Volatility Dynamics of Early-stage Firms with Jump Risk and Stage-Clearing

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Abstract

Early-stage firms usually have a major large Research and Development (R&D) project that requires multi-stage investment. Firms' volatility can dramatically change due to the evolvement of R&D efforts and stage clearing. First, the success (failure) of R&D efforts within each stage (jump risk) decreases (increases) the uncertainty (i.e. volatility) level of the firms' future returns – "jump effect". Second, at the end of each stage, firms decide whether to continue next stage investment upon re-evaluating the project prospect conditional on the resolution of technical uncertainty and other information; as firms survive each investment stage and are becoming mature, the uncertainty level of their future returns should eventually decrease in later investment stages that lead to maturity – "stage-clearing effect". Ignoring these effects results in incorrect estimation of firms' future volatility, an important element for early-stage firm valuation. In this paper, I develop a generalized Markov-Switching EARCH methodology for early-stage firms with discrete stage-clearing and jumps. My methodology can identify structural changes in the idiosyncratic volatility and also explore the relation between price changes and future volatility. Using a hand-collected dataset of early-stage biotech firms, I confirmed the existence of the "stage-clearing effect" and the "jump effect". In the second part of my paper, I model early-stage firms as sequences of nested call options with jumps that lead to mature firms. "Jump effect" arises because the early-stage firms are modeled as compound call options with jumps on the underlying cash flows, the volatility of the early-stage firms at each stage is determined by the compound call option elasticity to the underlying cash flows. If the downside (upside) jump happens, the value of the underlying cash flows decreases (increases), which makes the compound call option elasticity go up (down). As a result, the compound call option becomes riskier (less risky). "Stage-clearing effect" arises because as firms exercise their option to continue investment, the new options that firms enter into will eventually become a less risky option.

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1 Introduction

Early-stage firms have no current cash flow and tend to have a major large Research and Development (R&D) project. The process of R&D is lengthy, complex, and risky. First, jumps can arise at any time from R&D failure, breakthrough or legal-regulatory actions.[1] They can also arise from the actions of competitors or changes in the environment immediately before or soon after product introduction.[2] Second, the R&D project typically requires multi-stage investment. Firms have to make discrete decisions on whether to continue next stage investment or abandon the project. When the R&D project is successfully completed, the firm will turn in to mature firms and generate a stream of stochastic cash flows.

Early-stage firms are economically important to study since they predominate in any research-intensive industries such as biotech industry, hardware and software development industry and alternative energy industry. Early-stage firms are hold by venture capitalist type investors. For various reasons, venture capitalist type investors in reality may not hold perfectly diversified portfolios. For instance Gompers and Lerner (1999) found that “the typical venture fund makes one to two dozen investments over its lifespan.” Therefore, it is important to study the volatility dynamics of early-stage firms for risk management.

Both the jumps and the discrete investment decisions may significantly affect the uncertainty level (volatility) about the firms' future returns. On the one hand, jump risks, such as the technical success or failure of a specific R&D effort, may increase or decrease the technical uncertainty of the project which in turn affects the volatility of firms' future returns correspondingly – "jump risk effect". On the other hand, firms' decision to continue next stage investment upon re-evaluating the project prospect at

[1] The clinical trials of a new drug [e.g. DiMasi et al (1995a), DiMasi et al (1995b) and Tufts CSDD], marketing strategy of an internet firm, and exploratory drilling for oil provide powerful examples.

[2] For example, a new drug may be rendered unnecessary by a superior treatment option. A software product may fail because of technological advances in hardware.

the end of each stage should also decrease the project uncertainty in the long run. This is because decisions of continuing investment is conditional on the resolution of technical uncertainty and other information that supports positive outcomes of the project; as firms survive a few investment stages and are becoming mature, the uncertainty level of their future returns should eventually decrease in later investment stages that lead to maturity – "stage-clearing effect". Ignoring these two effects results in incorrect estimation of firms' future volatility, while volatility is an important element for early-stage firm valuation.

In this paper, we explore the effects of jumps and stage-clearing on early-stage firms' future volatility both empirically and theoretically. In the first part of my paper, we try to take into account of the effects of jumps and stage-clearing in estimating the volatility of early-stage firms. It is hard to empirically incorporate jumps and diffusion jointly. While the static OLS model has been extensively used in the literature, it cannot easily capture whatever time variation that may exist in a stock's variance. The traditional GARCH models do not adequately capture structural shifts in the data caused by low probability events. Early-stage firms have extremely large shocks which arise from different causes and have different consequences for subsequent volatility than do small shocks. Therefore, we develop a generalized Markov Switching EARCH methodology for early-stage firms with discrete stage-clearing and jumps. My methodology can estimate the multiple structural changes in the idiosyncratic volatility as early-stage firms clear investment stages by allowing for the possibility of discrete changes in the values of the parameters in the volatility estimation.[3] My methodology can also identify structural changes in the idiosyncratic volatility and explore the relation between value changes due to jumps and future volatility.

[3]The most popular approach for modeling conditional volatility is the GARCH family models as introduced by Engle (1982) and generalized by Bllerslev (1986) and Nelson (1991). Regime switching ARCH models were introduced by Hamilton and Susmel (1994), Cai (1994). There are several common features in these models. First, the conditional volatility process is allowed to switch between a finite numbers of regimes. Second, the timing of regime switch is usually assumed to be governed by a first-order Markov process. In this paper, I use a generalized Markov-Switching EARCH model that allows for regime changes in variance.

I use a unique hand-collected dataset that has all the early-stage biotech firms in the US equity market during the sample period from April 1996 to 2005. We also identify the pipeline of these 102 early-stage firms in the biotech industry and the events list of news release of those firms. We apply Markov Switching EARCH methodology to the 102 early-stage firms in the biotech industry. We find consistent evidence for the "jump risk effect" and "stage-clearing effect". Specifically, we find (1) discrete decreases in volatility when firms clear R&D stages; (2) significant negative correlation between value changes and future volatility, especially when there are news about the technical success or failure (jump) within each stage of these firms.[4] The dates of changing regimes identified by Markov Switching EARCH methodology are also consistent with my life-cycle investment stages dataset. We also find that for mature firms[5] the uncertainty level of their future returns does not decrease eventually, i.e. the "stage-clearing effect" does not hold for mature firms.

In the second part of my paper, we develop a model that models early-stage firms as sequences of nested call options with jumps that lead to mature firms. The business as usual uncertainty about the potential future cash flows of the project is modeled in a diffusion process; the technical success or failure, the rare firm-specific event, is given by the jump part which is a compound Poisson process with finite jumps in every time interval. Since this paper focuses on the firm-specific events on R&D efforts, jumps are modeled as non-market risk or private risk (or idiosyncratic risk). We build a theoretical model that considers different types of jump distribution. We derive an explicit formula of the volatility of an investment call option in a jump-diffusion frame work. We solve the problem by Monte Carlo simulation adapting the procedure developed by Longstaff and Schwartz (2001) for valuing American-type options.

[4] Other papers also documented a similar pattern in general firms (e.g.Black (1976), Christie[1982], Schwert[1989], Glosten, Jagannathan and Runkle [1992], Braun, Nelson and Sunier [1995], and many others). Fixed costs such as financial and operating leverage were offered as an explanation for this effect. However, it was shown that these costs can only partially explain this pattern. My paper shows that this pattern is related with the technical uncertainty for early stage firms.

[5] The mature firms are at least publicly traded for 10 years before my sample period and are not included in my sample during my sample period.

I am able to obtain a number of interesting results on joint existence of different sources of risk in determining the value and volatility of the early-stage firms. First, my model shows that when the early-stage firms have good news, the subsequent volatility will become smaller; when the early-stage firms have bad news, the subsequent volatility will become more volatile. This "jump risk effect" also exists in my previous empirical findings. The intuition for my model to explain this effect is as follows: Since the early-stage firms is a compound call option on the underlying cash flows, the volatility of the early-stage firms at each stage is determined by the compound call option elasticity to the underlying cash flows. If the downside jump happens, the value of the underlying cash flows decreases, which makes the compound call option elasticity go up. As a result, the compound call option becomes riskier. If the upside jump happens, the value of the underlying cash flows increases, which makes the compound call option elasticity go down. Consequently, the compound call option becomes less risky. Second, my model shows that as firms survive each investment stage and are becoming mature, the uncertainty level of their future returns eventually decrease in later investment stages that lead to maturity – "stage-clearing effect". This is because as technical uncertainty is resolved, the probability of successful completion changes. This changes the properties of the option to abandon the project, which is exercised with the potential future cash flow. If firms exercise their option to continue investment, the new options that firms enter into will eventually become a less risky option. Third, ignoring sudden and dramatically bad or extremely good jumps will lead to over-estimate the values of early-stage firms about 23 percent on average. The early-stage firm is a series of compound options on the underlying cash flows. The "strike price" of this option is the expected future investments required to complete the R&D. The impact on firm value depends on the volatility of the underlying cash flows and investment requirements (away-from-the-money or at-the-money). Without considering the technical risk results in over-estimates of the volatility of the underlying cash flows. The net result of over-estimated volatility of the underlying cash flows and in-the-money investment requirement is an over-estimate in the values of early-stage firms.

This study contributes to the literature in several ways. First, empirically, we use a new methodology-Markov Switching EARCH-to jointly estimate the jump risk effects and multiple structural changes in the idiosyncratic volatility as early-stage firms clear investment stages. The estimation of idiosyncratic risk plays a critical role in many areas of research.[6] Campbell, Lettau, Malkiel and Xu (2001) first documented the increase in idiosyncratic volatility during 1964-1997, nevertheless, Bekaert, Hodrick and Zhang (2008) showed that there is no upward trend in idiosyncratic volatility anywhere in the developed world when they include 8 more years of data. Fink, Fink, Grullon and Weston (2005) and Brown and Kapadia (2007) ascribe the rise in idiosyncratic volatility to the increasing propensity of firms to issue public equity at an earlier stage in their life cycle. They show that after controlling for the proportion of young firms in the market, there is no trend in the time series of idiosyncratic risk. In this paper, we show why early-stage firms have high idiosyncratic volatility in the early life cycle stage and have low idiosyncratic volatility in the later life cycle stage. This explains that why there is an increasing trend in idiosyncratic volatility when there is increasing propensity of early-stage firms to issue public equity, but there is no trend in idiosyncratic volatility when there is less early-stage firms issue public equity and early-stage firms in the early life cycle stage are turning into later life cycle stage. To the best my knowledge, there is no study that studies structural changes in the firm-level idiosyncratic risk. Markov Switching EARCH model is different from the usual structural change model. The former allows regular regime-switching at any random point of time, while the latter only permits occasional endogenous changes. By allowing the conditional variances of the firm to have abrupt shifts improves the model's in-sample fit and the out-of sample forecasts of volatility. My methodology therefore generates better estimates of idiosyncratic risk. The out-of-sample forecasts should result in improvements in financial risk management and trading strategies that involve volatility arbitrage. Second, theoretically, we derive

[6] For example, Campbell et al.(2001) use a variance decomposition method to study the dynamics of idiosyncratic risk, industry, and market components of the volatility of individual stock returns. Goyal and Santa-Clara (2003) study the relationship between average stock variance (largely idiosyncratic) and the return on the market. Spiegel and Wang (2005) use EGARCH model to study the role of idiosyncratic risk and liquidity in determining stock returns.

an explicit formula of the volatility of an investment call option in a jump-diffusion framework. This formula explains the discrete changes in volatility when firms clear R&D stages and negative relation between price changes due to jumps and future volatility. The proposed model can contribute to the evaluation of the real growth options of R&D investments in the research-intensive industries such as biotech, software or hardware development, and the alternative energy industry.

