2}$$

and the current basis is

$$v(t,T) = v(t-1,T) + a(t) \tag{3}$$

The purchase price can be above or below par, and the basis is updated each period. The value of the premium bond to the buy-and-hold investor can be expressed as

$$
\begin{aligned}
&U\left(t,T,v(t,T),\hat{t}\right) \\
&= E_t^P\left\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}) \right. \\
&\left. +(1-\tau)c_t - a(t+1)\tau]1_{[t^*>t+1]}\right\} \\
&+ E_t^P\left\{e^{-(1-\tau)r_t}\left[\delta_t + \tau_x(t+1,\hat{t})(v(t+1,T)-\delta_t)\right]1_{[t^*\in\{t,t+1\}]}\right\}
\end{aligned}
\quad (4)
$$

where E_t^p denotes the conditional expectation under a risk-neutral probability measure P at time t and $1_{\{.\}}$ is the point process indicating the events of no-default and default. The first and second components on the right-hand side represent the expected payoffs when there is no default and default, respectively.

At the initial purchase time t_0 when the investor first acquires the bond,

$$\hat{v}(t_0,T) = v(t_0,T) = U(t_0,T,v(t_0,T),t_0) \quad (5)$$

At maturity,

$$v(T,T) = 1 \quad (6)$$

$$U(T,T,v(T,T),\hat{t}) = 1 - a(t)\tau + (1-\tau)c_t \quad (7)$$

For discount bond, an investor will pay ordinary income tax on the total amortized amount of the discount when the bond matures. The value of the discount bond to the buy-and-hold investor can be expressed as

$$
\begin{aligned}
&U\left(t,T,v(t,T),\hat{t}\right) \\
&= E_t^P\left\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}) \right. \\
&\left. +(1-\tau)c_t 1_{[t^*>t+1]}\right\} \\
&+ E_t^P\left\{e^{-(1-\tau)r_t}\left[\delta_t + \tau_x(t+1,\hat{t})(\hat{v}(t_0,T)-\delta_t)\right]1_{[t^*\in\{t,t+1\}]}\right\}
\end{aligned}
\quad (8)
$$

At maturity the discount (the difference between face value and original purchase price, not the current basis) will be taxed with ordinary income tax rate

$$v(T,T) = 1 \quad (9)$$

$$U(T,T,\hat{v}(T,T),\hat{t}) = 1 - (1-\hat{v}(t_0,T))\tau + (1-\tau)c_t \quad (10)$$

3.2 Optimal Trading Strategy

The current tax law grants an investor an incentive to time their trades. The benefits of trading may come from several sources. Investors can get a tax rebate by realizing losses. Investors' incentive to realize losses is stronger if the market price is substantially lower than the basis and the holding period is less than a year. On the other hand, if there is a capital gain, investors may defer the realization of this gain to postpone tax payments. Alternatively, investors may realize capital gains in order to re-establish a short-term holding status so that they can realize future losses short-term.

In each period, investors must evaluate the market condition to see if they would be better off by selling their bonds. If the value of holding the bond is greater than that of selling it, the investor will continue to hold the bond. Otherwise, the investor will sell the bond and repurchase it back from the market. Thus, the worth of the bond to the investor will be the maximum of the no-trading (U_N) or trading (U_T) value:

$$\begin{aligned} &U(t,T,v(t,T),\hat{t}(t)) \\ &= \max\left\{U_N(t,T,v(t,T),\hat{t}(t)),U_T(t,T,v(t,T),\hat{t}(t))\right\} \end{aligned} \tag{11}$$

Again the value to the bondholder will depend on whether it is a discount bond or premium bond.

For a premium bond, if the bondholder chooses not to trade, the bond's value to this investor at time *t* would be:

$$\begin{aligned} &U_N(t,T,v(t,T),\hat{t}(t)) \\ &= E_t^P\left\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}(t+1)) \right. \\ &\left. + (1-\tau)c_t - a_N(t+1)\tau]1_{[t^*>t+1]}\right\} \\ &+ E_t^P\left\{e^{-(1-\tau)r_t}\left[\delta_t + \tau_x(t+1,\hat{t})(v(t+1,T)-\delta_t)\right]1_{[t^*\in\{t,t+1\}]}\right\} \end{aligned} \tag{12}$$

Note that since there is no trading at time *t*, the trading time indicator does not change; that is,

$$\hat{t}(t+1)=\hat{t}(t) \tag{13}$$

and consequently,

$$a_N(t+1)=\frac{1-\hat{v}(\hat{t},T)}{T-\hat{t}(t)} \tag{14}$$

$$v(t+1,T)=v(t,T)+a_N(t+1) \tag{15}$$

Conversely, if the bondholder sells the bond and repurchases it back immediately, the bond value to the investor at time *t* will be:

$$\begin{aligned}
&U_T(t,T,v(t,T),\hat{t}(t))\\
&=-\tau_x(t,\hat{t})[U_T(t,T,v(t,T),\hat{t}(t))-v(t,T)]\\
&+E_t^P\left\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}(t+1))+(1-\tau)c_t-a_T(t+1)\tau]1_{[t^*>t+1]}\right\}\\
&+E_t^P\left\{e^{-(1-\tau)r_t}\left[\delta_t+\tau_x(t+1,\hat{t})(v(t+1,T)-\delta_t)\right]1_{[t^*\in\{t,t+1]}\right\}
\end{aligned} \tag{16}$$

Since there is trading at time *t*, the purchase time is updated and we have

$$\hat{t}(t+1)=t \tag{17}$$

$$\hat{v}(\hat{t},T)=U_T(t,T,v(t,T),\hat{t}(t)) \tag{18}$$

$$a_T(t+1)=\frac{1}{T-t}\left(1-\hat{v}(\hat{t},T)\right) \tag{19}$$

$$v(t+1,T)=\hat{v}(\hat{t},T)+a_T(t+1) \tag{20}$$

where the basis is reinitialized to reflect the new purchase price. Both the basis, $\hat{v}(t)$, the amount of amortization, and the time at which the basis is set, $\hat{t}(t)$, are time-dependent. For example, the basis at time *t+1*, $\hat{v}(t+1,T)$, depends on whether the bond is traded at time *t*. If there is no trade at time *t*, the basis is adjusted at the old amortization rate, $\hat{v}(t+1,T)=\hat{v}(t,T)+a_N(t+1)$; otherwise, the basis will be adjusted based on the new price and new amortization rate, $\hat{v}(t+1,T)=v(t,T)+a_T(t+1)$. This example illustrates a complicated dynamic programming problem that the value of the bond to the investor, $U(t+1,T,\hat{v}(t+1),\hat{t}(t+1))$, depends not only on what happens after *t+1,* $1_{\{t^*>t+1\}}$, but also before *t+1,* $1_{\{t^*\in\{t,t+1\}\}}$.

Similar to the buy-and-hold case, the following boundary conditions parallel to (5)-(7) must hold:

$$\hat{v}(t_0,T)=v(t_0,T)=U(t_0,T,\hat{v}(t_0,T),t_0) \tag{21}$$

At maturity,

$$v(T,T)=1 \tag{22}$$

$$U(T,T,v(T,T),\hat{t}(T))=1-a(t)\tau+(1-\tau)c_t \tag{23}$$

For a discount bond, the holding value to an investor will be

$$\begin{aligned}&U_N(t,T,v(t,T),\hat{t}(t))\\&=E_t^P\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}(t+1))\\&+(1-\tau)c_t]1_{[t^*>t+1]}\}\\&+E_t^P\{e^{-(1-\tau)r_t}\left[\delta_t+\tau_x(t+1,\hat{t})(\hat{v}(t_0,T)-\delta_t)\right]1_{[t^*\in\{t,t+1\}]}\}\end{aligned} \tag{24}$$

Once the bondholder sells the bond and repurchases it back immediately, the bond value to the investor at time *t* will be:

$$\begin{aligned}&U_T(t,T,v(t,T),\hat{t}(t))\\&=-\tau_x(t,\hat{t})[(U_T(t,T,v(t,T),\hat{t}(t))-v(t,T)]+(1-\tau)[v(t,T)-\hat{v}(t_0,T)]\\&+E_t^P\{e^{-(1-\tau)r_t}[U(t+1,T,v(t+1,T),\hat{t}(t+1))+(1-\tau)c_t]1_{[t^*>t+1]}\}\\&+E_t^P\{e^{-(1-\tau)r_t}\left[\delta_t+\tau_x(t+1,\hat{t})(v(t+1,T)-\delta_t\right]1_{[t^*\in\{t,t+1\}]}\}\end{aligned} \tag{25}$$

Since there is trading at time *t*, the purchase time is updated and we have

$$\hat{t}(t+1) = t \tag{26}$$

$$\hat{v}(\hat{t},T) = U_T(t,T,v(t,T),\hat{t}(t)) \tag{27}$$

$$a_T(t+1) = \frac{1}{T-t}\left(1-\hat{v}(\hat{t},T)\right) \tag{28}$$

$$v(t+1,T) = \hat{v}(\hat{t},T) + a_T(t+1) \tag{29}$$

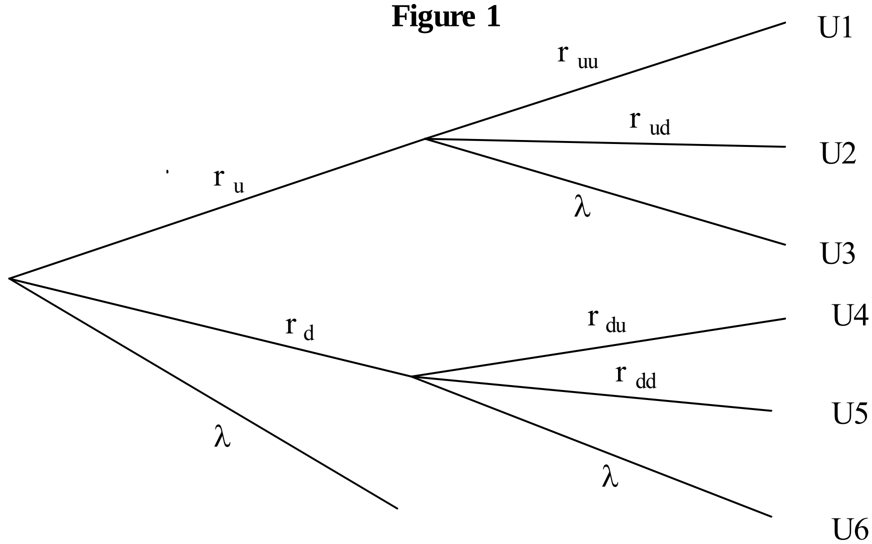
Let $v_{BH}(t_0,T)$ denote the bond price under the buy-and-hold strategy and $v_{OP}(t_0,T)$ denote the bond price under the optimal trading strategy. The tax timing option value of this bond in percentage terms is

$$TO(t_0,T) = \frac{\lfloor v_{OP}(t_0,T) - v_{BH}(t_0,T) \rfloor}{v_{OP}(t_0,T)} \tag{20}$$

4. Numerical Results

In this section we conduct numerical analyses of tax effects on the tax-timing option value and the pricing of corporate bonds. The short-term interest rate, *r*, is the only state variable in the model and we assume that it follows a driftless binomial random walk with two reflecting barriers (see Figure 1 below). In the beginning, we adopt the same parameter values (interest rates, coupons, etc.) and tax regime as in Constantinides and Ingersoll (1984) for direct comparison with their results. Later, we adopt parameters and the tax regime that are closer to the current economic environment.