The theoretical part of my paper is closest to Berk, Green and Naik (2004), which develops a dynamic model of multi-stage investment project that captures many features of R&D ventures and start-up companies. Their model studies how systematic and idiosyncratic technical risks determine the risk premium and value of ventures. My paper takes a different angle by focusing on the effect of jumps and discrete stage clearing on the volatility dynamics of early-stage firms and we show that ignoring the effect in estimating future volatility has economically and statistically important effect on valuation. Several papers in the economics and finance literature have studied the R&D process as a contingent claim on the value of an underlying cash flows, which is interpreted as the present value of the cash flows on completion of the R&D. Majd and Pindyck (1987) solve an investment problem in which the project requires a fixed total investment to complete, with a maximum instantaneous rate of investment. But all the uncertainty in their model comes from stochastic evolution of the value of the project upon completion. There is no jump/technical risk.

The remainder of this paper is organized as follows. Section 2 discusses the empirical methodology, describes the data, and presents the empirical results of my analysis. The theoretical model is developed in Section 3. Section 4 describes the method to solve the problem and presents the theoretical model results. Section 5 provides some concluding remarks.

2 Empirical Evidence

I develop a generalized Markov Switching EARCH methodology that takes into account of the effect of jumps and stage-clearing in estimating the volatility of early-stage firms. My methodology can identify structural changes in the idiosyncratic volatility and explore the relation between price changes and future volatility.

2.1 Data

I collect the list of early-stage firms' data from IPO central, Ivo Welch's website and IPO monitor. IPO Central has the data from April 1996 to 2001, Ivo Welch's website has data from 1993 to 1996, and IPO Monitor has data from 1997 to 2005. To be included in my sample, a firm must be not from basic materials; financial services; utility sectors. (I use SIC sector/industry category.) We also require the market values of the firms bigger than $50 million, the trading volumes of the firms bigger than 5000, and stock prices bigger than $1. These criteria result in 688 early-stage firms list.

With this early-stage firms list, we have identified the pipeline of 102 early-stage firms from the biotech industry, life cycle investment stages, and the events list of news releases of those firms. To identify the pipeline of early-stage firms from the biotech industry, we first identify all firms with drug candidates in clinical trials in Phase I, Phase II or Phase III during 1993 to 2005 at www.clinicaltrials.gov, which is a government database of clinical trials. We then exclude large, diversified companies from my study. We also exclude from my sample every company that is on this list from Yahoo, with market cap greater than $1B[7]. Furthermore, we exclude companies on this list with market cap greater than $5B. After that we have a list of Phase I, Phase II and Phase III clinical trials, start and stop dates, all smaller companies with very few products in development or in the market. The stock price of these companies will be most sensitive to news about the clinical trials. Next, we try to identify the life cycle investment stages (early development, late development, early revenue, shipping, mature A and mature B) of

[7]data source http://biz.yahoo.com/p/510mktu.html

each firm based on the news release from its website and its revenue per year, operation expenditure, operation income and net income. The detail life cycle investment stage criterion is reported in Table 1.

With this list, we obtain daily and weekly stock return data from CRSP from January of 1993 to December 2005. The weekly frequency of the risk-free rate is not directly available, so it is computed, using the continuously compounded formula, as the compound of the weekly risk-free rate. The weekly excess stock market return is the sum of the daily stock market return in excess of the daily risk-free rate within each week. The biotech industry portfolio returns are obtained from French's website and they are constructed according to the classification scheme in Fama and French (1997). Fama and French (1997) assign each NYSE, AMEX, and NASDAQ stock to an industry[8] portfolio at the end of June of year t based on its four-digit SIC code at that time. (They use Compustat SIC codes for the fiscal year ending in calendar year $t-1$. Whenever Compustat SIC codes are not available, they use CRSP SIC codes for June of year t.) They then compute industry portfolio returns from July of t to June of $t+1$.

2.2 Empirical Methodology

I decompose firm return and industry returns into the three components. For industry returns:

$$R_t = \alpha + \beta_m R_{mkt} + \eta_t \tag{1}$$

and for individual firm returns:

$$\begin{aligned} R_{jt} &= \alpha_j + \beta_j R_t + \varepsilon_{jt} \qquad (2) \\ &= \alpha_j + \beta_{jm} R_{mkt} + \beta_j \eta_t + \varepsilon_{jt} \qquad \varepsilon_{jt} | I_{t-1} \sim t(0, h_{jt}) \\ \frac{\ln h_{jt}}{\gamma_{s_t}} &= a_0 + \sum_{l=1}^{q} a_l \frac{\ln \varepsilon^2_{jt-l}}{\gamma_{s_{t-l}}} + \xi_1 \cdot d_{1,t-1} \cdot \frac{\ln \varepsilon^2_{j,t-1}}{\gamma_{s_{t-1}}} - \xi_2 \cdot d_{2,t-1} \cdot \frac{\ln \varepsilon^2_{j,t-1}}{\gamma_{s_{t-1}}} \qquad (3) \end{aligned}$$

[8] data source http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_49_ind_port.html

In equation (1)β_m denotes the beta for biotech industry with respect to the market return, and η_t is the biotech industry-specific residual. Similarly, in equation (2) β_{jm} denotes the beta for firm j with respect to the market return, β_j is the beta of firm j with respect to biotech industry and ε_{jt} is the firm-specific residual. ε_{jt} are assumed to follow a student-t distribution which allows for fatter tails than the Gaussian distribution. R_t is the industry excess return, R_{jt} is the firm excess return at time t, R_{mkt} is the market excess return at time t, h_t is the model's estimate of the ε_{jt}'s conditional variance. S_t denotes the regime of the volatility at time t. The constant γ_{s_t} captures the structural shift parameter, and the autoregressive coefficients $\frac{a_l}{\gamma_{s_{t-l}}}$ depend on the current and lagged regimes of the firm. For instance, a shift from a high to a low variance regime, would be captured in a change in the $\frac{a_l}{\gamma_{s_{t-l}}}{}'s$. Where the $\gamma_{s_t}' s$ are scale parameters that capture the change in regime. Here γ_{s_t} denotes the parameter γ_1 when the process is in the regime represented by $s_t = 1$, while γ_{s_t} indicates γ_2 when $s_t = 2$, and so on. One of the $\gamma_{s_t}' s$ is unidentified and, hence, the first regime γ_1 is set equal to 1 with $\gamma_{s_t} \geq 1$ for $s_t = 2, 3, ..., G$. To explore the technical success or failure (jump) impact on price changes and future volatility, we let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ and there is bad news about R&D project at time t-1, $d_{1,t-1} = 0$ if $\varepsilon_{jt-1} \geq -0.05$ or there is no bad news about R&D project. we let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ and there is good news about R&D project at time t-1, $d_{2,t-1} = 0$ if $\varepsilon_{jt-1} \geq 0.05$ or there is no good news about R&D project. If there is significant bad news about R&D project, the subsequent return volatility is bigger, it predicts that $\xi_1 > 0$. If there is significant good news about R&D project, the subsequent return volatility is smaller, it predicts that $\xi_2 > 0$. Following Hamilton (1989), we can use maximum likelihood estimation to solve the model.

Such a model also requires a formulation of the probability law that causes the idiosyncratic volatility level to switch among regimes. One simple specification is that the state of the idiosyncratic volatility level is the outcome of a G-state Markov chain that is independent of R_{jt} for all t: $Prob(s_t = u|s_{t-1} = v, s_{t-2} = w, ..., R_{jt}, R_{jt-1}, R_{jt-2,...}) = \Pr ob(s_t = u|s_{t-1} = v) = p_{uv}$. Under this specification the transition probabilities, the

p_{uv}'s, are constant. For example, if the idiosyncratic volatility level was in a high variance regime last period ($s_t = 2$), the probability of changing to the low variance regime ($s_t = 1$).is a fixed constant p_{21}.

I use the identified pipeline of 102 early-stage firms from biotech industry to decide the number of stages (regimes) for the Markov Switching EARCH(G, q) model. To decide the appropriate lag length of my Markov Switching EARCH(G, q) model, we compare the Akaike Information Criterion (AIC) and the Schawarz's Bayesian Criterion (SBC). There are several equivalent formulations for these criteria. We will use the following: $AIC = T\log(RRS) + 2NR$ and $SBC = T\log(RRS) + NR(\log T)$, where RRS is the sum of squared residuals. NR is the number of regressors and T is the number of observations.

2.3 Estimation Results

2.3.1 Main Results

In this section, we apply my estimation method to the 102 early-stage firms from biotech industry. Table 2 estimates Markov Switching EARCH(G, q) models of the relationship between the firm risk premium, market risk premium, and conditional firm variance using one sample firm KOSP weekly data for the period August 1997 to December 2005. The model estimated are nested within the Markov Switching EARCH model in the previous section. The parameter estimates are presented in Table 2 for both the cases Markov Switching EARCH$(2, 2)$ and Markov Switching EARCH$(2, 1)$. we am particularly interested in the results for the estimated $\gamma_{2.}$ As argued by Hamilton and Susmel (1994), γ_2 provides an estimate of the ratio of the conditional variance in regime 2, relative to low-volatility regime. That is, in my two regimes' case, γ_2 provides information on how high is moderate volatility relative to low volatility. In every specification at which we looked, the scale parameters that capture the change in regime γ_2 is strongly statistically significant on the basis of likelihood ratio test. These results indicate the evidence of discrete decreases in volatility when firms clear R&D stages. The jump risk effect [the parameter ξ_1 and ξ_2 in Eq. (3)] is likewise statistically significant. This means significant negative correlation between price changes and future volatility especially when there are significant news about the R&D project within each stage of these firms. The transition probability matrix is as follows

$$P = \begin{pmatrix} p & 1-p \\ 1-q & 1 \end{pmatrix}$$

where $p = Pr(s_t = 1|s_{t-1} = 1)$, and $q = Pr(s_t = 2|s_{t-1} = 2)$. The superscript s indicates regime 1 or 2. The estimated p and q probabilities are large (near one), indicating persistent regimes.

[Insert Table 2 about here]

Figure 1 plots the results of one sample firm KOSP from the above procedure. Panel A plots the weekly returns of firm KOSP from August 1997 to Dec 2005. Panel B plots conditional volatility estimates of firm returns from the student t Markov Switching EARCH $(2, 2)$ model. Panel C plots smoothed probability that firm was in regime 1 (lower volatility $\gamma_1 = 1$) for each week, as calculated from the student t Markov Switching EARCH $(2, 2)$. Panel D plots smoothed probability that firm was in regime 2 (higher volatility $\gamma_2 = 6.30$) for each indicated week, as calculated from the student t Markov Switching EARCH $(2, 2)$. The smoothed probability of being in regime 2 at time t is computed using information from the whole time-series up to time T, that is, $P[s_t = 2|R_T, ..., R_1]$. If we define R_t to be in regime 2 if the probability of being in regime 2 is higher than 0.5, and vice versa for regime 1. As can be expected from the parameter estimates, there are high-variance regime and low variance regime over the sample. The high-volatility period (regime 2 higher volatility $\gamma_2 = 6.30$) lasted from August $22, 1997$ to December $12, 2003$; the low-volatility period (regime 1 lower volatility $\gamma_1 = 1$) lasted from December $13, 2003$ to December 2005. I found strong evidence of regime-shifting behavior. This shows discrete decreases in volatility when KOSP clear R&D stages.

Around December $12, 2003$, when the early-stage firms changed stage, the subsequent volatility become smaller. It is interesting to note that this is also consistent with my life cycle investment stages dataset.