Figure 1



The short rate *r* in the binomial system will go up or down with an equal probability. We initially consider two volatility scenarios for the interest rate process. In the low-variance process, the interest rate takes on the twenty-one values, 0.04, 0.05, …, 0.24. At each point in time, the interest rate either increases or decreases by 0.01, each with a probability of one half. If the interest rate hits one of the reflecting barriers, then at the next time point it either remains unchanged or takes on the value of 0.05 or 0.23, each with a probability of one half. In the high-variance process, the interest rate takes on one of eleven values, 0.04, 0.06, …, 0.24. The interest rate increases or decreases by 0.02 and the probabilities of increase and decrease are the same as in the low-variance process with reflecting barriers set at 0.04 and 0.24.

We start with four tax scenarios similar to the setting in Constantinides and Ingersoll (1984), except that both premiums and discounts are now amortized for each bond. This allows us to evaluate the impact of default by controlling the effects of the tax regime. The four tax scenarios are summarized below.

(I) Under scenario I, the marginal investor is an individual. Coupon income is taxed at the marginal income tax rate $\tau = 0.5$. Realized short-term and long-term gains and losses are taxed at the same rate $\tau_s = \tau_l = 0.25$.

(II) Under scenario II, the marginal investor is an individual. Coupon income is taxed at the rate $\tau = 0.5$. Realized short-term gains and losses are taxed at the rate $\tau_s = 0.5$. Realized long-term gains and losses are taxed at the rate $\tau_l = 0.25$.

(III) Under scenario III, the marginal investor is an individual. Coupon income is taxed at the rate $\tau = 0.5$. Short- and long-term gains and losses are untaxed, i.e., $\tau_s = \tau_l = 0$.

(IV) Under scenario IV, the marginal bondholder is a bank or bond dealer. Coupon income and all capital gains and losses are taxed at the rate $\tau = \tau_s = \tau_l = 0.5$.

Scenario I assumes symmetric taxation for long and short term capital gains whereas Scenario II adopts an asymmetric tax treatment for capital gains. Scenario III assumes no capital gains taxes (e.g., individual retirement accounts) and Scenario IV is intended to capture the tax effect when the trader is a dealer or a bank. Note that Constantinides and Ingersoll (1984) consider the amortization of premium bonds but not discount bonds. This setting affects the path of investor's optimal trading strategy and generally leads to an overestimation of the tax-timing option values. Since the current U.S. tax laws require an amortization for both premium and discount bonds, we incorporate these amortization rules in each of the above four tax scenarios.[7] We keep

[7] Note that similar to our procedure, Constantinides and Ingersoll (1984) use the straight-line method to amortize premiums.

Constantinides and Ingersoll's (1984) assumption for amortization only when we replicate their results for the case of default-free taxable bonds.

4.1 Prices and Tax-timing Option Values of Default-free Bonds

We first replicate Constantinides and Ingersoll's (1984) results for default-free taxable bonds (i.e., $\lambda = 0$ in Figure 1) under their original setting. The parameter choices are the same as Constantinides and Ingersoll (1984). The initial short-term interest rate is 14% and its standard deviation σ_r is equal to 0.02 and 0.01 in the high and low variance cases, respectively. The coupon rates are 6%, 10%, 14% and 18%. Unlike their study, we assume no capital loss limitation.

[Insert Tables 1 and 2 here]

Tables 1 and 2 show simulated prices and the tax-timing option values of default-free bonds under Constantinides and Ingersoll's setting. These results are comparable to theirs.[8] Minor differences arise for two possible reasons. First, our model assumes that all realizations of returns are taxed separately and there is no capital loss limitation. Second, we calculate the buy-and-hold price based on a stochastic interest rate process identical to that used to calculate bond prices under the optimal trading strategy. Under the optimal trading strategy we iterate in each period to obtain the "trading" price and compare that with "no-trading" price to determine the equilibrium price of the bond. Under the buy-and-hold strategy we only iterate in the initial period and take "no-trading" prices in all other periods. In contrast, Constantinides and Ingersoll employed the following formulas to calculate the buy-and-hold prices:

$$P = (1 - \tau_c) c \sum_{t=1}^{T} \pi_t + (1 - \tau_l + \tau_l P) \pi_T \text{, } for \quad P \le 1 \tag{21a}$$

$$P = [(1-\tau_c)c + \tau_c(P-1)]\sum_{t=1}^{T}\pi_t + \pi_T, \ for \quad P \geq 1 \qquad (21b)$$

where π_t is the price at time zero of the after-tax cash flow at time *t*. The main difference between this method and ours is that we allow the interest (or discount) rate to be stochastic.

4.2 Equilibrium Prices and Tax-timing Option Values for Defaultable Bonds

We next examine the effects of default risk on the equilibrium price and tax-timing option value. At each node of the binomial process in Figure 1, there is a probability λ that the bond will default with a recovery rate equal to δ. In the event of default, the investor receives a residual value plus a tax rebate due to the capital loss. We set λ equal to 1% and δ equal to 50%.

The equilibrium prices of defaultable bonds and their tax timing option values are reported in Tables 3 and 4, respectively. Bond prices decrease after incorporating the effect of default because default risk reduces the expected payoff of the bond. Bond prices are only slightly higher when the interest rate volatility is higher. Comparing Table 3 with Table 1, we find that the percentage decrease in bond value is large under Scenario III, especially for par and premium bonds. Since both short- and long-term capital gain taxes are zero under this scenario, an investor has no opportunity to receive a tax rebate from the loss of default in his investment. Thus, there is a larger drop in bond price under this scenario.

Bond prices in Scenario II are higher than in other scenarios. Two factors may have contributed to higher prices in Scenario II. First, realization of short-term losses provides valuable rebates and a short-term holding period is not difficult to establish.

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[8] The slight difference under optimal trading strategy may be due to the dynamic programming estimation procedure.

Second, compared to Scenario IV, Scenario II gives investors an opportunity to realize long-term capital gains with a lower tax rate whenever a realization of capital gains is optimal. In some circumstances investors may want to realize capital gains in order to establish a short-term holding status. Alternatively, investors may postpone capital gains until maturity. At any rate, these benefits are lowest for short-maturity bonds because their market prices are least volatile. For this reason, prices of short-maturity bonds under Scenario II are very close to prices under other scenarios, particularly when interest rate volatility is low.

[Insert Table 3 here]

Default risk tends to lower the tax-timing option value. As shown in Table 4, the tax-timing option value all decreases under Scenarios I, II, III and IV, compared to the results in Table 2. The reason is straightforward. Once a bond defaults, it will stop an investor from taking tax advantage in the future trading periods. Therefore, tax timing option value will decrease.

Timing option values under Scenario II are expected to be highest for two reasons. First, investors are able to choose their positions optimally to fully exploit all available tax benefits since the restrictive offset rule doesn't apply here. Second, asymmetric capital gain taxes in this scenario allow investors to benefit most from the tradeoff between the tax rebate from short-term capital loss realization and the tax payment on long-term capital gains at a lower rate.

Scenario III allows investors to maneuver their amortization basis without incurring any tax penalty on capital gain realization. But investors lose the opportunity to receive tax rebates from capital losses. The former effect dominates the latter for par and premium bonds. This explains why timing option values for discount bonds under Scenario III in Table 4 are close to zero. But for par bond and premium bonds the tax timing option values are much larger under this scenario. Conversely, for discount bonds under scenario I and IV, investors have a greater chance to realize capital losses to receive tax rebates because the basis of discount bonds increases over time. Under Scenario IV, the tax rebate on capital losses is larger due to a higher capital gain tax rate. This is why tax-timing option values for discount bonds are higher under Scenario IV than under Scenario III.

In general, the tax-timing option value increases with the coupon rate. This is because the basis of premium bonds decreases by the amount amortized each period, giving investors an incentive to trade to establish a higher basis. For example, whenever bond price exceeds the current basis, investors are inclined to establish a higher basis to reduce taxes via future amortization. The situation is different for discount bonds. Under Scenario III, investors still want to trade whenever trading price exceeds the current basis. But their main purpose is to reduce the income tax burden since the amortized amount of the discount bond is taxed at the regular income tax rate. Our results show that the tax advantage of premium bonds outweighs that of discount bonds.

[Insert Table 4 here]

In summary, the tax-timing option value remains sizable when the default effect is accounted. This is particularly the case when there is an asymmetric tax treatment (e.g., Scenario II). The tax-timing option value is higher for long-term discount bonds when interest rate volatility is higher. The tax-timing option value can be more than 14% for long-maturity premium bonds under asymmetric taxation. On the other hand, the tax-timing option value is modest for short-maturity (five-year) bonds for all scenarios.

4.3 Effects of Transaction Costs

The analysis above does not consider the effects of transaction cost. Transaction cost may reduce the frequency of trades and the tax-timing option value. To incorporate the effects of transaction cost, we set trading cost at .5%, 1% and 2% of transaction price. In the interest of brevity, we report the results only for par bonds at high interest rate volatility (σ_r = 02).

Table 5 reports timing option values in the presence of transaction cost. For ease of comparison, we also report the tax-timing option values with no transaction cost in the first column of each scenario. The results show that transaction cost has only a modest effect on the tax-timing option value when it is low. For example, when transaction cost is .5%, the tax-timing option value decreases only very slightly. Even at the 1% level of transaction cost, the tax-timing option value drops a little more but remains sizable, especially under Scenarios II and III and IV. As transaction cost increases to 2%, the timing option value drops substantially. For example, under Scenario II the timing option value drops from 14.2% to 4.2% for 30-year bonds. Results show that the timing option value is relatively small when transaction cost is high. The sensitivity of the timing-option value to transaction cost is higher for the case of asymmetric taxation (Scenario II).

Schultz (2001) finds that round-trip trading costs average about 0.27% for institutional trades. Hong and Warga (1998) report institutional bond trading costs of 0.13% for investment-grade bonds and 0.19% for non-investment-grade bonds. Edward, Harris and Piwowar (2004) report an average round-trip cost of 0.54% for a representative institutional trade and 1.38% for a retail trade. Within the range of these transaction costs, our results show that the tax-timing option value is nontrivial.

4.4 Changes in Interest Rate Volatility and Tax Regime

Tax regimes and interest rates change over time. Changes in tax rates and interest rates affect investors' trading strategies, which in turn change the value of tax-timing option. We next examine the sensitivity of bond price and the tax-timing option value to changes in interest and tax rates.

Short-term interest rates are low in recent years. Table 8 provides summary statistics of one-month Treasury bill rates from 1981 to 2001. Average short rate drops from 7.75% in the period of 1981-1991 to 4.51% in the period of 1991-2001. In addition, the standard deviation of interest rates in the latter period (0.95%) is less than a half of that in the former (2.6%). In the subperiod of 1996-2001, it further decreases to 0.59%. To accommodate the recent trend, we change the initial short-term interest rate to 6% and lower coupon rates to 2%, 4%, 6% and 8%. In addition, the standard deviation of interest rates is reduced to 1% (high) and 0.5% (low).

[Insert Table 6 here]

The U.S. tax laws have changed several times since 1981. The highest individual income tax bracket is 39.6% during the Clinton Administration. The current maximum federal income tax rate (τ_F) is reduced to 35% for both individuals and corporations,[9] and maximum state income tax rate (τ_S) ranges from 5% to 10%. The effective income

[9] According to the tax bill passed May 23, 2003, the highest income tax rate for individuals is reduced to 35% retroactive to January 1, 2003. The next three rates are 33%, 28% and 25%. This tax act accelerates the tax reduction scheduled for 2004 through 2006 by the Economic Growth and Tax Relief Reconciliation Act of 2001 (see the report of the Wall Street Journal May 23, 2003).

tax rate for corporate bond investors is $\tau = \tau_F + \tau_S(1-\tau_F)$ as the state tax is a deduction against the federal income tax. Combining both federal and stat taxes, the maximum effective tax rate is about 40%. We hence lower the marginal income tax rate to 40% and the capital gains tax rate to 20% in Scenario I, to 20% and 40% for long- and short-term gains tax rates in Scenario II, and to 40% for both short- and long-term gain tax rates in Scenario IV. Both short- and long-term capital gains tax rates remain zero in Scenario III. Default probability λ is kept at 1% and recovery rate δ at 50%.

[Insert Tables 7 and 8 here]

Table 7 shows equilibrium bond prices under the new tax and interest rate setting. Compared to the results in Table 3, prices for discount bonds are higher whereas prices for par and premium bonds are somewhat lower. These results hold even if the volatility level is fixed at 1%. Prices are not very sensitive to interest rate volatility in the set range. As shown, an increase in interest rate volatility from .5% to 1% affects bond prices only marginally.

Figure 2

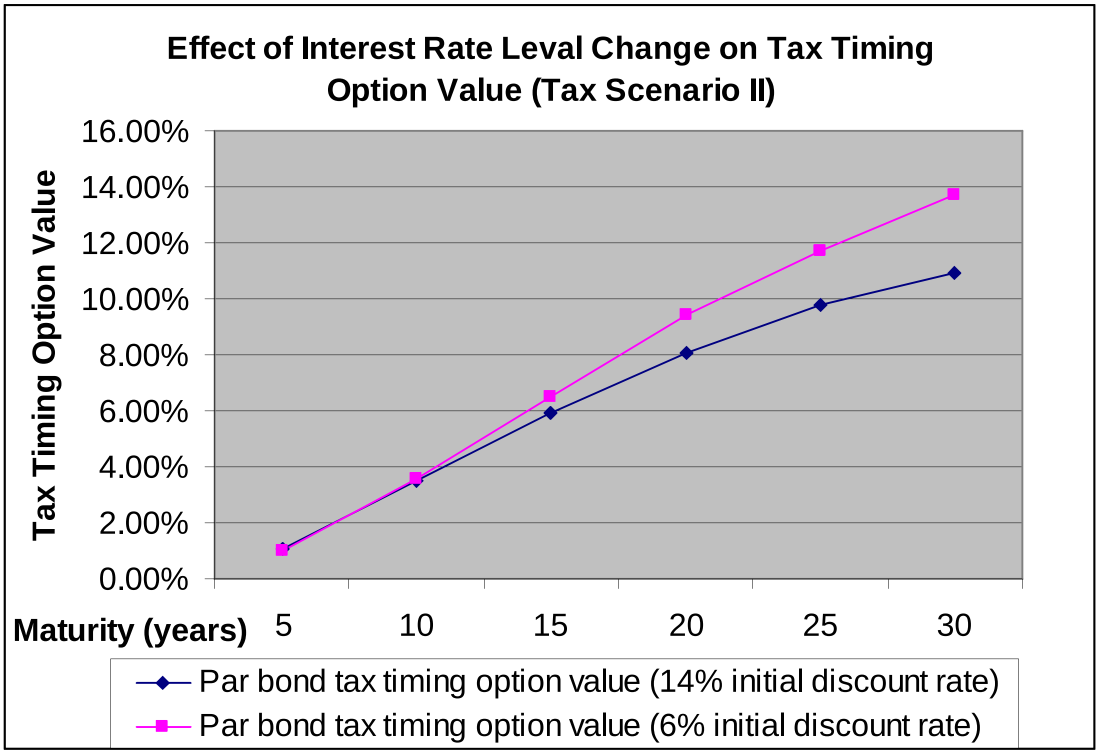


Table 8 reports tax-timing option values. Compared to those in Table 6 with the same interest rate volatility (σ_r = 1%), lower tax and interest rates generally increase the timing option value. The effect on the tax-timing option value is higher for long-term bonds, premium bonds and for the case of asymmetric taxation. Figure 2 shows the tax-timing option values for par bonds (6% coupon rate and 14% coupon rate) given the 1% interest rate volatility under Scenario II. The tax-timing option value increases about 0% to 3% as maturity increases. Nevertheless, tax-timing option values continue to account for a notable portion of bond prices under all scenarios. Unlike the effect on bond price, interest rate volatility strongly affects the tax-timing option value. Lowering the interest rate volatility from 1% to .5% reduces the value of timing option substantially. The impact of interest rate volatility on the timing option value is larger for longer-term bonds.

Summarizing, lower tax and interest rates increase the timing option value for individual investors, when taxes are asymmetric and for dealers and banks (Scenario IV). The tax-timing option value is negligible for shorter-maturity bonds. For example, for five-year bonds, the timing option value is less than 1.5%. However, despite low interest rates and volatility, the tax-timing option value remains sizable for longer-maturity bonds under asymmetric taxation. The tax-timing option value is above 10% for par bond and premium bonds with maturity longer than 20 years when interest rate volatility is around 1%. To the extent that individual income taxes are characterized by asymmetric taxation, the tax-timing option value cannot be ignored for long-term bonds.

4.5 Sensitivity of Tax-timing Option Values to Default Risk

An important question is how sensitive the tax-timing option value is to default risk. We next assess the impact of changes in default probability and recovery rates on tax-timing option values. The initial short rate is set at 6%, its standard deviation is 1%, and the recovery rate is 50%. The tax regime is identical to that in Table 8. We vary default probabilities from 0% to 4% and calculate the tax-timing option value associated with these default probabilities.

Other things being equal, the tax-timing option value should decrease as default probability increases since default forces investors to close their position before maturity and stops the trading process. This default effect is expected to be stronger for longer-maturity bonds since tax-timing option is a compounding option. However, the situation becomes more complicated when default-related tax effects are taken into account. For example, investors may sell the bond to establish a short-term status to receive a higher tax rebate upon default.

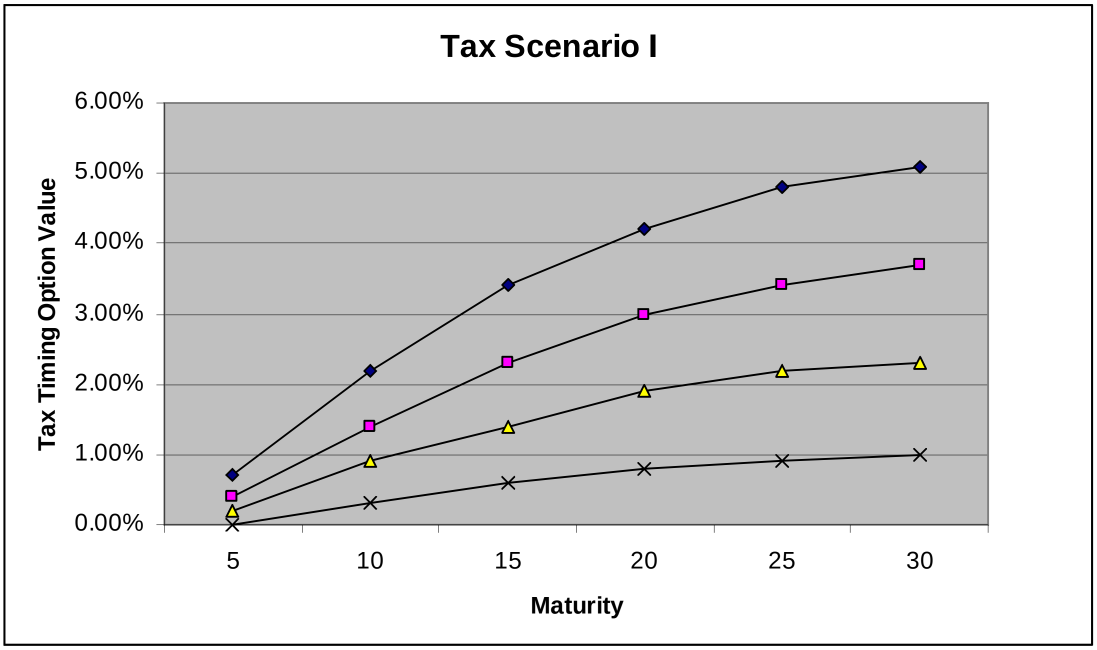
Table 9 reports the tax-timing option values associated with different default probabilities. For ease of illustration, we plot the timing option values for bonds with a 6% coupon rate for all scenarios in Figure 3. The curve with diamonds represents the results for zero default probability, squares represent 1% default probability, triangles represent 2% default probability and crosses represent 4% default probability.

[Insert Table 9 here]

As shown, the timing option value decreases with default probability for all four scenarios.

Default risk has a larger impact on the timing option value for Scenarios I and III and for longer-maturity bonds. For example, for 6% coupon bonds with 30-year maturity (see Figure 4), the tax timing option value under Scenario III decreases from 10.7% to 0.1% as default probability increases from zero to 4%. Default risk puts investors under Scenario III at the most disadvantageous position because they cannot claim tax rebates from default losses. In contrast, investors under Scenarios II and IV are in a better position to shield the burden of default risk due to relatively high capital gain tax rates

Figure 3


Tax Scenario I
Tax Timing Option Value
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
5
10
15
20
25
30
Maturity