[Insert Figure 1 about here]

I summarize the cross sectional results in table 3. Table 3 estimates Markov Switching EARCH $(2, 2)$ models of the relation between the firm risk premium, market risk premium, and conditional firm variance using all the sample firms for the period 1993 to 2005. First, we estimated the Markov Switching EARCH model for each firm in my sample separately. The number of regimes G for each firm is decided by its life-cycle investment stages. Second, we sort the estimated volatility level $\gamma'_{s_t}s$ of each firm by its life-cycle stages in time. We let γ_{first} denote the estimated $\gamma'_{s_t}s$ in the first life-cycle investment stage in the sample period, and γ_{last} denotes the estimated $\gamma'_{s_t}s$ in the last life-style investment stage in the sample period. For example, for the KOSP case, since the estimated γ in the first life-cycle investment stage from August $22, 1997$ to December $12, 2003$ is γ_2, we let γ_{first} equals $\gamma_2 = 6.30$; since the estimated γ in the last life-cycle investment stage from December $13, 2003$ to December 2005 is γ_1, we let γ_{last} equals γ_1. We report the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} in the first column of table 3. The difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the early-stage firm group is 4.73 and unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$, is 1.19. The t test indicate that the scale parameters γ_{first} is significant different from he scale parameters γ_{last}. This shows the volatility level change between the first stage and the last stage is significant different from zero. It indicates the uncertainty level of their future returns decrease eventually.

Table 3 also reports the sample mean of ξ_1 and ξ_2 (the jump risk effect) and the standard deviation of ξ_1 and ξ_2. The sample mean of ξ_1 is 0.11 and the the standard deviation of ξ_1 is 0.03. The t test indicate that ξ_1 is statistically significant different from zero. It means when there is extremely bad news about the R&D project, firm's expected return volatility increases by 11%. The sample mean of ξ_2 is 0.06 and the the standard deviation of ξ_2 is 0.03. The t test indicate that ξ_2 is statistically significant different from zero. It means when there is extremely good news about the R&D project, firm's expected return volatility decreases by 6%. This shows the uncertainty level of their future returns eventually decrease in later investment stages that lead to maturity – "stage-clearing effect" and significant negative correlation between price changes and future volatility especially when there are significant news about the R&D project within each stage of these firms.

[Insert Table 3 about here]

Table 4 summarizes the estimation results using the Markov Switching EARCH model for the biotech firms which are not included in my sample during the sample period. We divide them into two groups: mature firms and died early-stage firms. The mature biotech firms are at least publicly traded for 10 years before my sample period and are continuously existed during my sample period. There are 153 biotech mature firms. The dead early-stage firms are the early-stage firms which died during my sample period. There are 36 dead early-stage firms. We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ as a proxy for extremely bad news about R&D project at time t-1, $d_{1,t-1} = 0$ if $\varepsilon_{jt-1} \geq -0.05$. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ as a proxy for extremely good news about R&D project at time t-1, $d_{2,t-1} = 0$ if $\varepsilon_{jt-1} \leq 0.05$. The second column reports the number of firms in each group. The third column reports the difference between the sample mean of the volatility level at the first stage $\gamma_{_{first}}$ and the last stage $\gamma_{_{last}}$. The fourth column reports the unbiased estimator of the standard deviation of the two samples $\gamma_{_{first}}$ and $\gamma_{_{last}}$, $std(\overline{\gamma}_{_{first}} - \overline{\gamma}_{_{last}})$. Based on the estimation results, all the early stage firm in my sample have more than one regime. The third row reports that the difference between the sample mean of the volatility level at the first stage $\gamma_{_{first}}$ and the last stage $\gamma_{_{last}}$ of the early-stage firm group is 4.53 and unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$ is 1.34. The t test indicate that the scale parameters γ_{first} is significant different from he scale parameters γ_{last}. This shows the volatility level change between the first stage and the last stage is significant different from zero. It also shows the uncertainty level of their future returns decrease eventually. based on the estimation results, all the dead early-stage firm group in the third row, there is no regimes changes. The smoothed probability that firm is in high volatility regime is always above 90%. The uncertainty level of their future returns does not change. Of all the mature biotech firms, 121 biotech mature firms do not have volatility level changes at all. The smoothed probability that firm is in high volatility regime is always above 90%. The remaining 32 firms have volatility level changes. The last row reports that the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the mature firm group is 1.22 and unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$, is 6.0. The t test indicate that the scale parameters γ_{first} is significant different from he scale parameters γ_{last}. It shows that the difference between the scale parameters $\gamma_{first} - \gamma_{last}$, that captures the changes in regime, is not significant different from zero at all. It indicates uncertainty level of their future returns did not decrease.

[Insert Table 4 about here]

To summarize the results so far, the regime switching autoregressive conditional heteroskedasticity model appears to provide an accurate description of the data. We use a generalized Markov Switching EARCH methodology that allows for the possibility of discrete changes in the values of the parameters in the volatility estimation. Because early-stage firms have extremely large shocks, arise from different causes and have different consequences for subsequent volatility than do small shocks. We have two findings. First, the discrete decrease of the uncertainty level of the early-stage firms' future returns when the resolution of the technical uncertainty in each stage enables firms to invest for the next stage. Second, the success or failure of R&D efforts within each stage (jump risk) also decreases or increases the uncertainty level of the early-stage firms' future returns. We confirmed the existence of the "stage-clearing effect" and the "jump risk effect".

2.3.2 Robustness

This section summarizes the estimation results using the Markov Switching EARCH model for the biotech firms which are not included in my sample during the sample period. We divide them into two groups: mature firms and died early-stage firms. The mature biotech firms are at least publicly traded for 10 years before my sample period and are continuously existed during my sample period. There are 153 biotech mature firms. The dead early-stage firms are the early-stage firms which died during my sample period. There are 36 dead early-stage firms. We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ as a proxy for extremely bad news about R&D project at time t-1, $d_{1,t-1} = 0$ if $\varepsilon_{jt-1} \geq -0.05$. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ as a proxy for extremely good news about R&D project at time t-1, $d_{2,t-1} = 0$ if $\varepsilon_{jt-1} \leq 0.05$.

Table 5 Panel A is the estimation result using Markov Switching EARCH model with 2 regimes. We report the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} in the second column of table 4 Panel A. The third column reports the unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$. The second row reports that the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the early-stage firm group is 4.13 and unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$ is 1.07. The t test indicate that the scale parameters γ_{first} is significant different from he scale parameters γ_{last}. This shows the volatility level change between the first stage and the last stage is significant different from zero. It also shows the uncertainty level of their future returns decrease eventually.

For the dead early-stage firm group in the third row, there is no regimes changes. The smoothed probability that firm is in high volatility regime is always around 1. The uncertainty level of their future returns does not change. Of all the mature biotech firms, 121 biotech mature firms do not have volatility level changes at all. The smoothed probability that firm is in high volatility regime is always around 1. The remaining 32 firms have volatility level changes. The fourth row reports that the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the mature firm group is 0.02 and unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$, is 5.8. The t test indicate that the scale parameters γ_{first} is significant different from he scale parameters γ_{last}. It shows that the difference between the scale parameters $\gamma_{first} - \gamma_{last}$, that captures the changes in regime, is not significant different from zero at all. It indicates uncertainty level of their future returns did not decrease.

[Insert Table 5 Panel A about here]

Table 5 Panel *B* is the estimation result using Markov Switching EARCH model with 3 regimes. Similarly, it shows that for mature firms the uncertainty level of their future returns does not decrease eventually. And for dead early-stage firms there is no regimes changes. Of all the early-stage biotech firms in my sample, 20 biotech early-stage firms have three regimes in the volatility level. The second row reports that the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the early-stage firm group is 5.53 and the unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$, is 1.49. It indicates uncertainty level of their future returns decrease eventually. It shows that the difference between the scale parameters $\gamma_{first} - \gamma_{last}$, that captures the changes in regime, is significant different from zero. Of all the biotech mature firms, 10 biotech early-stage firms have three regimes in the volatility level. The fourth row reports that the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} of the mature firm group is 0.01 and the unbiased estimator of the standard deviation of the two samples γ_{first} and γ_{last}, $std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$, is 6.2. The t test indicate that the scale parameters γ_{first} is not statistically significant different from the scale parameters γ_{last} It indicates the uncertainty level of mature firms's future returns did not decrease.

[Insert Table 5 Panel B about here]

3 The Model

In this section, we develop a model that models early-stage firms as sequences of nested call options with jumps that lead to mature firms. The early-stage firm has no current cash flows and consists of a single R&D project, but through its R&D has acquired a compound option to launch a product at some future time. Launching the product needs sequential investment. When the R&D project is successfully completed, it becomes like an established firm. The established firm will generate a stream of stochastic cash flows, which we model as a geometric Brownian motion process that has both idiosyncratic and systematic risks. In order to realize these cash flows, the firms must first complete N discrete stages of R&D. We can then value the early-stage firm simply as a compound call option on the established firm.

A typical example is an investment cycle project, where R&D is followed by an early development phase, late development phase, early revenue phase, shipping phase, and mature phase[9]. The life cycle of an early-firm is based on its revenue per year, operation expenditure, operation income, and net income. The Figure 2 is an example of a early-stage firm with two-stage sequential investments to complete its R&D project.

[9] The life cycle investment stages (early development, late development, mature) of each firm based on its revenue per year, operation expenditure, operation income, and net income. In the empirical part of my paper, I have detail definitions for each stage. The detail life cycle stage criterion is reported in data description.

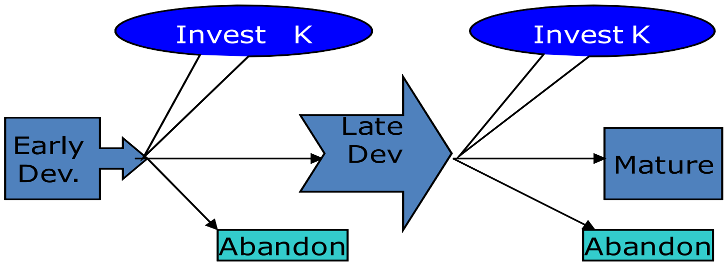

Figure 2 a early-stage firm with two-stage sequential investments to complete its R&D project. The life cycle investment stages are late development phase and mature phase.

3.1 Set Up

I consider a firm working in continuous time on an R&D project to bring a new product to the market. Completion of the project involves passing through a sequence of N discrete stages of development successfully. At the end of each stage, the firm must decide whether or not to invest for the next stage. If the firm chooses not to invest for the next stage, it will exit and the early-stage firm value becomes zero. In particular, we assume that, upon investing an amount K at time T_n, the early-stage firm faces the opportunity to continue the R&D project, $n = 1, 2, ..., N$. The investment cost K at each stage is a constant.

The existence of good news, bad news, booms and busts in investments cause discontinuities in returns. A R&D-based investment has such high instability in outcomes. There is private risk in the success or failure of the R&D that is unrelated to the value of the established firm. The sources of private risk can be R&D failure or breakthrough, competitor action, and legal or regulatory actions. We decompose the uncertainty in the early-stage firm into two components:

- Continuously evolving uncertainty coming from the business. i.e., levered position in the established firm.

- Discrete jumps coming from the technical failure/success.