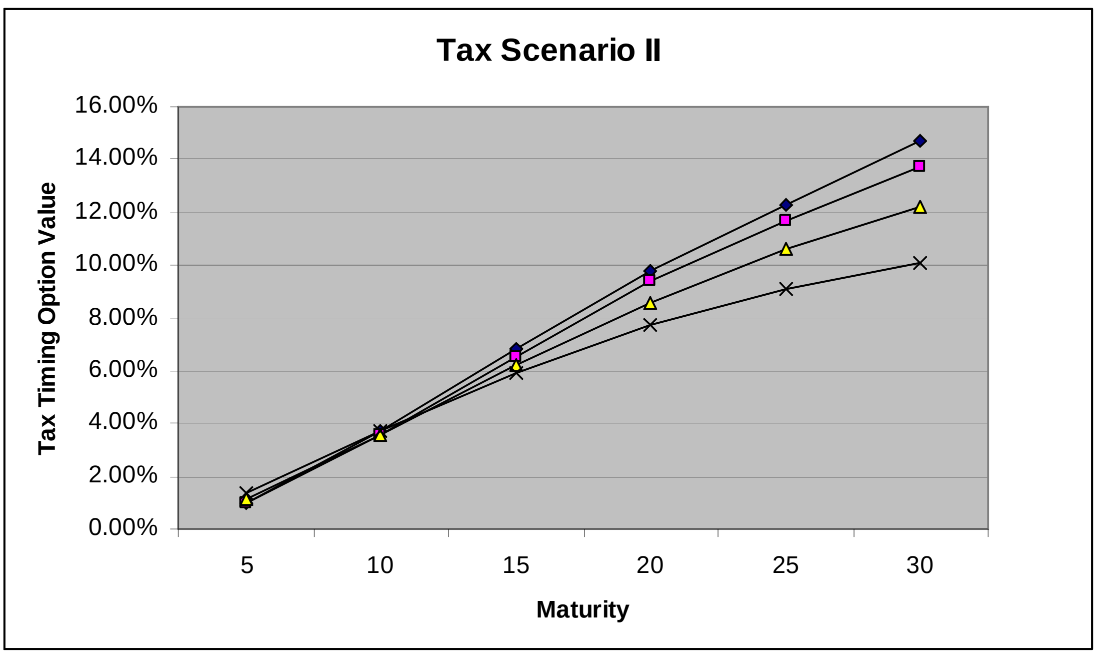
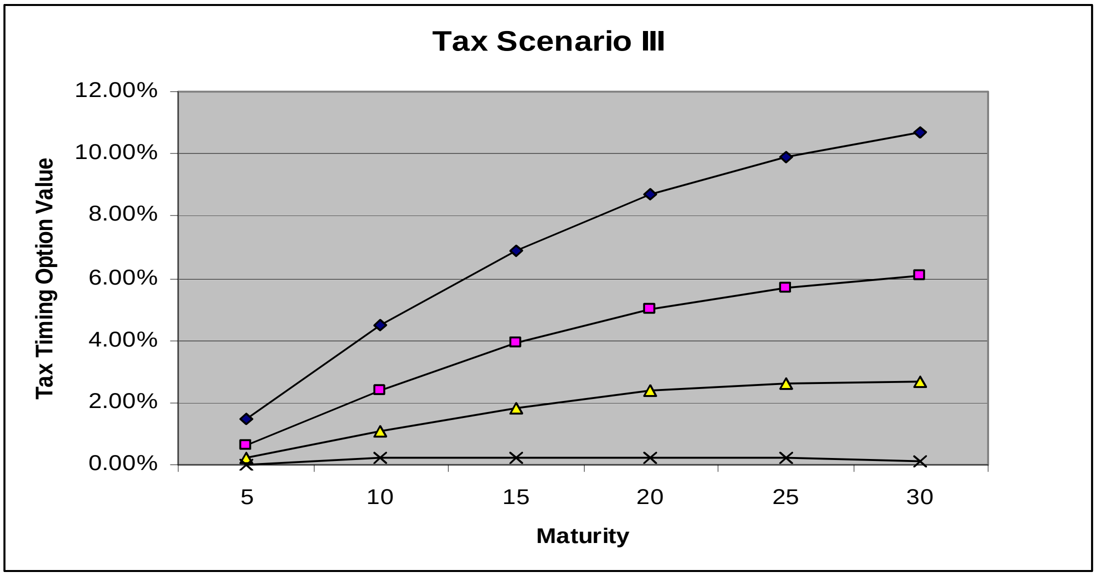
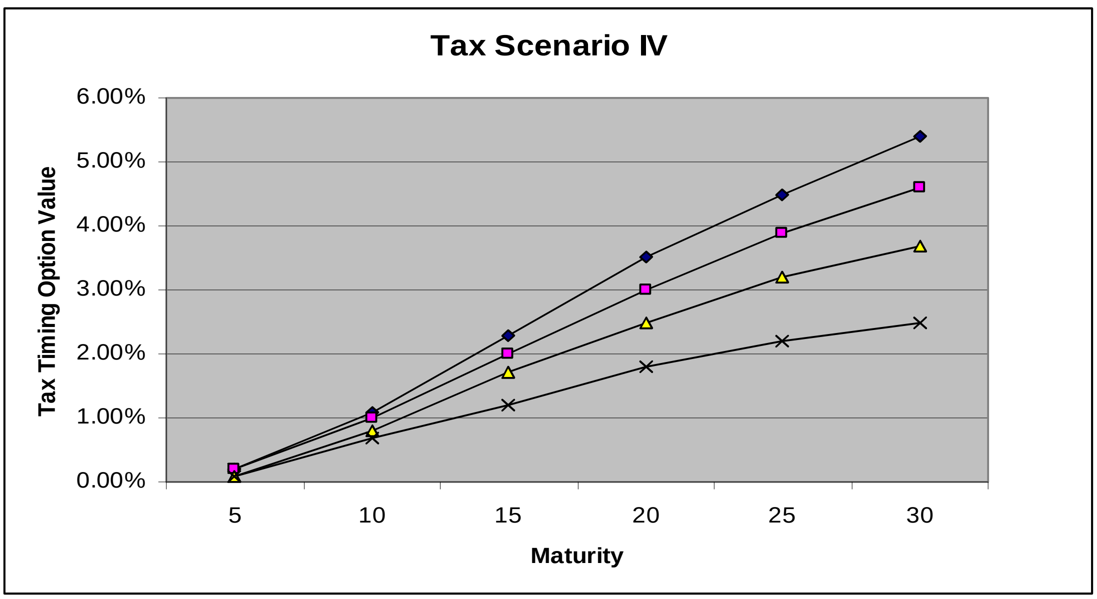

Tax Scenario II
Tax Timing Option Value
16.00%
14.00%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
5
10
15
20
25
30
Maturity

Figure 3 (Continued)





Diamonds-0% default probability, squares-1% default probability, triangle-2% default probability and crosses-4%default probability. The recovery rate δ equals to 50%.

Coupon rate equals 6%, The initial short-term interest rate is 6%. σ_r = 1% is the annual standard deviation of changes in the short-term interest rate.

The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Four tax scenarios are defined as in Tables 8 and 9.

The value of a tax-timing option is defined as $TO(t_0,T)=\frac{[v_{OP}(t_0,T)-v_{BH}(t_0,T)]}{v_{OP}(t_0,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond price under the buy-and-hold and optimal trading strategies respectively.

and larger expected tax rebates from default losses. This may explain why the effect of default risk on the tax-timing option value is milder for these two scenarios.

Results suggest that the tax-timing option value is lower for speculative-grade bonds. For short-maturity bonds, timing option value is low when default probability is high. By contrast, the tax-timing option is still above 10% for bonds with maturity longer than 20 years even when default probability is high. Not surprisingly, for high-quality bonds with negligible default risk, the tax-timing option becomes more valuable.

Table 10 reports the effects of changes in recovery rates on the tax-timing option value. Given maturity, the tax-timing option is almost flat across recovery rates. In some cases, there are slight increases or decreases but the magnitude is generally small, indicating that the recovery rate doesn't have a material effect for these two tax regimes.

[Insert Table 10 here]

4.6 Multiple Trading Dates

The preceding analyses assume that there is only one transaction a year similar to the setting in Constantinides and Ingersoll (1984). We next extend our analysis to allow for two transactions a year. Table 11 reports the tax timing option values when investors can trade twice a year (or two trading periods). Due to the computational burden, we only report tax-timing option values for bonds with maturity up to 15 years. As shown, the tax timing option values under all scenarios are larger than those reported in Table 8 as investors now have more opportunities to trade. The increase in the tax-timing option values tends to be higher for Scenario II and for low-coupon (discount) bonds.

In general, the timing option value increases substantially as the number of transactions doubles.[10] For example, compared to the results for Scenario II in Table 10, the timing option value increases from 6.5% to 8.3% for 15-year bonds when interest rate volatility is equal to 1% and coupon rate is 8%. In fact, the timing option value is almost doubled for all bonds with different coupon rates. The increase in timing option values in percentage terms is even higher for shorter-term bonds. The increase in timing option value is not as high for other tax scenarios but it is notable. For 15-year 6% coupon bonds, the timing option value increases from 2.3%, 3.9% and 2.0% for Scenarios I, III and IV to 3.8%, 6.4% and 3.2%, respectively. Similar patterns are found for bonds with lower coupon rates. Thus, the timing option value can increase substantially when trading frequency increases.

[Insert Table 11 here]

4.6.1 The Optimal Cutoff Level in the Short-term Trading Region

Dammon and Spatt (1996) show that the optimal cutoff level, below which investors realize all capital gains, is time variant. Since the structure of our model and the security that we are dealing with is quite different from theirs, it is not clear whether a similar conclusion will hold, or that a unique cutoff level would even exist. In this subsection, we examine the effect of multiple transactions on the realization region in the short term using an approach similar to Dammon and Spatt (1996).

In the interest of brevity, we focus on Scenario II for two main reasons. First, this

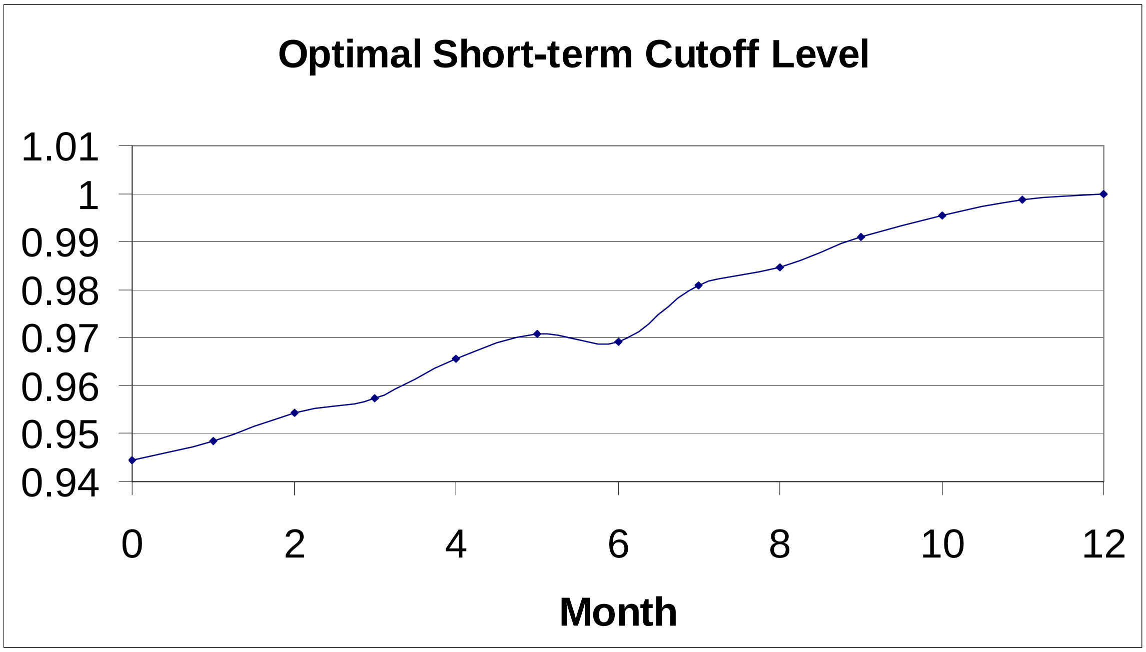
[10] This result holds even with moderate transaction costs. The timing option value remains quite high when the transaction cost is around .5%. For brevity, the results with transaction costs are not reported here.

tax regime grants individual investors the maximum benefit from optimal trading. Second, this tax regime is comparable to Dammon and Spatt's setting, which assumes asymmetric capital gain taxes.

To determine the short-term optimal realization region, we extend the number of trades to 12 trading dates a year so that investors have an opportunity to trade at the end of each month. We set the initial short-term interest rate and coupon rate to 0.05% per month (or 6% per annum) and the standard deviation of short rates to 0.0028% (1% per annum). There are no transaction costs, and default probability and recovery rate are 1% and 50%, respectively. We set the highest price at which investors will trade the bond in each period as the cutoff level for capital gain realization.