Therefore, we consider the value of the early-stage firms' underlying cash flows $X = \{X_t, 0 \leqslant t \leqslant N\}$ follow a mixed jump diffusion process.

$$\frac{dX}{X} = \alpha dt + \sigma_X dz + (Y-1)dq \tag{4}$$

where α is the expected return on the cash flow, σ_X is the volatility of the underlying cash flows, conditional on no arrivals of important new information (i.e. the Poisson event does not occur) and dz denotes the increments of Wiener Process. The jump component is a "Poisson-driven" process. The Poisson-distributed "event" is the arrival of an important piece of information about the early-stage firm. It is assumed that the arrivals are independently and identically distributed. λ is the mean number of arrivals per unit time; q is the cumulative number of jumps in the underlying cash flows value $X = \{X_t, 0 \leqslant t \leqslant N\}$ to date. Then the change in q called dq indicates a jump; dq becomes a dummy variable that can take on a value of 0 (usually) or 1 (occasionally). Over short time periods, the probabilities are:

$$dq = \begin{cases} 0 & \Pr\text{ob(the event does not occur in the time interval } (t, t+dt)) = 1 - \lambda dt \\ 1 & \Pr\text{ob(the event occurs once in the time interval } (t, t+dt)) = \lambda dt \end{cases} \tag{5}$$

Given that the Poisson event occurs (i.e. some important information on early-stage firm arrives), there is a "drawing" from a distribution to determine the impact of this information on the value of cash flow, i.e., if X_t is the cash flow value at time t and $Y-1$ is the random variable accounts for the relative jump amplitude. Neglecting the continuous part, the cash flow value at time t+dt, X_{t+dt} , will be the random variable $X_{t+dt} = X_t Y$, given that one such arrival occurs between t and $t+dt$. The value of cash flow changes by some amount $\gamma(X, t, Y) = X_t Y - X_t = X_t(Y-1)$. Conditional on a jump, the expected capital gain or loss as a percentage of the underlying cash flow X_t is $E(Y-1) \equiv g$, where E is the expectation operator over the distribution function of

Y under the objective probability measure P. It is assumed throughout that Y has a probability measure with compact support and $Y \geq 0$. Moreover, the $\{Y\}$ from successive drawings are independently and identically distributed. Y, dq and dz are assumed to be independent of each other.

The underlying cash flow process $X = \{X_t, 0 \leqslant t \leqslant N\}$ as given by (4) has two sources of uncertainty. The term σdz corresponds to "business-as usual" uncertainty, while the term dq describes rare events. If the Poisson event does not occur (i.e. $dq = 0$) , then the return dynamics would be identical to the process presented by Black and Scholes (1973) and Merton (1973). If on the other hand the Poisson event occurs, then $(Y - 1)$ is an impulse function which takes the project value from X_t to $X_t Y$. Although the firm does not receive X_t prior to the completion of the R&D project, it is assumed that the firm's decision makers can still observe X_t - they know what the firm's cash flows would be were the project complete today. Similarly to Merton (1976), we are interested in the case where jumps are firm specific and uncorrelated with the market as a whole. If we assume that the conclusions of the CAPM hold, then this nonsystematic risk has a premium of zero. Although this assumption may be too strong for industries where firms may place an important premium jump risk, it has been widely used in the real options literature (see, for instance, Martzoukos and Trigeorgis (2002)).

Following standard arguments in the mathematical finance literature we can construct the risk-neutral pricing measure under which we work for the remaining part of the paper. The process for the underlying cash flow under $\mathbb{Q}$ is given by:

$$\frac{dX}{X} = \alpha^* dt + \sigma_X dz^* + (Y - 1)dq \tag{6}$$

where dz^* a standard Wiener process, dq and Y are as above, independently distributed of $dz^*$and α^* is chosen such that the discounted project value is a martingale under $\mathbb{Q}$:

$$\alpha^* = r - \lambda g \tag{7}$$

where r denotes the risk-free rate.

Sequential investment opportunities may be viewed as compound options, and the option value may be derived in a recursive way. Let

$$V_N(T_N, X) = \max\{X_N - K, 0\} \tag{8}$$

$e^{r(T_N - T_{N-1})} E_{N-1}[V_N(T_N, X)]$ denote the value of an European call option at time T_{N-1} with exercise price K and expiration date T_N, we now define a sequence of call options, with value

$$V_n(T_n, X) = \max\{e^{-r(T_{n+1}-T_n)} E_n[V_{n+1}(T_{n+1}, X)] - K, 0\} \tag{9}$$

on the call option whose value is $e^{-r(T_{n+1}-T_n)} E_n[V_{n+1}(T_{n+1}, X)]$, with exercise price K and expiration date T_{n+1}, for $n = 1, 2.., N-1$, where we assume $T_1 \leqslant T_2... \leq T_N$. we want to determine the value of the early-stage firm $V_n(T_n, X)$ at each stage $T_n, n = 0, 1, 2, ..., N-1$. Let X_n^* denote the value of X such that $e^{-r(T_{n+1}-T_n)} E_n[V_{n+1}(T_{n+1}, X)] - K = 0$ if $n < N$ and $X_N^* = K$.

Because all the calls are a function of the value of the underlying cash flow X and the time t, the following partial differential equation holds for $V_n(t, X)$:

$$0 = \frac{\partial V_n(t,X)}{\partial t} + (r - \lambda g) X \frac{\partial V_n(t,X)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_n(t,X)}{\partial X^2} + \lambda E[V_n(t, XY) - V_n(t,X)] - rV_n(t,X) \tag{10}$$

$t \leqslant T_n, n = 0, 1, 2, ..., N, T_1 \leqslant T_2... \leq T_N$. The the boundary condition is

$$V_n(T_n, X)) = \max\{e^{-r(T_{n+1}-T_n)} E_n[V_{n+1}(T_{n+1}, X)] - K, 0\} \tag{11}$$

$$V_N(T_N, X) = \max\{X_N - K, 0\} \tag{12}$$

where $e^{-r(T_{n+1}-T_n)} E_n[V_{n+1}(T_{n+1}, X)]$ stands for the value of the underlying compound option at T_n.

The jump size can have different types of distributions; jump size may be negative, positive, big, or small. we have different specification on g; for example $g = -1$, means if the jump occurs, it is only to bankruptcy. The jump intensity (jump frequency, rate of jumps per period) can also change over time. We use numerical techniques to value the early-stage firm in section 3.

3.2 Volatilities of the Early-stage firms

Empirical data shows that when the early-stage firms have good news, the subsequent volatility at the next stage will become smaller; if the early-stage firms have bad news, the subsequent volatility at the next stage will become more volatile. This phenomenon "Jump risk effect"can also be replicated in my model. The intuition that my model can replicate this feature in the data is as follows: Since the early-stage firms is a compound call option V on the underlying cash flows X , the volatility of the early-stage firms at each stage is decided by the compound call option elasticity ($\frac{\partial V}{\partial X} \times X/V$) and the value of the underlying cash flows X. If the downside jump happens, the value of the underlying cash flows X decreases, which makes the compound call option elasticity go up. As a result, the subsequent volatility of the compound call option will go up. If the upside jump happens, the value of the underlying cash flows X increases, which makes the compound call option elasticity go down. As a result, the subsequent volatility of the compound call option will go down.

"Stage clearing effect" arises because as firms exercise their option to continue investment, the new options that firms enter into will eventually become a less risky option. If the investment stages are more than two stages, then in stages between the first stage and the last stage, the model allows firms to enter into an option potentially riskier than the options in previous stages (i.e., to continue investments even when temporarily get bad results in middle stages). But the firm cannot always do that over all stages since option value maximization mechanism restricts the firm's net present value to be positive. Therefore, the firm has to either stop investment after a series of bad results or survive the R&D development if there are enough good results to resolve the technical uncertainty

Now, we analyze the volatility of early-stage firm in the general case when there are N nested compound call option, where $N > 2$. We first want to determine the value of early-stage firms $V_n(T_n, X)$ at each stage n, $n = 0, 1, 2, ..., N-1$. Let X_1^* denote the value of X such that $e^{r(T_2-T_1)}E_1[V_2(T_2,X)] - K = 0$. Then, if the value of underlying cash flow X is greater than X_1^*, the Nth-compound call option will be exercised (firms choose to invest K to continue the project), while for the values less than X_1^* it will remain unexercised (firms choose not to invest K to continue the project)

I define J_i the number of Possion arrivals in the time interval $[T_i, T_{i+1}]$, $i = 0, 1, 2, ..., N-1$, and $T_0 = 0$. Consequently, let $m_n = \sum_{i=1}^{n} J_i$ be the total number of arrivals in the interval $[0, T_n]$ for $n = 1, 2, ..., N$ and $v_n = T_{n+1} - T_n$. Following Gukhal (2004), which derives analytical valuation formulas for compound options when the underlying asset follows a jump-diffusion process, We derive analytical valuation formulas for n nested multicompound options on jump-diffusions.

Proposition 1 *The valuation of early-stage firms at each stage n is*

$$V_n(T_n, X) = \prod_{j=n}^{N}[\sum_{J_j=0}^{\infty} \frac{e^{-\lambda v_j(\lambda v_j)^{J_j}}}{J_j!} X_0 M_n(a_{m1}, a_{m2}, ..., a_{mn}, \rho_n^n)] + \tag{13}$$

$$-\sum_{j=n}^{N}\{\prod_{p=j}^{n}[\sum_{J_p=0}^{\infty} \frac{e^{-\lambda v_p(\lambda v_p)^{J_p}}}{J_p!} K e^{-rT_j} M_{n+1-p}(b_{m1}, b_{m2}, ..., b_{mn,}\rho_{n+1p}^n)]$$

where

$$b_{m_n} = \frac{\ln(X_0/X_n^*) + [r - \frac{1}{2}(\sigma_X^2 + m_n\frac{\sigma_J^2}{T_n})]T_n}{\sqrt{(\sigma_X^2 + m_n\frac{\sigma_J^2}{T_n})T_n}}$$

$$a_{m_n} = b_{m_n} + \sqrt{(\sigma_X^2 + m_n\frac{\sigma_J^2}{T_n})T_n}$$

Let $M_k(a_{m1}, a_{m2}, ..., a_{mn}, \rho_n^n)$ denote the n-dimensional cumulative distribution function, with upper limits of integration $a_{m1}, a_{m2}, ..., a_{mn}$ and correlation matrix ρ_n^n. Conditional on observing V_i and V_j jumps, ρ_n^n is a ndimension symmetric correlation matrix with typical element

$$\rho_{s_i,V_j} = \frac{\sqrt{(\sigma_X^2 + m_i\frac{\sigma_J^2}{T_i})T_i}}{\sqrt{(\sigma_X^2 + m_j\frac{\sigma_J^2}{T_j})T_j}}$$

for 1$\leq i \leq j \leq n$ and $n = 1, 2, ..., N$. The volatility of the early-stage firm at each stage n is

$$\frac{\partial V}{\partial X} \times \frac{X}{V} \times \sigma_X = \frac{E \times \sigma_X}{E - F} > \sigma$$

where

$$E \equiv \frac{\partial V}{\partial X} \times X = \prod_{j=1}^{n}[\sum_{J_j=0}^{\infty} \frac{e^{-\lambda v_j}(\lambda v_j)^{J_j}}{J_j!} X_0 M_n(a_{m1}, a_{m2}, ..., a_{mn}, \rho_n^n)$$

$$F \equiv \sum_{j=1}^{n}\{\prod_{p=j}^{n}[\sum_{J_p=0}^{\infty} \frac{e^{-\lambda v_p}(\lambda v_p)^{J_p}}{J_p!} K_j e^{-rT_j} M_{n+1-p}(b_{m1}, b_{m2}, ..., b_{mn,}\rho_{n+1p}^n)]$$

$$\partial(\frac{\partial V}{\partial X} \times \frac{X}{V})/\partial X < 0 \tag{14}$$

The elasticity of early-stage firms $V_X \times X/V$ is a decreasing function in underlying cash flows X. If the downside jump happens, the value of the underlying cash flows X decreases, which makes the compound call option elasticity go up. As a result, the subsequent volatility of the compound call option will go up. Consequently, there are multiple structural changes in the volatility as early-stage firms clear investment stages. In the model, we assume the uncertainty from the business (market risk and industry risk) does not have structural change, therefore there are multiple structural changes in the idiosyncratic volatility as early-stage firms clear investment stages.