Figure 6 shows that the cutoff level increases over time. Due to amortization, the basis used to calculate capital gain/loss increases if the bond is purchased at a discount. Although the coupon rate is set equal to the initial short-term interest rate in this case, the bond is priced at a discount due to default risk. As the basis increases over time, the optimal cutoff level increases as time passes. Below the cutoff level, investors realize the short-term loss whereas above the cutoff level, they realize the short-term gain. As shown in Figure 6, the optimal trading strategy involves the deferral of short-term losses as large as 6%. Thus, corporate bond investors may not always realize short-term losses to take advantage of a loss deduction. A main reason is that if the investor sells the bond, the short-term holding period is restarted. This is disadvantageous because it extends the length of time the investor has to wait to receive more favorable long-term tax treatment on any subsequent capital gains. The results in Figure 6 show that this disadvantage of restarting the short-term holding period may outweigh the benefit of realizing short-term losses.

Figure 4



5. Implications for Empirical Estimation

5.1 Effects of Ignoring Tax-timing Options on Estimation of Default Probability

One of the important applications of the term structure model is to retrieve the risk-neutral default probability from observed bond yields. Previous studies have often assumed away tax effects in inferring the default probability from bond yields (e.g., Jarrow, Lando and Turnbull, 1997; Duffee, 1999). If the tax effect is not trivial, estimates of the default parameter would be biased if taxes are ignored by the model. Although some studies have attempted to capture the tax effect, they have typically assumed a buy-and-hold strategy (see, for example, Yawitz et al., 1985; Elton et al., 2001). This assumption ignores the tax-timing option value associated with the optimal trading strategy.

To assess the effect of the tax timing option value on the estimation of default probability, we conduct simulation analysis. We first simulate the prices of defaultable bonds with known default probability, recovery rate, tax rate, and interest rate under the optimal trading strategy. Given these bond prices, we solve for default probabilities by assuming a buy-and-hold strategy. We then compare the estimated default probability with the true probability to determine the estimation bias when the tax-timing option value is ignored.

Table 12 reports the estimates of default probabilities for bonds with different maturities and ex ante (true) default probabilities. The coupon rate and the short-term interest rate are set equal to 6%. The standard deviation of annualized interest rates is 1% and 0.5% for the high and low volatility cases. Equilibrium bond prices are calculated based on the tax regime under Scenario II and the trading process is set up as in Table 9. The recovery rate is fixed at 50% and the true default probabilities are set at 2%, 4% and 6%, respectively.

Results show that estimation error for default probability is quite substantial when the tax-timing option value is ignored, particularly for long-maturity bonds. The tax-timing option increases the bond value and reduces yield to maturity. When this tax timing option value is ignored, its effect will be factored into the estimated default parameter. To fit the bond with higher price (or lower yield) due to the timing option, the default rate estimate is forced to be lower to bring up the price. The longer the maturity, the higher the tax-timing option value and therefore, the lower the estimated default probability. In addition, results in Table 14 show that the bias increases with default probability, suggesting that the estimation will be more serious for speculative-grade bonds.

[Insert Table 12 here]

5.2 Effects of Ignoring Tax-timing Option on Estimation of Implied Tax Rates

Ignoring the tax-timing option value can also cause underestimation of the marginal investor's income tax rates. If bond prices reflect the investor's optimal trading strategy, estimates of marginal tax rates based on the assumption of buy-and-hold strategy will be biased downward. As shown earlier, the tax-timing option value often accounts for a substantial portion of bond price especially under asymmetric tax rates. Failure to take this option value into account would lead to a substantial underestimation of the implicit marginal tax rate. The tax-timing option increases the bond price and decreases the bond yield. When the value of the timing option is ignored, this higher price (or lower yield) must be accommodated by a lower tax rate in empirical fitting. Thus, the estimates of marginal investor's tax rates will be lower when the tax timing option value is ignored.

To explore the effect of timing option on the implied marginal tax rate, we conduct simulations by setting default probability to 1% (low), and 4% (high), respectively. We focus on the scenario of asymmetric taxation (II); that is, ordinary income tax rate is set equal to 40% and the long-term capital gain tax rate is 20%. Again, we first simulate bond prices under the optimal trading strategy and then use these (true) prices to estimate the implied marginal tax rates by ignoring the tax timing option value or assuming that investors follow a buy-and-hold strategy.

[Insert Table 13 here]

Table 13 reports the estimates of implicit marginal income tax rates for bonds of different maturities and default probabilities.[11] For comparison, we also report the results without default risk ($\lambda = 0$). As expected, ignoring the tax-timing option value results in underestimation of the true marginal income tax rate. The bias in estimation increases with maturity and default risk. For example, when default probability is close to zero, the estimated marginal tax rates are 36.2%, 34.7%, 27.1% and 23.3% for bonds with maturities equal to 5, 10, 20 and 30 years. By contrast, the estimated marginal tax rates are 32.8%, 30.5%, 24.7% and 20.7%, respectively, when default probability is increased to 4%. The true marginal income tax rate is 40% for all maturities. The results show that the bias in marginal income tax rate can be quite substantial when the maturity is long.

Green (1993) shows that the estimated implicit marginal tax rates decline with maturity.[12] Although he studied the marginal tax rate for municipal bonds, it will still be interesting to compare with his results. Green's estimates of the marginal income tax rate are 36.2% for five-year bonds, 30.8% for ten-year bonds, and 23.2% for 20-year bonds. These estimates are remarkably similar to the pattern shown in Table 15 for the case of $\lambda = 1\%$ and $\sigma = 0.01$. As indicated earlier, the tax-timing option value increases with maturity. When the value of the timing option is ignored, the higher price (or lower yield) induced by optimal trading must be accommodated by a lower tax rate in empirical fitting. Our results show that neglecting the tax-timing option value contributes to the

[11] These estimates are close to previous estimates. McCulloch's (1975) estimates of the marginal tax rate on Treasury securities range between 22% and 33%. His finding is similar to our numerical results under the high variance process with zero probability of default. As shown in the first column of the high-variance case, when default probability λ equals 0, the implied tax rates range from 23.3% for 30-year Treasury bonds to 36.2% for 5-year Treasury notes. Pye (1969) finds that the effective tax rate of the marginal bondholder ranges between 10% and 36% and Litzenberger and Rolfo's (1984) estimates have an average of 28%. All these estimates fall into the range of our numerical estimates.

increasing underestimation bias of implicit marginal tax rates over maturities. Thus, the empirical puzzle of declining marginal investors' income tax rate with bond maturity may well be due to the omission of the timing option value in the pricing model.

6. Conclusion

Personal taxes have been shown to be an important determinant of cost of debt and capital structure (see Elton et al., 2001; Graham, 1999, 2000, 2003). However, there is almost no study on the issue of tax-timing option for corporate bonds. As a consequence, how big is the tax-timing option value embedded in corporate bonds is unknown. This paper attempts to fill this gap.

We examine the impact of default risk on the trading behavior of bond investors and its effect on the tax-timing option value. It is found that the tax-timing option value tends to decrease with default risk and this effect is larger for long-maturity bonds. The effect of default risk on the timing option value also depends on coupon size, and volatility of interest rates. The timing option value increases with interest rate volatility whereas the effect of coupon size depends on the structure of income taxes.

We find that tax timing option value accounts for a sizable portion of corporate bond price. Under asymmetric taxation, which represents the prevailing tax structure for individuals, the tax timing option value ranges from 15% to 24% for bonds with maturity longer than 20 years when the level of interest rate is high. The timing option value remains sizable, ranging from 10% to 16%, even when both the level and volatility of interest rates are low. Ignoring the tax-timing option value leads to biased estimation of

[12] See Table 3 of Green (1993, p. 239).

default probabilities, implicit income tax rate and corporate bond spread. Thus, the tax-timing option should be considered in pricing corporate bond and estimating spreads.

While it has been shown that the tax-timing option can be quite important for pricing corporate bonds, there is no closed-form formula to estimate its value. The tax-timing option value depends on bond characteristics and the interest rate environment and there is no easy way to prescribe a simple formula to quantify it. For bonds with maturity less than or equal to five years, the timing option value is generally low (less than 1.5%). The effect of ignoring the tax-timing option value is therefore relatively small. However, for long-maturity bonds, the tax-timing option value is typically greater than 10% under a normal level of interest rate volatility. Neglecting the tax-timing option value would lead to serious mispricing of long-term bonds. One possible remedy is to estimate the timing-option value for a particular bond of interest using a numerical method based on the bond characteristics and tax regime. Bond price data can then be adjusted for the tax-timing option value and fitted to the simpler buy-and-hold model. This two-step procedure should provide much reasonable parameter estimation for the term structure model of defaultable bonds.

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Table 1
Treasury bond prices under optimal trading strategy

	High Variance Process σ_r = 0.02 per year				Low Variance Process σ_r = 0.01 per year			
Maturity	I	II	III	IV	I	II	III	IV
				Coupon = 0.06				
5	0.803	0.804	0.838	0.749	0.801	0.801	0.837	0.746
10	0.689	0.699	0.726	0.640	0.682	0.683	0.722	0.629
15	0.619	0.641	0.648	0.586	0.608	0.614	0.642	0.569
20	0.573	0.607	0.593	0.554	0.563	0.579	0.587	0.536
25	0.543	0.586	0.554	0.534	0.533	0.561	0.550	0.518
30	0.522	0.572	0.526	0.521	0.514	0.553	0.524	0.507
				Coupon = 0.10				
5	0.905	0.912	0.923	0.879	0.901	0.901	0.919	0.874
10	0.856	0.890	0.879	0.839	0.845	0.855	0.864	0.822
15	0.828	0.889	0.849	0.825	0.814	0.843	0.832	0.801
20	0.811	0.892	0.826	0.819	0.797	0.847	0.813	0.794
25	0.801	0.896	0.809	0.818	0.789	0.858	0.804	0.793
30	0.795	0.898	0.796	0.817	0.785	0.869	0.798	0.795
				Coupon = 0.14				
5	1.020	1.036	1.037	1.012	1.009	1.017	1.017	1.005
10	1.047	1.103	1.087	1.041	1.023	1.055	1.047	1.020
15	1.065	1.160	1.119	1.070	1.037	1.098	1.076	1.039
20	1.076	1.201	1.135	1.092	1.049	1.138	1.100	1.057
25	1.083	1.230	1.141	1.109	1.061	1.174	1.118	1.074
30	1.087	1.248	1.142	1.121	1.070	1.203	1.131	1.088
				Coupon = 0.18				
5	1.162	1.175	1.182	1.150	1.150	1.155	1.161	1.143
10	1.268	1.329	1.325	1.254	1.245	1.275	1.281	1.233
15	1.331	1.441	1.416	1.328	1.303	1.369	1.366	1.295
20	1.369	1.519	1.469	1.381	1.340	1.442	1.425	1.340
25	1.392	1.570	1.497	1.416	1.367	1.499	1.465	1.375
30	1.407	1.604	1.510	1.441	1.387	1.543	1.491	1.401

The initial short-term interest rate is 14%. σ_r is the annual standard deviation of changes in the short-term interest rate.