3.3 An Example with Two Stages

In this section, we use a simple example with two stages to illustrate how the volatility changes when the underlying cash flows changes.

Consider the early-stage firm is a compound call option with expiration date T_1 and strike price K written on the value of early-stage firm $V_1(T_1, X)$ at time T_1 where $T_1 < T_2$. The value of early-stage firm at time T_1 can be thought as an European option $V_1(T_1, X)$ at T_1 with expiration date T_2 and strike price K. Let $V_0(T_{0,}X)$ denote this compound option, the present value of the early-stage firm. The early-stage firm choose to invest K to continue the project (i.e. this compound option is exercised) at time T_1 when the expected value of early-stage firm $V_2(T_2, X)$ (i.e. the value of the underlying asset) at time T_1, exceeds the investment cost K (strike price). When $e^{r(T_2-T_1)}E_1[V_2(T_2, X)] < K$, it is not optimal to invest K to continue the project and hence exit the project. The value of the underlying cash flow at which early-stage firm is indifferent between investing and not investing is specified by the following relation: $V_1(T_1, X_1^*) = K$. This value of indifference is denoted by X_1^*.

When it is optimal to exercise the compound option at time T_1, the early-stage firm pays K and receives the value of early-stage firm (an European option) $V_2(T_2, X)$. At time T_2, the early-stage firm can in turn continue to invest K to continue the project when the value of the underlying cash flow X_2 exceeds K and expires worthless otherwise. Hence, the cash flows to the early-stage firm are an outflow of K at time T_1 when $X_1 > X_1^*$, a net cash flow at time T_2 of $X_2 - K$ when $X_1 > X_1^*$ and $X_2 > K$ and none in the other states.

The value of the early-stage firm is the expected present value of these cash flows and is given by

$$E_0[e^{-rT_2} \cdot (X_2 - K) \cdot 1_{\{X_2 \geq K_2\}} \cdot 1_{\{X_1 \geq X^*\}}] + E_0[e^{-rT_1}(-K) \cdot 1_{\{X_1 \geq X^*\}}] \tag{15}$$

Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). This is similar to the process specified in Merton (1976) in that the jump risk is diversifiable, and hence not priced in equilibrium. To simplify notation, we also assume g equals zero[10], i.e.

$$\mu_J = -\frac{1}{2}\sigma_J^2 \tag{16}$$

J_1 is the number of jumps in the intervals $[0, T_1]$, and J_2 is the number of jumps in the intervals $(T_1, T_2]$. Conditioning on the number of jumps J_1 and J_2, the logarithmic return $r_{T_1} = \log(X_{T_1}/X_0)$ has a normal distribution with mean $\mu_J T_1 = (r - \sigma_X^2/2)T_1$ and variance $\sigma_{JD1}^2 T_1 = \sigma_X^2 T_1 + J_1\sigma_J^2$ and $r_{T_2} \sim N(\mu_{JD}, \sigma_{JD}^2 T_2)$ where $\mu_{JD}T_2 = (r - \sigma_X^2/2)T_2$

[10] That is, $E[ln(Y)] = \mu_J$ and $Var[\ln(Y)] = \sigma_J^2$, so $E[Y] = \exp(\mu_J + \frac{1}{2}\sigma_J^2)$.

and $\sigma_{JD}^2 T_2 = \sigma_X^2 T_2 + (J_1 + J_2)\sigma_J^2$. The correlation coefficient between $r_{T_1}$and r_{T_2} is given by $\rho_{1T} = \frac{cov(x_{T_1}, x_{T_2})}{\sqrt{var(x_{T_1})(var(x_{T_2})}}$. The valuation of the early-stage firms V is known from Gukhal (2004)

$$E_0[e^{-rT_2} \cdot (X_2 - K) \cdot 1_{\{X_2 \geq K_2\}} \cdot 1_{\{X_1 \geq X^*\}}] + E_0[e^{-rT_1}(-K) \cdot 1_{\{X_1 \geq X^*\}}]$$
$$= \sum_{J_1=0}^{\infty}\sum_{J_2=0}^{\infty} \frac{e^{-\lambda T_1}(\lambda T_1)^{J_1}}{J_1!}\frac{e^{-\lambda v}(\lambda v)^{J_1}}{J_2!} \times \{X_0 M(a_1, b_{1,}\rho_{1T}) - Ke^{-rT}M(a_2, b_2, \rho_{1T})\} \quad (17)$$
$$+ \sum_{J_1=0}^{\infty} \frac{e^{-\lambda T_1}(\lambda T_1)^{J_1}}{J_1!} Ke^{-rT_1}N(a_2) \quad (18)$$

where

$$a_1 = \frac{\ln(X_0/X_1^*) + (\mu_{JD1} + \sigma_{JD1}^2/2)T_1}{\sigma_{JD1}\sqrt{T_1}},$$
$$a_2 = a_1 - \sigma_{JD1}\sqrt{T_1},$$
$$b_1 = \frac{\ln(X_0/K) + (\mu_{JD} + \sigma_{JD}^2/2)T_2}{\sigma_{JD}\sqrt{T_2}},$$
$$b_2 = b_1 - \sigma_{JD}\sqrt{T_2}$$

and $M(x, y, \rho)$ is the $Pr[X < x, Y < y]$ where X and Y are bivariate normal with a correlation coefficient of ρ.

The volatility of the early-stage firm is

$$\frac{\partial V}{\partial X} \times \frac{X}{V} \times \sigma_X = \frac{D}{D - A - B} \times \sigma_X > 1 \times \sigma_X \quad (19)$$

where

$$D \equiv \frac{\partial V}{\partial X}X_0 = \sum_{J_1=0}^{\infty}\sum_{J_2=0}^{\infty} \frac{e^{-\lambda T_1}(\lambda T_1)^{J_1}}{J_1!}\frac{e^{-\lambda v}(\lambda v)^{J_1}}{J_2!} M(a_1, b_{1,}\rho_{1T})X_0$$
$$A \equiv \sum_{J_1=0}^{\infty}\sum_{J_2=0}^{\infty} \frac{e^{-\lambda T_1}(\lambda T_1)^{J_1}}{J_1!}\frac{e^{-\lambda v}(\lambda v)^{J_1}}{J_2!} \times K_2 e^{-rT}M(a_2, b_2, \rho_{1T})$$
$$B \equiv \sum_{J_1=0}^{\infty} \frac{e^{-\lambda T_1}(\lambda T_1)^{J_1}}{J_1!} K_1 e^{-rT_1}N(a_2)$$

$$\partial(\frac{\partial V}{\partial X} \times \frac{X}{V})/\partial X < 0 \quad (20)$$

The elasticity of early-stage firms $\frac{\partial V}{\partial X} \times X/V$ is a decreasing function in underlying cash flows X. If the downside jump happens, the value of the underlying cash flows X decrease, which makes the compound call option elasticity go up. As a result, the subsequent volatility of the compound call option will go up.

4 Numerical Results

In this paper, we solves the problem with Monte Carlo simulation by adapting the procedure developed by Longstaff and Schwartz (2001) for valuing American-type options. The simulation approach used is very intuitive, flexible, and transparent. In addition, it provides insights about the volatilities of early-stage firms at each stage that are lost in the more complex numerical solution to the partial differential equation.

To solve the model numerically, we use the following (annualized) baseline parameter values: the risk-free interest rate $r = 0.02$, the expected growth rate of cash flow $\mu_x = 0.05$. Market volatilities will be used for cash flow simulation along the new drug's commercial life. We have selected a representative group of ten biggest biotechnological firms whose volatility parameters are shown in figure 3.

Figure 3 Volatilities for ten of the biggest biotechnological firms[11]

	Volatility
Amgen	26.47%
Genentech	36.94%
Serono	32.30%
Biogen	37.27%
Genzyme	36.96%
Chiron	47.68%
Medimune	30.76%
Celltech group	47.64%
Average	37.07%

[11] Source: Top Biophamaceutical companies report. 2003.

Volatilities are provided according to annual terms. However, Yahoo Finance volatility is calculated on four-monthly data. It is calculated on monthly data within the website of financial analysis "Focus.com". Results shown by "Volatility.com" correspond to a n implicit volatility over the ordinary price of a call option. We consider five values of the volatility of the underlying cash flow $\sigma_X \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$. We obtained the jump density parameter λ from DiMasi, J.A. (1995a and 1995b) and Myers and Howe (1997), and we consider five values $\in \{0, 0.1, 0.2, 0.3, 0.4\}$. In the next section We perform $500,000$ simulations of the 8 stage life cycle of a single company.[12]

4.1 Valuation of Early-stage Firms

In this section, we compare the impact of the jump on the valuation of the early-stage firms.

Valuation of Early-stage Firms ignoring Jump Risk Table 6 shows the value of the early-stage firm that requires 8 stages to complete. The original state is $X_0 = 100$, σ_X is the volatility of the established firm, conditional on no jump. The cost is flat cost, which means the launching costs of the early-stage firms at each stages are $K_t = 10$, $\forall t = 0, 1, ..., 8$. Risk free rate is 2%. We perform $500,000$ simulations of the 8 life cycle stage of a single company. We found that as the volatility of the established firm increases, the value of the early-stage firms at each stage also increases. Since we model the early-stages firm as a series of compound options on the underlying established firm, a higher the volatility of the established firm σ_X will lead to a higher value of the early-stage firms ($V_t, t = 0, 1, ...8$, compound options on the established firm). When a stage is completed the expected future investment is decreased, thus reducing the "strike price" and therefore increasing the "in the moneyness." The net result is an increase in the value of the firm.

[12] See DiMAsi et al. (1991) and Schwartz (2004).

[Insert Table 6 about here]

Valuation of Early-stage Firms considering Jump Risk We now turn to the impact of the jump on the valuation of the early-stage firms. R&D in the early-stage firms frequently has upward jumps or downward jumps. For example, drugs can turn in to mega-selling block-buster products, or, alternatively, pipeline compounds suffer clinical trial failures and withdrawal from the markets. Hence, we consider different distribution of the jump size in the following section, lognormal and fixed size jump. We use the same parameterization in this section as previously. In addition we also need to specify the jump distribution and parameters.

The importance of uncertainty in R&D productivity can be gauged by contrasting the case with jump to the case with no jump. Table 7 reports the jump size impact on the value of the early-stage firms considering jump. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY (In other words, the function for the change in value is $\gamma(X,t,Y) = XY - X = X(Y-1)$.) It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500,000$ independent simulations of firm. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. To facilitate this comparison, Table 6 Panel A only provides the value of the early-stage firm V_0 with jump frequency $\lambda = 0.3$, at different jump size σ_J; Panel B provides value of the early-stage firm at each stage with jump frequency $\lambda = 0.3, \sigma_X = 0.3$ at different jump size σ_J .One fact stand out in the table. The value of the early-stage firms at each stage when considering jump is orders of smaller magnitude. The extra uncertainty from jump reduces considerable value to the project in its early- stages, because the dominant downside jump risk decreases the value of growth option. Ignoring such jumps due to these innovations or failure results in over-estimates of the volatility of the underlying cash flows σ_X and inaccurate estimates of the valuation of the early-stage firms.