If the trading price of the bond is above par, this difference is amortized linearly to the maturity date and the basis is increased by the amount amortized. If the trading price of the bond is below par, the difference is treated as capital gain at the next trading time or at maturity and the basis is equal to the previous trading price.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.25$
II. $\tau_s = 0.50, \tau_l = 0.25$
III. $\tau_s = \tau_l = 0.$
IV. $\tau_s = \tau_l = 0.5.$

Table 2
Tax-timing option value of Treasury bonds under optimal trading strategy

	High Variance Process σ_r = 0.02 per year				Low Variance Process σ_r = 0.01 per year			
Maturity	I	II	III	IV	I	II	III	IV
				Coupon = 0.06				
5	0.0%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%
10	0.6%	2.0%	0.1%	1.6%	0.1%	0.3%	0.0%	0.4%
15	1.3%	4.8%	0.2%	3.4%	0.5%	1.3%	0.1%	1.4%
20	1.8%	7.3%	0.1%	4.6%	0.9%	3.7%	0.1%	2.4%
25	2.3%	9.5%	0.1%	5.6%	1.4%	6.3%	0.2%	3.4%
30	2.7%	11.2%	0.1%	6.2%	1.8%	8.6%	0.2%	4.2%
				Coupon = 0.10				
5	0.2%	1.0%	0.3%	0.4%	0.0%	0.0%	0.0%	0.0%
10	1.1%	4.9%	1.4%	2.2%	0.3%	1.5%	0.2%	0.8%
15	1.8%	8.5%	2.2%	4.0%	0.8%	4.3%	0.8%	2.0%
20	2.3%	11.1%	2.3%	5.2%	1.4%	7.2%	1.6%	3.1%
25	2.7%	12.9%	2.3%	6.1%	2.0%	9.8%	2.4%	4.0%
30	2.9%	14.1%	2.0%	6.7%	2.4%	11.8%	2.9%	4.7%
				Coupon = 0.14				
5	1.5%	3.1%	3.2%	0.8%	0.8%	1.5%	1.5%	0.4%
10	3.4%	8.3%	6.9%	2.8%	1.8%	4.7%	4.1%	1.5%
15	4.2%	12.0%	8.7%	4.6%	2.5%	7.9%	6.1%	2.7%
20	4.3%	14.3%	9.3%	5.7%	3.0%	10.5%	7.4%	3.7%
25	4.3%	15.7%	9.2%	6.5%	3.2%	12.6%	8.2%	4.4%
30	4.1%	16.5%	8.7%	7.0%	3.4%	14.0%	8.5%	5.0%
				Coupon = 0.18				
5	1.6%	2.7%	3.3%	0.6%	0.9%	1.3%	1.8%	0.3%
10	3.0%	7.4%	7.2%	1.9%	1.9%	4.2%	4.7%	0.9%
15	3.4%	10.8%	9.2%	3.2%	2.2%	7.0%	6.8%	1.6%
20	3.3%	12.9%	9.9%	4.2%	2.3%	9.2%	8.1%	2.3%
25	3.2%	14.2%	10.0%	4.8%	2.4%	11.0%	8.9%	2.9%
30	3.0%	14.9%	9.7%	5.3%	2.5%	12.3%	9.3%	3.4%

The initial short-term interest rate is 14%. σ_r is the annual standard deviation of changes in the short-term interest rate.

If the trading price of the bond is above par, this difference is amortized linearly to the maturity date and the basis is increased by the amount amortized. If the trading price of the bond is below par, the difference is treated as capital gain at the next trading time or at maturity and the basis is equal to the previous trading price.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.25$
II. $\tau_s = 0.50, \tau_l = 0.25$
III. $\tau_s = \tau_l = 0.$
IV. $\tau_s = \tau_l = 0.5.$

Tax timing option is defined as $TO(t_0,T) = \frac{v_{OP}(t_0,T) - v_{BH}(t_0,T)}{v_{OP}(t_o,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

Table 3
Defaultable bond prices under optimal trading strategy

	High Variance Process σ_r = 0.02 per year				Low Variance Process σ_r = 0.01 per year			
Maturity	I	II	III	IV	I	II	III	IV
Coupon = 0.06								
5	0.789	0.790	0.821	0.738	0.787	0.787	0.820	0.735
10	0.670	0.678	0.703	0.627	0.664	0.666	0.699	0.616
15	0.600	0.614	0.624	0.572	0.591	0.596	0.619	0.557
20	0.556	0.575	0.571	0.541	0.546	0.555	0.566	0.525
25	0.528	0.552	0.536	0.522	0.519	0.532	0.531	0.507
30	0.509	0.537	0.511	0.510	0.502	0.519	0.508	0.497
Coupon = 0.10								
5	0.887	0.893	0.902	0.866	0.884	0.886	0.899	0.860
10	0.828	0.851	0.842	0.818	0.820	0.828	0.833	0.803
15	0.795	0.833	0.802	0.800	0.784	0.801	0.792	0.779
20	0.777	0.825	0.774	0.792	0.764	0.792	0.768	0.769
25	0.765	0.821	0.755	0.788	0.754	0.792	0.753	0.766
30	0.758	0.819	0.742	0.786	0.749	0.794	0.744	0.766
Coupon = 0.14								
5	0.994	1.004	0.998	0.993	0.984	0.992	0.983	0.987
10	1.003	1.040	1.018	1.011	0.982	1.006	0.986	0.992
15	1.010	1.073	1.029	1.031	0.986	1.026	0.994	1.003
20	1.013	1.095	1.031	1.047	0.991	1.048	1.003	1.016
25	1.015	1.111	1.030	1.058	0.997	1.067	1.011	1.027
30	1.016	1.120	1.025	1.066	1.002	1.082	1.016	1.037
Coupon = 0.18								
5	1.127	1.135	1.135	1.127	1.116	1.122	1.115	1.120
10	1.208	1.249	1.239	1.214	1.187	1.212	1.200	1.194
15	1.253	1.329	1.298	1.274	1.227	1.277	1.256	1.244
20	1.278	1.381	1.328	1.315	1.253	1.324	1.292	1.279
25	1.292	1.414	1.340	1.341	1.270	1.359	1.314	1.305
30	1.299	1.434	1.342	1.359	1.282	1.384	1.326	1.324

The initial short-term interest rates is 14%. σ_r is the annual standard deviation of changes in the short-term interest rate.

The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.25$
II. $\tau_s = 0.50$, $\tau_l = 0.25$
III. $\tau_s = \tau_l = 0$.
IV. $\tau_s = \tau_l = 0.5$.

The default process is exogenously specified, with default probability λ equal to 1% and recovery rate δ equal to 50%.

Table 4
Tax-timing option value of defaultable bonds under optimal trading strategy

	High Variance Process σ_r = 0.02 per year				Low Variance Process σ_r = 0.01 per year			
Maturity	I	II	III	IV	I	II	III	IV
				Coupon = 0.06				
5	0.1%	0.2%	0.0%	0.1%	0.1%	0.1%	0.0%	0.0%
10	0.5%	1.8%	0.1%	1.5%	0.2%	0.4%	0.0%	0.4%
15	1.2%	3.6%	0.2%	3.2%	0.5%	1.3%	0.0%	1.3%
20	1.6%	5.2%	0.1%	4.3%	0.7%	2.3%	0.0%	2.2%
25	2.1%	6.8%	0.1%	5.2%	1.2%	3.7%	0.0%	3.1%
30	2.4%	7.9%	0.1%	5.9%	1.5%	5.0%	0.0%	3.8%
				Coupon = 0.10				
5	0.1%	0.8%	0.1%	0.4%	0.0%	0.2%	0.0%	0.1%
10	0.8%	3.5%	0.6%	2.1%	0.2%	1.2%	0.1%	0.8%
15	1.4%	6.2%	0.9%	3.7%	0.7%	2.9%	0.3%	1.8%
20	1.9%	8.3%	0.8%	4.9%	1.1%	4.9%	0.7%	2.8%
25	2.3%	9.8%	0.6%	5.8%	1.6%	6.6%	1.1%	3.6%
30	2.5%	10.8%	0.5%	6.3%	1.9%	8.0%	1.3%	4.2%
				Coupon = 0.14				
5	1.1%	2.1%	1.7%	0.6%	0.3%	1.1%	0.4%	0.3%
10	2.7%	6.5%	4.8%	2.6%	1.1%	3.5%	2.0%	1.3%
15	3.5%	9.9%	6.4%	4.4%	1.8%	5.9%	3.5%	2.4%
20	3.6%	12.1%	6.8%	5.6%	2.3%	8.1%	4.6%	3.4%
25	3.6%	13.4%	6.6%	6.4%	2.6%	9.8%	5.4%	4.1%
30	3.6%	14.2%	6.0%	6.9%	2.8%	10.9%	5.7%	4.6%
				Coupon = 0.18				
5	1.4%	2.1%	3.1%	0.5%	0.7%	1.3%	1.6%	0.2%
10	2.8%	6.3%	6.9%	1.8%	1.6%	3.8%	4.3%	0.8%
15	3.1%	9.4%	8.7%	3.1%	1.9%	6.0%	6.1%	1.5%
20	3.0%	11.4%	9.1%	4.0%	2.0%	7.8%	7.2%	2.2%
25	2.8%	12.6%	8.8%	4.6%	2.0%	9.2%	7.7%	2.7%
30	2.6%	13.2%	8.2%	5.1%	2.0%	10.1%	7.7%	3.1%

The initial short-term interest rate is 14%. σ_r is the annual standard deviation of changes in the short-term interest rate.

The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.25$
II. $\tau_s = 0.50, \tau_l = 0.25$
III. $\tau_s = \tau_l = 0.$
IV. $\tau_s = \tau_l = 0.5.$

Tax timing option is defined as $TO(t_0, T) = \frac{v_{OP}(t_0, T) - v_{BH}(t_0, T)}{v_{OP}(t_o, T)}$ in percentage, where $v_{BH}(t_0, T)$, $v_{OP}(t_0, T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

The default process is exogenously specified, with default probability λ equal to 1% and recovery rate δ equal to 50%.

Table 5
Effects of transaction costs on the tax-timing option value of par bonds

	Timing option values at different transaction costs				Timing option values at different transaction costs			
Transaction cost	0.0%	0.5%	1%	2%	0.0%	0.5%	1%	2%
	Tax Scenario I				Tax Scenario II			
5	1.1%	0.6%	0.5%	0.6%	2.1%	1.6%	1.2%	0.8%
10	2.7%	1.8%	1.3%	1.1%	6.5%	5.3%	3.7%	2.3%
15	3.5%	2.6%	2.0%	1.5%	9.9%	8.1%	5.8%	3.4%
20	3.6%	2.8%	2.2%	1.7%	12.1%	9.9%	7.1%	3.9%
25	3.6%	2.9%	2.3%	1.8%	13.4%	11.1%	7.8%	4.2%
30	3.6%	2.8%	2.2%	1.8%	14.2%	11.7%	8.2%	4.2%
	Tax Scenario III				Tax Scenario IV			
5	1.7%	1.5%	1.2%	0.9%	0.6%	0.4%	0.3%	0.5%
10	4.8%	4.1%	3.2%	2.4%	2.6%	2.2%	1.7%	1.4%
15	6.4%	5.4%	4.3%	3.3%	4.4%	3.9%	3.3%	2.6%
20	6.8%	5.7%	4.5%	3.6%	5.6%	5.1%	4.4%	3.5%
25	6.6%	5.4%	4.3%	3.5%	6.4%	5.9%	5.2%	4.2%
30	6.0%	4.9%	3.9%	3.2%	6.9%	6.4%	5.7%	4.6%

Computed at midpoint of interest rate range, r = 0.14. Interest rate follows high-variance process with standard deviation of 0.02 per year. Coupon rate is 0.14.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.25$
II. $\tau_s = 0.50$, $\tau_l = 0.25$
III. $\tau_s = \tau_l = 0$.
IV $\tau_s = \tau_l = 0.5$.