The total expected return r in equation (6) has two components: One part comes from the diffusion process, $\alpha^* = r - \lambda g$, and the other comes from the jump process λg. The second moment $\overline{\sigma}$ in equation (6) has two components: One part comes from the diffusion process, σ_X and the other, denoted σ_{jump}, comes from the jump process: $\sigma_{jump} = \sqrt{\lambda Var(Y)} = \sqrt{\lambda(e^{\sigma_J^2} - 1)e^{2\mu_J + \sigma_J^2}}$. We want to compare the value of early-stage firm using the pure-diffusion process with using the jump-diffusion process. Thus, we need to choose the parameters of the jump-diffusion process in such a way that the first two moments for this process given in equation (6) match exactly the first two moments of the pure-diffusion process. The interpretation of the compensation of the parameters is that the total expected return on the underlying process X, r and the second moment $\overline{\sigma}$ substracts from them λg and σ_{jump}, respectively, with the understanding that this will be added back through jump term, $(Y - 1)dq$. In this way, the expected return and second moment coming from the diffusion terms in order to offset exactly the contribution of the jump. Even though the unconditional expected return and second moment under the compensentated jump-diffusion process will match those from the pure-diffusion process, the two process will not lead to identical valuations. This is because the jump also introduces skewness and kurtosis into the return process. For example, the second moment of a firm $\overline{\sigma}$ equals to 0.5. If ignoring jumps due to innovations or failure, (i.e. using the pure-diffusion process), the diffusion volatility is $\sigma_X = 0.5$, and the estimated value of the early-stage firms without jump is 23.23 from Table 6 Panel A; If considering the jumps, by matching the second moment and expected return, we can decompose the second moment into diffusion process $\sigma_X = 0.4$ (the average volatility of the biggest biotech firms obtained from the bottom line of Figure 3) and jump process $\sigma_{jump,}$ which has a jump frequency $\lambda = 0.3$ [the average jump frequency obtained from DiMasi, J.A. (1995a and 1995b) and Myers and Howe (1997)] and jump size $\sigma_J = 0.2$ [obtained from Martzoukos and Trigeorgis (2002)], $\mu_J = -\frac{1}{2}\sigma_J^2 = -0.02$ [from equation (16)], the estimated value of the early-stage firms is 18.77. To sum up, ignoring sudden and dramatically bad or extremely good jumps will lead to over-estimate the values of early-stage firms about 23 percent on average.

[Insert Table 7 about here]

The importance of jump frequency impact on the value of the early-stage firms considering jump can be seen in Table 8. Table 8 Panel A is the value of the early-stage firm V_0 with jump different frequency. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The results show that the higher probability of the jump to occur during a certain period of time, the lower the valuation of the firm. The higher jump intensity λ pushes trigger values higher and early-stage firms value (compound option values) lower. In R&D under uncertainty, this confirms the adage of the more discoveries or successful clinical trials, the more valuable the project. Table 8 Panel B is the value of the early-stage firm V_0 with different jump frequency. Assume fixed size jump Y, $Y = 0.1$ means when jump happens, X changes immediately to $0.1X$. $Y = 0.5$ means when jump happens, X changes immediately to $0.5X$. σ_X is the volatility of the established firm. The result is similar to the lognormal jump case.

[Insert Table 8 about here]

4.2 Volatility of Early-stage firms

In this section, we compare the impact of the jumps and stage-clearing on the volatility of the early-stage firms.

Volatility of Early-stage Firms ignoring Jump Risk Another interesting feature illustrated by table 9 is that even though the firm's investment experience is idiosyncratic, resolution of this uncertainty still affects the volatility. At each stage, we divide the whole sample into two subsamples based on underlying cash flows value change. If the value of the underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt - X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; if the value of the underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. we calculate the volatility of each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 8$. The net result changes in the volatility of early-stage firms depends on the change of the underlying cash flows. Since the early-stage firms is a compound call option on the underlying cash flows, the volatility of the early-stage firms at each stage is decided by the compound call option elasticity and the value of the underlying cash flows. At each stage, if the downside jump happens, which makes the value of the underlying cash flows X decrease, the firm belongs to the "Down" subsample. The value of the underlying cash flows X decrease makes the compound call option elasticity go up. As a result, the subsequent volatility of the compound call option will go up. If the upside jump happens, which makes the value of the underlying cash flows increase, the firm belongs to the "Up" subsample. The value of the underlying cash flows increase makes the compound call option elasticity go down. Consequently, the subsequent volatility of the compound call option will go down.

[Insert Table 9 about here]

Volatility of Early-stage Firms considering Jump Risk We now turn to the impact of the jump and stage-clearing on the volatility of the early-stage firms. R&D in the early-stage firms frequently jumps up or jumps down. For example, drugs can turn in to mega-selling block-buster products, or, alternatively, pipeline compounds suffer clinical trial failures and withdrawal from the markets. Hence, we consider different distribution of the jump size in the following section, lognormal and fixed size jump. We use the same parameterization in this section as the previous section specified in the valuation of the early-stage firm with jump. The importance of uncertainty in R&D productivity can be gauged by contrasting the case with jump to the case with no jump. To facilitate this comparison, Table 10 Panel A only reports the jump size impact on the volatility of the early-stage firms considering with jump frequency $\lambda = 0.3$ at different jump size σ_J. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal

(μ_J, σ_J^2). To simplify notation, we also assume that g equals zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$ At each stage, we divide the whole sample into two subsamples based on the underlying cash flows value change as we stated at the beginning of this section. We calculate the volatility of each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 8$ within subsamples. Two facts stand out in the table. First, the volatility of the early-stage firms at each stage when considering jump is orders of larger magnitude. Second, the volatility of early-stage firms at each stage is more sensitive to the value of the underlying cash flows' change in the presence of the jump. The extra uncertainty, in which downside risk dominates upside risk (based on the assumption), adds considerable uncertainty to the project in its early-stages because it makes the value of the underlying cash flows fluctuate more. Ignoring such jumps due to these innovations or failure results in under-estimates of volatility of the early-stage firms at each stage. Since the early-stage firms is a compound call option on the underlying cash flows, the volatility of the early-stage firms at each stage is decided by the compound call option elasticity and the value of the underlying cash flows. At each stage, if the downside jump happens, the value of the underlying cash flows X decrease, which makes the compound call option elasticity go up. As a result, the subsequent volatility of the compound call option will go up. For example, the total volatility of a firm $\overline{\sigma}$ equals to 0.6. If ignoring jumps due to innovations or failure, (i.e. using the pure-diffusion process), the diffusion volatility is $\sigma_X = 0.6$, and the estimated volatility of the early-stage firms from "Up" subsample at stage 3 is 1.17 (result from Table 10); If considering the jumps, by matching the second moment and expected return, we can decompose the second moment into diffusion process $\sigma_X = 0.4$ and jump process σ_{jump} (jump frequency $\lambda = 0.3$ and jump size $\mu_J = -0.08, \sigma_J = 0.4$), the estimated volatility of the early-stage firms is 0.99. The net result is when the early-stage firms have good news, the subsequent volatility will become smaller; when the early-stage firms have bad news, the subsequent volatility will become more volatile. This phenomenon also exists in the empirical findings. Third, the volatility discrete decreases when firms clear R&D stages. This is because technical uncertainty is resolved, the probability of successful completion changes.

[Insert Table 10 about here]

The importance of jump frequency impact on the volatility of the early-stage firms considering jump can be seen in Table 11. Table 11 Panel A is the volatility of the early-stage firm at each stage with jump different frequency. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The results show that the higher probability of the jump to occur during a certain period of time, the larger the volatility of the firm. The higher jump intensity λ push trigger values higher and early-stage firm's volatility larger. In R&D under uncertainty, this confirms the adage of the more discoveries or successful clinical trials, the more volatile the project.

[Insert Table 11 about here]

Figure 4 shows the jump size impact on the volatility of the early-stage firms considering jump. Figure A is the volatility of the "Up" subsample early-stage firm at each stage with jump frequency $\lambda = 0.3$. The x axis is the stage number; the y axis is the estimated volatility of the early-stage firms. "–" line is for the σ_j the volatility of the of the log normal jump = 0.1 "o" line is for the σ_j the volatility of the of the log normal jump = 0.2, "+" line is for the σ_j the volatility of the of the log normal jump = 0.3. Figure B is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$,when the underlying firm value increases. Figure C is the volatility of the "Down" subsample early-stage firm at each stage with jump frequency $\lambda = 0.3$. The x axis is the stage number; the y axis is the estimated volatility of the early-stage firms. "–" line is for the σ_j the volatility of the of the log normal jump = 0.1. "o" line is for the σ_j the volatility of the of the log normal jump = 0.2, "+" line is for the σ_j the volatility of the of the log normal jump

= 0.3. Figure D is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$, when the underlying firm value decreases.

[Insert Figure 4 about here]

Figure 5 shows when the early-stage firms have good news, the subsequent volatility will become smaller; if the early-stage firms have bad news, the subsequent volatility will become more volatile. It also shows the volatility discrete decreases when firms clear R&D stages. Figure A is the volatility of the "Up" subsample early-stage firm at each stage with different jump frequency and $\sigma_J = 0.3$. Figure B is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$, when the underlying firm value increases. Figure C is the volatility of the "Down" subsample early-stage firm at each stage with different jump frequency $\sigma_J = 0.3$. Figure D is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$, when the underlying firm value decreases.

[Insert Figure 5 about here]

5 Conclusions

Early-stage firms' success and failures are usually prevalent during clinical studies or after they are launched in the market. Investment cost, asset returns, optimal operating policy, and probabilities of success or failure are key issues and primary concerns of staring a R&D early-stage firms. To date, the effects of both positive and negative effects caused by extreme rare events on the volatility and valuation of early-stage firms has rarely been considered. The aim of this paper is to investigate impacts of mixed jumps for the early-stage firms.

I use a new empirical methodology to estimate the multiple structural changes in the idiosyncratic volatility as early-stage firms clear investment stages. To the best of my knowledge, there is no study that connects structural changes in the estimation of idiosyncratic risk with firm investment stages and jumps. We use a generalized Markov Switching EARCH methodology that allows for the possibility of discrete changes in the values of the parameters in the volatility estimation. we find discrete decreases in volatility when firms clear R&D stages ("stage-clearing effect"); significant negative correlation between price changes and future volatility especially when there are news about the technical success or failure (jump) within each stage of these firms ("jump risk effect"). My empirical results is consistent with my theoretical results. My results have broad implication in valuation, risk management, government policy in alternative energy industry, and trading strategies that involve volatility arbitrage.

I study the real options in a R&D project of an early-stage firm associated with uncertainty from the project and possibility of jumps which may either end the investment opportunity or boost its prospects. This specific model evaluates investment made in risky project through a jump distribution. The fat tails of this statistical process serve as an alternative change to measure "value at risk" for R&D discoveries from financial risk applications.

The resolution of idiosyncratic uncertainty can dramatically alter the volatility on the early-stage firms. This explains the discrete changes in volatility when it clears R&D stages. My model explains that when the early-stage firms have good news, the subsequent volatility will become smaller; if the early-stage firms have bad news, the subsequent volatility will become more volatile. This phenomenon also exists in the empirical findings.