Tax timing option is defined as $TO(t_0,T) = \frac{v_{OP}(t_0,T) - v_{BH}(t_0,T)}{v_{OP}(t_o,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

The default process is exogenously specified, with default probability λ equal to 1% and recovery rate δ equal to 50%.

Table 6

Summary statistics of one-month Treasury bill rates (%)

Years	**1981-1991**	**1991-2001**	**1996-2001**
Mean	7.75	4.51	4.86
Standard Deviation	2.60	0.95	0.59

Table 7

Defaultable bond prices under optimal trading strategy

	High Variance Process σ_r = 0.01 per year				Low Variance Process σ_r = 0.005 per year			
Maturity	I	II	III	IV	I	II	III	IV
				Coupon = 0.02				
5	0.853	0.854	0.873	0.823	0.852	0.853	0.872	0.822
10	0.747	0.754	0.774	0.709	0.743	0.744	0.772	0.703
15	0.670	0.684	0.697	0.634	0.662	0.665	0.692	0.623
20	0.612	0.635	0.637	0.582	0.602	0.608	0.630	0.567
25	0.569	0.598	0.590	0.544	0.557	0.568	0.582	0.527
30	0.535	0.571	0.553	0.516	0.524	0.540	0.545	0.499
				Coupon = 0.04				
5	0.916	0.920	0.926	0.902	0.915	0.917	0.925	0.900
10	0.860	0.875	0.872	0.844	0.854	0.859	0.866	0.837
15	0.820	0.849	0.832	0.810	0.809	0.821	0.821	0.795
20	0.792	0.835	0.801	0.788	0.778	0.799	0.787	0.769
25	0.771	0.826	0.777	0.775	0.756	0.786	0.763	0.752
30	0.756	0.822	0.757	0.766	0.740	0.780	0.745	0.741
				Coupon = 0.06				
5	0.983	0.989	0.984	0.982	0.979	0.984	0.978	0.980
10	0.980	1.001	0.986	0.980	0.967	0.981	0.966	0.971
15	0.981	1.022	0.990	0.986	0.962	0.987	0.962	0.969
20	0.985	1.045	0.995	0.996	0.960	0.998	0.962	0.972
25	0.988	1.067	0.998	1.007	0.962	1.014	0.965	0.978
30	0.991	1.087	1.001	1.017	0.965	1.031	0.970	0.985
				Coupon = 0.08				
5	1.062	1.066	1.062	1.064	1.057	1.061	1.053	1.062
10	1.118	1.139	1.127	1.119	1.103	1.118	1.102	1.110
15	1.162	1.207	1.180	1.168	1.140	1.169	1.145	1.150
20	1.197	1.268	1.221	1.210	1.169	1.215	1.180	1.184
25	1.223	1.319	1.252	1.246	1.193	1.258	1.210	1.213
30	1.244	1.363	1.275	1.275	1.213	1.296	1.235	1.238

The initial short-term interest rates is 6%. σ_r is the annual standard deviation of changes in the short rate. The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.2$

II. $\tau_s = 0.40, \tau_l = 0.2$

III. $\tau_s = \tau_l = 0.$

IV. $\tau_s = \tau_l = 0.4.$

The default process is exogenously specified, with default probability λ equal to1% and recovery rate δ equal to 50%.

Table 8
Tax-timing option value of defaultable bonds under optimal trading strategy

	High Variance Process σ_r = 0.01 per year				Low Variance Process σ_r = 0.005 per year			
Maturity	I	II	III	IV	I	II	III	IV
Coupon = 0.02								
5	0.0%	0.1%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%
10	0.3%	1.1%	0.1%	0.6%	0.0%	0.2%	0.0%	0.1%
15	0.6%	2.7%	0.1%	1.3%	0.1%	0.6%	0.0%	0.4%
20	0.9%	4.6%	0.2%	2.2%	0.3%	1.4%	0.0%	0.8%
25	1.2%	6.4%	0.1%	3.0%	0.5%	2.5%	0.1%	1.4%
30	1.4%	8.1%	0.1%	3.7%	0.8%	4.0%	0.1%	2.0%
Coupon = 0.04								
5	0.0%	0.4%	0.0%	0.1%	0.0%	0.2%	0.0%	0.0%
10	0.5%	2.2%	0.5%	0.8%	0.1%	0.7%	0.0%	0.2%
15	0.9%	4.4%	0.8%	1.7%	0.2%	1.7%	0.1%	0.6%
20	1.4%	6.8%	1.1%	2.6%	0.5%	3.2%	0.3%	1.2%
25	1.6%	8.9%	1.3%	3.5%	0.9%	4.8%	0.6%	1.8%
30	1.9%	10.8%	1.3%	4.2%	1.2%	6.6%	1.0%	2.4%
Coupon = 0.06								
5	0.4%	1.0%	0.6%	0.2%	0.1%	0.6%	0.1%	0.0%
10	1.4%	3.6%	2.4%	1.0%	0.4%	1.8%	0.6%	0.4%
15	2.3%	6.5%	3.9%	2.0%	0.9%	3.5%	1.5%	0.9%
20	3.0%	9.4%	5.0%	3.0%	1.4%	5.4%	2.4%	1.6%
25	3.4%	11.7%	5.7%	3.9%	1.9%	7.3%	3.3%	2.2%
30	3.7%	13.7%	6.1%	4.6%	2.3%	9.2%	4.1%	2.8%
Coupon = 0.08								
5	0.7%	1.1%	1.4%	0.1%	0.3%	0.8%	0.7%	0.1%
10	1.9%	3.8%	4.0%	0.8%	1.0%	2.3%	2.1%	0.3%
15	2.6%	6.6%	5.9%	1.5%	1.5%	4.0%	3.5%	0.7%
20	3.1%	9.2%	7.2%	2.3%	1.8%	5.9%	4.8%	1.1%
25	3.3%	11.4%	8.0%	2.9%	2.1%	7.6%	5.7%	1.6%
30	3.4%	13.2%	8.4%	3.5%	2.3%	9.3%	6.5%	2.0%

The initial short-term interest rates is 6%. σ_r is the annual standard deviation of changes in the short-term interest rate.

The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Tax scenarios are described by their marginal income tax rate $\tau = 0.50$, short-term capital gains tax rates, τ_s, and long-term rate, τ_l:

I. $\tau_s = \tau_l = 0.2$
II. $\tau_s = 0.40$, $\tau_l = 0.2$
III. $\tau_s = \tau_l = 0.$
IV. $\tau_s = \tau_l = 0.4.$

Tax timing option is defined as $TO(t_0, T) = \dfrac{v_{OP}(t_0, T) - v_{BH}(t_0, T)}{v_{OP}(t_o, T)}$ in percentage, where $v_{BH}(t_0, T)$, $v_{OP}(t_0, T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

The default process is exogenously specified, with default probability λ equal to 1% and recovery rate δ equal to 50%.

Table 9
Tax-timing option value with different default probabilities

	Tax Timing Option															
	Default = 0%				***Default = 1%***				***Default = 2%***				***Default = 4%***			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
	Coupon = 2%															
5	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.6%	0.0%	0.0%
10	0.3%	1.0%	0.1%	0.7%	0.3%	1.1%	0.1%	0.6%	0.2%	1.3%	0.1%	0.5%	0.2%	1.6%	0.1%	0.4%
15	0.6%	2.7%	0.2%	1.5%	0.6%	2.7%	0.1%	1.3%	0.4%	2.7%	0.1%	1.1%	0.3%	2.8%	0.1%	0.9%
20	1.1%	4.8%	0.3%	2.6%	0.9%	4.6%	0.2%	2.2%	0.8%	4.4%	0.2%	1.9%	0.6%	3.9%	0.1%	1.4%
25	1.4%	6.9%	0.2%	3.6%	1.2%	6.4%	0.1%	3.0%	1.0%	5.9%	0.1%	2.5%	0.7%	4.9%	0.1%	1.8%
30	1.9%	9.0%	0.2%	4.6%	1.4%	8.1%	0.1%	3.7%	1.2%	7.3%	0.1%	3.1%	0.8%	5.6%	0.1%	2.2%
	Coupon = 4%															
5	0.1%	0.3%	0.1%	0.1%	0.0%	0.4%	0.0%	0.1%	0.0%	0.6%	0.0%	0.1%	0.0%	1.0%	0.0%	0.0%
10	0.7%	2.1%	1.1%	0.9%	0.5%	2.2%	0.5%	0.8%	0.3%	2.3%	0.2%	0.7%	0.2%	2.6%	0.1%	0.5%
15	1.4%	4.6%	2.2%	1.9%	0.9%	4.4%	0.8%	1.7%	0.6%	4.3%	0.2%	1.4%	0.4%	4.3%	0.1%	1.1%
20	2.1%	7.3%	3.1%	3.1%	1.4%	6.8%	1.1%	2.6%	0.9%	6.4%	0.3%	2.2%	0.6%	6.0%	0.1%	1.6%
25	2.6%	9.8%	3.8%	4.2%	1.6%	8.9%	1.3%	3.5%	1.1%	8.2%	0.2%	2.9%	0.7%	7.2%	0.0%	2.0%
30	3.0%	12.2%	4.3%	5.2%	1.9%	10.8%	1.3%	4.2%	1.3%	9.7%	0.2%	3.4%	0.9%	8.1%	0.0%	2.4%
	Coupon = 6%															
5	0.7%	1.0%	1.5%	0.2%	0.4%	1.0%	0.6%	0.2%	0.2%	1.1%	0.2%	0.1%	0.0%	1.4%	0.0%	0.1%
10	2.2%	3.7%	4.5%	1.1%	1.4%	3.6%	2.4%	1.0%	0.9%	3.6%	1.1%	0.8%	0.3%	3.7%	0.2%	0.7%
15	3.4%	6.8%	6.9%	2.3%	2.3%	6.5%	3.9%	2.0%	1.4%	6.2%	1.8%	1.7%	0.6%	5.9%	0.2%	1.2%
20	4.2%	9.8%	8.7%	3.5%	3.0%	9.4%	5.0%	3.0%	1.9%	8.6%	2.4%	2.5%	0.8%	7.7%	0.2%	1.8%
25	4.8%	12.3%	9.9%	4.5%	3.4%	11.7%	5.7%	3.9%	2.2%	10.6%	2.6%	3.2%	0.9%	9.1%	0.2%	2.2%
30	5.1%	14.7%	10.7%	5.4%	3.7%	13.7%	6.1%	4.6%	2.3%	12.2%	2.7%	3.7%	1.0%	10.1%	0.1%	2.5%
	Coupon = 8%															
5	0.7%	0.7%	1.5%	0.2%	0.7%	1.1%	1.4%	0.1%	0.6%	1.6%	1.3%	0.1%	0.2%	2.0%	0.2%	0.2%
10	2.1%	3.1%	4.4%	0.8%	1.9%	3.8%	4.0%	0.8%	1.7%	4.5%	3.5%	0.7%	0.9%	4.9%	1.0%	0.8%
15	3.0%	5.8%	6.8%	1.6%	2.6%	6.6%	5.9%	1.5%	2.4%	7.4%	5.0%	1.4%	1.4%	7.6%	1.5%	1.4%
20	3.6%	8.5%	8.6%	2.4%	3.1%	9.2%	7.2%	2.3%	2.8%	9.8%	5.9%	2.2%	1.6%	9.6%	1.7%	2.0%
25	3.9%	10.8%	9.8%	3.2%	3.3%	11.4%	8.0%	2.9%	2.9%	11.8%	6.2%	2.7%	1.7%	11.1%	1.8%	2.3%
30	4.0%	12.9%	10.7%	3.9%	3.4%	13.2%	8.4%	3.5%	2.9%	13.3%	6.3%	3.2%	1.7%	12.1%	1.7%	2.6%

The initial short-term interest rate is 6%. σ_r = 1% is the annual standard deviation of changes in the short-term interest rate.
The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.
Four tax scenarios are defined as in Table 9 and 10.

Tax timing option is defined as $TO(t_0,T)=\frac{v_{OP}(t_0,T)-v_{BH}(t_0,T)}{v_{OP}(t_o,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

The default process is exogenously specified, with default probability λ equal to 1%, 2%, 4% , 6%, respectively and the recovery rate δ equal to 50%.

Table 10
Tax-timing option value with different recovery rates

	Tax Timing Option															
	Recovery = 50%				*Recovery = 40%*				*Recovery = 30%*				*Recovery = 20%*			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
	Coupon = 2%															
5	0.0%	0.1%	0.0%	0.0%	0.0%	0.2%	0.0%	0.0%	0.0%	0.2%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%
10	0.3%	1.1%	0.1%	0.6%	0.3%	1.2%	0.1%	0.6%	0.3%	1.3%	0.1%	0.6%	0.3%	1.4%	0.1%	0.6%
15	0.6%	2.7%	0.1%	1.3%	0.5%	2.8%	0.1%	1.3%	0.5%	2.9%	0.1%	1.2%	0.5%	3.1%	0.1%	1.3%
20	0.9%	4.6%	0.2%	2.2%	0.9%	4.8%	0.2%	2.2%	0.9%	5.0%	0.2%	2.2%	0.9%	5.2%	0.2%	2.2%
25	1.2%	6.4%	0.1%	3.0%	1.1%	6.7%	0.1%	3.0%	1.2%	6.9%	0.1%	2.9%	1.1%	7.2%	0.1%	2.9%
30	1.4%	8.1%	0.1%	3.7%	1.5%	8.5%	0.1%	3.7%	1.5%	8.9%	0.1%	3.7%	1.5%	9.2%	0.1%	3.7%
	Coupon = 4%															
5	0.0%	0.4%	0.0%	0.1%	0.0%	0.4%	0.0%	0.1%	0.0%	0.5%	0.0%	0.1%	0.0%	0.6%	0.0%	0.1%
10	0.5%	2.2%	0.5%	0.8%	0.4%	2.3%	0.4%	0.8%	0.4%	2.4%	0.3%	0.7%	0.4%	2.4%	0.3%	0.7%
15	0.9%	4.4%	0.8%	1.7%	0.8%	4.5%	0.7%	1.6%	0.8%	4.6%	0.5%	1.6%	0.7%	4.7%	0.4%	1.6%
20	1.4%	6.8%	1.1%	2.6%	1.3%	6.9%	0.9%	2.6%	1.2%	7.1%	0.8%	2.6%	1.1%	7.2%	0.6%	2.6%
25	1.6%	8.9%	1.3%	3.5%	1.5%	9.1%	1.0%	3.4%	1.5%	9.3%	0.8%	3.4%	1.4%	9.5%	0.6%	3.4%
30	1.9%	10.8%	1.3%	4.2%	1.8%	11.0%	1.0%	4.2%	1.8%	11.3%	0.8%	4.2%	1.7%	11.5%	0.5%	4.1%
	Coupon = 6%															
5	0.4%	1.0%	0.6%	0.2%	0.3%	1.0%	0.4%	0.1%	0.2%	1.0%	0.3%	0.1%	0.2%	1.1%	0.3%	0.2%
10	1.4%	3.6%	2.4%	1.0%	1.3%	3.7%	2.1%	1.0%	1.2%	3.7%	1.9%	0.9%	1.1%	3.8%	1.6%	0.9%
15	2.3%	6.5%	3.9%	2.0%	2.2%	6.6%	3.5%	1.9%	2.0%	6.7%	3.2%	1.9%	1.9%	6.7%	2.8%	1.9%
20	3.0%	9.4%	5.0%	3.0%	2.9%	9.4%	4.6%	3.0%	2.7%	9.5%	4.2%	3.0%	2.6%	9.6%	3.8%	2.9%
25	3.4%	11.7%	5.7%	3.9%	3.3%	11.8%	5.3%	3.8%	3.1%	11.9%	4.8%	3.8%	3.0%	12.1%	4.4%	3.8%
30	3.7%	13.7%	6.1%	4.6%	3.6%	13.9%	5.7%	4.6%	3.4%	14.0%	5.2%	4.6%	3.3%	14.2%	4.7%	4.5%
	Coupon = 8%															
5	0.7%	1.1%	1.4%	0.1%	0.7%	1.2%	1.4%	0.1%	0.7%	1.3%	1.4%	0.1%	0.7%	1.4%	1.4%	0.1%
10	1.9%	3.8%	4.0%	0.8%	1.9%	4.0%	4.0%	0.8%	1.9%	4.2%	4.0%	0.8%	1.9%	4.4%	4.0%	0.8%
15	2.6%	6.6%	5.9%	1.5%	2.7%	6.9%	5.9%	1.5%	2.7%	7.1%	5.9%	1.6%	2.7%	7.4%	5.9%	1.6%
20	3.1%	9.2%	7.2%	2.3%	3.1%	9.5%	7.2%	2.3%	3.2%	9.8%	7.2%	2.4%	3.2%	10.1%	7.2%	2.4%
25	3.3%	11.4%	8.0%	2.9%	3.4%	11.7%	8.0%	3.0%	3.4%	12.1%	7.9%	3.0%	3.4%	12.4%	7.9%	3.1%
30	3.4%	13.2%	8.4%	3.5%	3.4%	13.6%	8.3%	3.6%	3.5%	14.0%	8.3%	3.6%	3.6%	14.3%	8.3%	3.7%

The initial short-term interest rate is 6%. σ_r = 1% is the annual standard deviation of changes in the short-term interest rate.
The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.
Four tax scenarios are defined as in Table 9 and 10.
Tax timing option is defined as $TO(t_0,T)=\frac{v_{OP}(t_0,T)-v_{BH}(t_0,T)}{v_{OP}(t_o,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.
The default process is exogenously specified, with default probability λ equal to 1% and the recovery rate δ equal to 50%, 40%, 30%, and 20% , respectively.

Table 11
Tax-timing option value with two trading intervals per year

	High Variance Process σ_r = 0.01 per year				Low Variance Process σ_r = 0.005 per year			
Maturity	I	II	III	IV	I	II	III	IV
				Coupon = 0.02				
5	0.1%	1.8%	0.1%	0.3%	0.0%	1.7%	0.0%	0.0%
10	0.4%	4.1%	0.1%	0.9%	0.1%	3.2%	0.0%	0.3%
15	0.7%	6.2%	0.1%	1.7%	0.3%	4.8%	0.0%	0.8%
				Coupon = 0.04				
5	0.2%	1.1%	0.1%	0.3%	0.0%	0.7%	0.0%	0.1%
10	0.5%	3.1%	0.8%	1.1%	0.2%	1.8%	0.1%	0.4%
15	0.8%	4.8%	1.2%	2.0%	0.5%	3.5%	0.2%	1.1%
				Coupon = 0.06				
5	0.5%	1.2%	0.7%	0.3%	0.1%	0.2%	0.1%	0.1%
10	2.3%	5.6%	3.8%	2.2%	0.5%	3.7%	1.7%	0.6%
15	3.8%	8.3%	6.4%	3.2%	1.1%	5.7%	2.5%	1.3%
				Coupon = 0.08				
5	1.2%	1.8%	2.4%	0.3%	0.6%	1.7%	1.2%	0.1%
10	2.3%	5.4%	5.8%	2.1%	2.4%	4.5%	3.1%	0.5%
15	2.9%	8.6%	8.0%	3.0%	3.1%	6.7%	4.7%	1.2%

We set two trading intervals each year. The initial short-term interest rates is 6%. The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.

Tax scenarios are described by their marginal income tax rate $\tau = 0.40$, capital gains tax rates τ_s, short-term, τ_l, long-term:

I. $\tau_s = \tau_l = 0.2$
II. $\tau_s = 0.40$, $\tau_l = 0.2$
III. $\tau_s = \tau_l = 0.$
IV. $\tau_s = \tau_l = 0.4.$

Tax timing option is defined as $TO(t_0,T) = \frac{v_{OP}(t_0,T) - v_{BH}(t_0,T)}{v_{OP}(t_o,T)}$ in percentage, where $v_{BH}(t_0,T)$, $v_{OP}(t_0,T)$ are bond prices under the buy-and-hold and optimal trading strategies, respectively.

The default process is exogenously specified, with default probability λ equal to 1% and recovery rate δ equal to 50%.

Table 12
Estimates of default probabilities when tax-timing options are ignored

	High Variance process σ_r = 0.01 per year				Low Variance Process σ_r = 0.005 per year		
Maturity	λ = 0.02	λ = 0.04	λ = 0.06		λ = 0.02	λ = 0.04	λ = 0.06
5	0.012	0.025	0.038		0.014	0.031	0.047
10	0.010	0.020	0.033		0.011	0.027	0.043
15	0.008	0.017	0.029		0.009	0.024	0.040
20	0.005	0.012	0.025		0.007	0.022	0.037
25	0.003	0.009	0.022		0.004	0.020	0.035
30	0.001	0.007	0.019		0.003	0.018	0.033

The initial short-term interest rate and coupon rate are both equal to 6%. σ_r is the annual standard deviation of changes in the short-term interest rate.
The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.
Tax scenarios are described by their marginal income tax rate $\tau = 0.40$, capital gains tax rates $\tau_s = 0.40$ short-term, $\tau_l = 0.2$ long-term.
The default process is exogenously specified, with default probability λ equal to 2%, 4% , and 6%, respectively and the recovery rate δ equal to 50%.

Table 13
Estimates of implied tax rates when tax-timing options are ignored [a, b, c, d]

	High Variance process σ_r = 0.01 per year				Low Variance Process σ_r = 0.005 per year		
Maturity	λ = 0	λ = 0.01	λ = 0.04		λ = 0	λ = 0.01	λ = 0.04
5	0.362	0.360	0.328		0.388	0.375	0.346
10	0.347	0.316	0.305		0.366	0.355	0.322
15	0.315	0.283	0.278		0.346	0.334	0.304
20	0.271	0.252	0.247		0.326	0.313	0.288
25	0.254	0.227	0.224		0.304	0.293	0.272
30	0.233	0.215	0.207		0.285	0.276	0.261

The initial short-term interest rate and coupon rate are set equal to 6%. σ_r is the annual standard deviation of changes in the short-term interest rate.
The difference between purchase price and par value is amortized linearly to the maturity date and the basis is increased by the amount amortized.
Tax scenarios are described by their marginal income tax rate $\tau = 0.40$, capital gains tax rates $\tau_s = 0.40$ short-term, $\tau_l = 0.2$ long-term.
The default process is exogenously specified, with default probability λ equal to zero, 1%, and 4%, respectively, and the recovery rate δ is 50%.