Appendix

A Proof of Proposition 1

Proof. The value of the early-stage firm is the expected present value of the resulting payoffs on the completed project:

$$
\begin{aligned}
& V_0(X,T_0) = \max\{e^{-rT_1}\cdot E_0^{\mathbb{Q}}[V_1(X,T_1)-K],0\} \\
= \; & E_0^{\mathbb{Q}}[e^{-rT_1}\cdot V_1(X,T_1)1_{\{X_1\geqslant X_1^*\}}(X_1)] - E_0^{\mathbb{Q}}[e^{-rT_1}\cdot K\cdot 1_{\{X_1\geqslant X_1^*\}}(X_1)] \\
= \; & ... \\
= \; & E_0^{\mathbb{Q}}[e^{-rT_N}\cdot X_N\cdot 1_{\{X_1\geqslant X_1^*\}}(X_1)\cdot ...\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)] \\
& -\sum_{j=1}^{N} E_0^{\mathbb{Q}}[e^{-rT_j}\cdot K\cdot 1_{\{X_1\geqslant X_1^*\}}(X_1)\cdot ...\cdot 1_{\{X_j\geqslant X_j^*\}}(X_j)]
\end{aligned}
$$

where $1_{\{X_n\geqslant X_n^*\}}(X_n)$ an indicator function with $1_{\{X_n\geqslant X_n^*\}}(X_n)=1$ if $X_n\geqslant X_n^*$, the early-stage firm chooses to invest K to continue the R&D project (i.e. the option is exercised), and $1_{\{X_n\geqslant X_n^*\}}(X_n)=0$ otherwise, the early-stage firm chooses not to investment for the next stage, it will exit and the value of the early-stage firm becomes zero, for $n=1,2,...,N$.

$$
\begin{aligned}
= \; & E_0^{\mathbb{Q}}[e^{-rT_1}\cdot E_{T_1}^{\mathbb{Q}}[...e^{-rv_N}\cdot V_N\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)]...1_{\{X_1\geqslant X_1^*\}}(X_1)] - \\
& -\sum_{j=1}^{N} E_0^{\mathbb{Q}}[e^{-rT_1}\cdot E_{T_1}^{\mathbb{Q}}[...e^{-r\tau_1}\cdot K\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)]...1_{\{X_1\geqslant X_1^*\}}(X_1)] \\
= \; & E_0^{\mathbb{Q}}[\sum_{J_1=0}^{\infty}[...\{\sum_{J_N=0}^{\infty}[e^{-rv_N}\cdot V_N\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)|J_N]\Pr(J_N)\}...1_{\{X_1\geqslant X_1^*\}}(X_1)|J_1]\Pr(J_1)\} - \\
& -\sum_{j=1}^{N} E_0^{\mathbb{Q}}[\sum_{J_1=0}^{\infty}[...\{\sum_{J_N=0}^{\infty}[e^{-rv_N}\cdot K\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)|J_N]\Pr(J_N)\}...1_{\{X_1\geqslant X_1^*\}}(X_1)|J_1]\Pr(J_1)\}
\end{aligned}
$$

and by using Poisson probabilities and rearranging terms, it simplifies to

$$
\begin{aligned}
&= \prod_{j=1}^{N}(\sum_{J_j=0}^{\infty}\frac{e^{-\lambda v_j}(\lambda v_j)^{J_j}}{J_j!}E_0^{\mathbb{Q}}[e^{-rT_1}E_{T_1}^{\mathbb{Q}}[...\{e^{-r\tau_1}\cdot V_N\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)\}...]1_{\{X_1\geqslant X_1^*\}}(X_1)|J_1,.,J_j,.,J_N\}) \\
&\quad -\sum_{j=1}^{N}\prod_{n=1}^{N}(\sum_{J_n=0}^{\infty}\frac{e^{-\lambda v_n}(\lambda v_n)^{J_n}}{J_n!}E_0^{\mathbb{Q}}[e^{-rT_1}E_{T_1}^{\mathbb{Q}}[...\{e^{-rv_1}\cdot K\cdot 1_{\{X_N\geqslant X_N^*\}}(X_N)\}...]1_{\{X_1\geqslant X_1^*\}}(X_1)|J_1,.,J_j,.,J_N \\
&= \prod_{j=1}^{n}[\sum_{J_j=0}^{\infty}\frac{e^{-\lambda v_j}(\lambda v_j)^{J_j}}{J_j!}X_0M_n(a_{m1},a_{m2},...,a_{mn},\rho_n^n)]+ \\
&\quad -\sum_{j=1}^{n}\{\prod_{p=j}^{n}[\sum_{J_p=0}^{\infty}\frac{e^{-\lambda v_p}(\lambda v_p)^{J_p}}{J_p!}nKe^{-rT_j}M_{n+1-p}(b_{m1},b_{m2},...,b_{mn,}\rho_{n+1p}^n)]
\end{aligned}
$$

where

$$
\begin{aligned}
b_{m_n} &= \frac{\ln(X_0/X_n^*)+[r-\frac{1}{2}(\sigma_X^2+m_n\frac{\sigma_J^2}{T_n})]T_n}{\sqrt{(\sigma_X^2+m_n\frac{\sigma_J^2}{T_n})T_n}} \\
a_{m_n} &= b_{m_n}+\sqrt{(\sigma_X^2+m_n\frac{\sigma_J^2}{T_n})T_n}.
\end{aligned}
$$

Let $M_n(a_{m1},a_{m2},...,a_{mn},\rho_n^n)$ denote the n-dimension cumulative distribution function, with upper limits of integration $a_{m1},a_{m2},...,a_{mn}$ and correlation matrix ρ_n^n. Conditional on observing V_i and V_j jumps, ρ_n^n is a n-dimention symmetric correlation matrix with typical element $\rho_{s_i,V_j} = \frac{\sqrt{(\sigma_X^2+m_i\frac{\sigma_J^2}{T_i})T_i}}{\sqrt{(\sigma_X^2+m_j\frac{\sigma_J^2}{T_j})T_j}}$ for $1\leq i\leq j\leq n$ and $n=1,2,...,N$. The volatility of the early-stage firm at each stage n is

$$
\frac{\partial V}{\partial X}\times\frac{X}{V}\times\sigma_X=\frac{E\times\sigma_X}{E-F}>\sigma
$$

where

$$
\begin{aligned}
E &\equiv \frac{\partial V}{\partial X}\times X=\prod_{j=1}^{N}[\sum_{J_j=0}^{\infty}\frac{e^{-\lambda v_j}(\lambda v_j)^{J_j}}{J_j!}X_0M_n(a_{m1},a_{m2},...,a_{mn},\rho_n^n) \\
F &\equiv \sum_{j=1}^{N}\{\prod_{p=j}^{n}[\sum_{J_p=0}^{\infty}\frac{e^{-\lambda v_p}(\lambda v_p)^{J_p}}{J_p!}K_je^{-rT_j}M_{n+1-p}(b_{m1},b_{m2},...,b_{mn,}\rho_{n+1p}^n)]
\end{aligned}
$$

Following Ekstrom and Tysk (2006) Theorem 3.1, the early-stage firm value V is log-concave as a function of the log value of the underlying cash flow X. Thus the elasticity of the compound option value X relative to the underlying cash flow X decreases with increasing underlying cash flow X.

$$\partial(\frac{\partial V}{\partial X} \times \frac{X}{V})/\partial X < 0$$

■

B Regression-Based Methods

The value of early-stage firm can be solved through dynamic programming approach. The dynamic programming formulation for early-stage firms is:

$$V_N(T_N, X) = \max\{X_N - K, 0\} \quad \text{(B.1)}$$

$$V_n(T_n, X) = \max\{E_n[e^{T_{n+1}-T_n} \cdot V_{n+1}(T_{n+1}, X)] - K, 0\},\ n = 1, ..., N-1 \quad \text{(B.2)}$$

Equation B.1 states the early-stage firms value at the last investment stage N. Equation B.2 states that at the t- th stage the early-stage firms value is the maximum of the expected present value of continuing to finish the R&D project (i.e. investing the launching cost K for the next stage) and the immediate exit value 0.

The recursive formulation in (B.2) allows for an efficient and scalable numerical implementation. We use regression-based methods to estimate conditional expectation values from simulated paths and thus to price the early-stage firms by simulation. Carrière(1996), Longstaff and Schwartz (2001), and Tsitsiklis and Van Roy (1999, 2001) have proposed the use of regression method to estimate continuation values from simulated paths and thus to price American options by simulation. Each continuation value is the regression of the option value on the current state, suggesting an estimation procedure: approximate continuation value by a linear combination of known functions of the current state and use regression (typically least-squares) to estimate the best coefficients in this approximation. This approach is relatively fast and broadly applicable; its accuracy depends on the choice of functions used in the regression.

In my model, each continuation value $V_n(T_n, X)$ is the regression of the next period early-stage firms value $V_{n+1}(T_{n+1}, X)$ on the current state X_t, which suggests an estimation procedure: approximate $V_n(T_n, X)$ by a liner combination of know functions of the current state and use least squares regression to estimate the best coefficients in this approximation. An expression for the continuation value is:

$$E[V_{n+1}(X_{n+1})|X_n] = \sum_{r=1}^{M} \beta_{ir}\psi_r(X_n) \tag{B.3}$$

for some basis functions $\psi_r : R^d \longrightarrow R$ and constants $\beta_{ir}, r = 1, ..., M$. We can rewrite (B.3) as

$$E[V_{n+1}(X_{n+1})|X_n] = \beta_i^T \psi(X_n) \tag{B.4}$$

with $\beta_i^T = (\beta_{i1,...,}\beta_{iM}), \psi(x) = (\psi_1(x), ..., \psi_M(x))^T$. Assuming (B.3) holds, the vector β_i is given by

$$\beta_i = (E[\psi(X_n)\psi(X_n)^T])^{-1}E[\psi(X_n)V_{n+1}(X_{n+1})] \equiv B_\psi^{-1}B_{\psi S} \tag{B.5}$$

The coefficients β_{ir} could be estimated from observations of pairs $(X_{nj}, V_{n+1}(X_{n+1,j}))$, $j = 1, .., b$.each consist of the state at time t and corresponding continuation value at time t+1.

I simulate b independent paths $\{X_{1j}, ..., X_{mj}\}, j = 1, ..., b$, of the mixed jump-diffusison process. At the termination nodes, set

$$\widehat{V}_{Nj}(x) = \max\{X_N - K, 0\}, j = 1, ..., b, \tag{B.6}$$

Apply backward induction: for $n = m - 1, ..., 1$, given estimated $\widehat{S}_{t+1,j}, j = 1, ..., b$, use regression as above to calculate $\widehat{\beta}_i = \widehat{B}_\psi^{-1}\widehat{B}_{\psi S}$; set

$$\widehat{V}_{nj} = \max\{e^{-r} \cdot \widehat{V_{n+1}}(X_{n+1})|X_n] - K, 0\}\}, j = 1, ..., b \tag{B.7}$$

Finally, we can get $V_0 = (\widehat{V}_{0,1} + ... + V_{0,b})/b$. The volatility of early-stage firm at each stage is the standard deviation of the early-stage firm return $\frac{\widehat{V}_n, j}{\widehat{V}_{n-1,j}}$, which survived from previous stages, i.e. $\sigma_n = \sqrt{Var(\frac{\widehat{V}_n}{\widehat{V}_{n-1}}|\widehat{V}_{n-1,j} > 0, j = 1, ..., b)}$.

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Table 1 Life Cycle Stage Criterion

1996

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3MM	$3-$46MM	$3-$46MM	$46MM+
Op Exp	< $3.8MM	=> $3.8MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

1997

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.2MM	$3.2-$48MM	$3.2-$48MM	$48MM+
Op Exp	< $4MM	=> $4MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

1998

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.3MM	$3.3-$50MM	$3.3-$50MM	$50MM+
Op Exp	< $4.2MM	=> $4.2MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

1999

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.5MM	$3.5-$52MM	$3.5-$52MM	$52MM+
Op Exp	< $3.7MM	=> $3.7MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2000

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.6MM	$3.6-$54MM	$3.6-$54MM	$54MM+
Op Exp	< $4.5MM	=> $4.5MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2001

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.6MM	$3.6-$55MM	$3.6-$55MM	$55MM+
Op Exp	< $4.6MM	=> $4.6MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2002

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.7MM	$3.7-$56MM	$3.6-$56MM	$56MM+
Op Exp	< $4.6MM	=> $4.6MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2003

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $3.8MM	$3.8-$57MM	$3.8-$57MM	$57MM+
Op Exp	< $4.8MM	=> $4.8MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2004

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	< $4MM	$4-$60MM	$4-$60MM	$60MM+
Op Exp	< $5MM	=> $5MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

2005

	Early Dev	Late Dev	Early Rev	Shipping	Mature A	Mature B
Rev / Yr	None	None	4.2MM	$4.2-$62.5MM	$4.2-$62.5MM	$62.5MM+
Op Exp	< $5.2MM	=> $5.2MM				
Op Inc				Neg	Pos for 2yrs	
Net Inc				Neg	Pos for 2yrs	

Table 2: Maximum likelihood estimates of the Markov Switching EARCH model

This table estimates Markov Switching EARCH(G, q) models of the relationship between the firm risk premium, market risk premium and conditional firm variance using weekly data for the period 1993 to 2005. The models estimated are nested within the following Markov Switching EARCH model:

$R_t = \alpha + \beta_m R_{mkt} + \eta_t$

$R_{jt} = \alpha_j + \beta_{jm} R_{mkt} + \beta_j \eta_t + \varepsilon_{jt} \quad \varepsilon_{jt}|I_{t-1} \sim t(0, h_{jt})$

$\frac{\ln h_{jt}}{\gamma_{s_t}} = a_0 + \sum_{l=1}^{q} a_l \frac{\ln \varepsilon_{it-l}^2}{\gamma_{s_{t-l}}} + \xi_1 \cdot d_{1,t-1} \cdot \frac{\ln \varepsilon_{jt-1}^2}{\gamma_{s_{t-1}}} - \xi_2 \cdot d_{2,t-1} \cdot \frac{\ln \varepsilon_{jt-1}^2}{\gamma_{s_{t-1}}}$

G denotes the regime number, β_m denotes the beta for biotech industry with respect to the market return, and η_t is the biotech industry-specific residual. Similarly, β_j is the beta of firm j with respect to biotech industry and ε_{jt} is the firm-specific residual. ε_{jt} are assumed to follow a student-t distribution which allows for fatter tails than the Gaussian distribution. R_t is the industry excess return, R_{jt} is the firm excess return at time t, R_{mkt} is the market excess return at time t, h_{jt} is the model's estimate of the ε_{jt}'s conditional variance. S_t denotes the regime of the firm at time t. The constant γ_{s_t} captures the structural shift parameter. T-ratios are reported in parentheses. L is the likelihood function. We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ and there is bad news about R&D project at time t-1. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ and there is good news about R&D project at time t-1. Significance on a ten percent(*), five percent (**), or one percent level (***) based on t test is indicated.

Parameter	Markov Switching EARCH$(2,2)$	Markov Switching EARCH$(2,1)$
Panel A: Mean Equation Parameters		
α_j	-0.81*	-0.80*
(t ratio)	-1.97	-1.92
β_{jm}	0.74*	0.74*
(t ratio)	2.01	2.28
β_j	0.57**	0.58**
(t ratio)	2.98	2.90
Panel B:Conditional Second Moment Equation Parameters		
a_0	4.47***	4.38***
(t ratio)	18.44	19.86
a_1	0.02	0.02
(t ratio)	1.13	1.13
a_2	-0.02	
(t ratio)	-1.01	
ξ_1	0.07*	0.08*
(t ratio)	1.81	1.83
ξ_2	0.05*	0.05*
(t ratio)	1.92	1.87
γ_2	6.30***	6.99***
(t ratio)	4.70	4.44
p	0.97	0.97
q	0.98	0.98
L	-1722.76	-1723.24

Table 3: Summary Statistics of Maximum likelihood estimates of the Markov Switching EARCH model

This table summarize the estimation results using the Markov Switching EARCH model. We estimated the Markov Switching EARCH model for each early-stage firm in my sample separately. The models estimated are nested within the following Markov Switching EARCH model:

$R_t = \alpha + \beta_m R_{mkt} + \eta_t$

$R_{jt} = \alpha_j + \beta_{jm} R_{mkt} + \beta_j \eta_t + \varepsilon_{jt} \quad \varepsilon_{jt}|I_{t-1} \sim t(0, h_{jt})$

$\frac{\ln h_{jt}}{\gamma_{s_t}} = a_0 + \sum_{l=1}^{q} a_l \frac{\ln \varepsilon^2_{it-l}}{\gamma_{s_{t-l}}} + \xi_1 \cdot d_{1,t-1} \cdot \frac{\ln \varepsilon^2_{jt-1}}{\gamma_{s_{t-1}}} - \xi_2 \cdot d_{2,t-1} \cdot \frac{\ln \varepsilon^2_{jt-1}}{\gamma_{s_{t-1}}}$

β_m denotes the beta for biotech industry with respect to the market return, and η_t is the biotech industry-specific residual. Similarly, β_j is the beta of firm j with respect to biotech industry and ε_{jt} is the firm-specific residual. ε_{jt} are assumed to follow a student-t distribution which allows for fatter tails than the Gaussian distribution. R_t is the industry excess return, R_{jt} is the firm excess return at time t, R_{mkt} is the market excess return at time t, h_{jt} is the model's estimate of the ε_{jt}'s conditional variance. S_t denotes the regime of the firm at time t. The constant γ_{s_t} captures the structural shift parameter. The number of regimes G for each firm is decided by its life-cycle stages. We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ and there is bad news about R&D project at time t-1. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ and there is good news about R&D project at time t-1. We sort the estimated volatility level $\gamma'_{s_t}s$ of each firm by its life-cycle investment stages in time. We let γ_{first} denote the estimated $\gamma'_{s_t}s$ in the first life-cycle investment stage in the sample period, and γ_{last} denotes the estimated $\gamma'_{s_t}s$ in the last life-cycle investment stage in the sample period. We report the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} in the first column of table 3. The second column reports the standard deviation of the difference between γ_{first} and γ_{last}. We also report the sample mean of ξ_1 and ξ_2 (the jump risk effect) and the standard deviation of ξ_1 and ξ_2. Significance on a ten percent (*), five percent (**), or one percent level (***) based on t test is indicated.

$\overline{\gamma}_{first} - \overline{\gamma}_{last}$	$std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$	$\overline{\xi_1}$	$std(\xi_1)$	$\overline{\xi_2}$	$std(\xi_2)$
4.73***	1.19	0.11**	0.03	0.06**	0.03

Table 4: Summary Statistics of Maximum likelihood estimates of the Markov Switching EARCH model

This table summarize the estimation results using the Markov Switching EARCH model. We estimated the Markov Switching EARCH model for each early-stage firm in my sample separately. The models estimated are nested within the following Markov Switching EARCH model:

$$R_t = \alpha + \beta_m R_{mkt} + \eta_t$$
$$R_{jt} = \alpha_j + \beta_{jm} R_{mkt} + \beta_j \eta_t + \varepsilon_{jt} \qquad \varepsilon_{jt}|I_{t-1} \sim t(0, h_{jt})$$
$$\frac{\ln h_{jt}}{\gamma_{s_t}} = a_0 + \sum_{l=1}^{q} a_l \frac{\ln \varepsilon^2_{it-l}}{\gamma_{s_{t-l}}} + \xi_1 \cdot d_{1,t-1} \cdot \frac{\ln \varepsilon^2_{jt-1}}{\gamma_{s_{t-1}}} - \xi_2 \cdot d_{2,t-1} \cdot \frac{\ln \varepsilon^2_{jt-1}}{\gamma_{s_{t-1}}}$$

β_m denotes the beta for biotech industry with respect to the market return, and η_t is the biotech industry-specific residual. Similarly, β_j is the beta of firm j with respect to biotech industry and ε_{jt} is the firm-specific residual. ε_{jt} are assumed to follow a student-t distribution which allows for fatter tails than the Gaussian distribution. R_t is the industry excess return, R_{jt} is the firm excess return at time t, R_{mkt} is the market excess return at time t, h_{jt} is the model's estimate of the ε_{jt}'s conditional variance. S_t denotes the regime of the firm at time t. The constant γ_{s_t} captures the structural shift parameter. The number of regimes G for each firm is decided by its life-cycle stages. We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ at time t-1. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ at time t-1. We sort the estimated volatility level $\gamma'_{s_t}s$ of each firm by its life-cycle investment stages in time. We let $\gamma_{_{first}}$ denote the estimated $\gamma'_{s_t}s$ in the first life-cycle investment stage in the sample period, and $\gamma_{_{last}}$ denotes the estimated $\gamma'_{s_t}s$ in the last life-cycle investment stage in the sample period. We report the difference between the sample mean of the volatility level at the first stage $\gamma_{_{first}}$ and the last stage $\gamma_{_{last}}$ in the second column of table 3. The third column reports the standard deviation of the difference between $\gamma_{_{first}}$ and $\gamma_{_{last}}$. We also report the sample mean of ξ_1 and ξ_2 (the jump risk effect) and the standard deviation of ξ_1 and ξ_2.The first column N is the number of the firms. Significance on a ten percent (*), five percent (**), or one percent level (***) based on t test is indicated.

Group	N	$\overline{\gamma}_{_{first}} - \overline{\gamma}_{_{last}}$	$std(\overline{\gamma}_{_{first}} - \overline{\gamma}_{_{last}})$
Early-stage with one regime	0	-	-
Early-stage with more than one regime	102	4.53***	1.34
Dead with one regime	36	-	-
Dead with more than one regime	0	-	-
Mature with one regime	121	-	-
Mature with more than one regime	32	1.22	6.0

Table 5: Maximum likelihood estimates of mature firms and dead firms

This table summarize the estimation results using the Markov Switching EARCH model for firms which are not included in my sample during the sample period. We divide them into two groups: mature firms and died early-stage firms. The mature firms are at least publicly traded for 10 years and not included in my sample during my sample period. The dead early-stage firms are the early-stage firms which died during my sample period. The models estimated are nested within the following Markov Switching EARCH model:

$R_t = \alpha + \beta_m R_{mkt} + \eta_t$

$R_{jt} = \alpha_j + \beta_{jm} R_{mkt} + \beta_j \eta_t + \varepsilon_{jt} \quad \varepsilon_{jt}|I_{t-1} \sim t(0, h_{jt})$

$\frac{\ln h_{jt}}{\gamma_{s_t}} = a_0 + \sum_{l=1}^{q} a_l \frac{\ln \varepsilon_{it-l}^2}{\gamma_{s_t-l}} + \xi_1 \cdot d_{1,t-1} \cdot \frac{\ln \varepsilon_{jt-1}^2}{\gamma_{s_{t-1}}} - \xi_2 \cdot d_{2,t-1} \cdot \frac{\ln \varepsilon_{jt-1}^2}{\gamma_{s_{t-1}}}$

We let $d_{1,t-1} = 1$ if $\varepsilon_{jt-1} \leq -0.05$ at time t-1. We let $d_{2,t-1} = 1$ if $\varepsilon_{jt-1} \geq 0.05$ at time t-1. Panel A is the estimation results using Markov Switching EARCH model with 2 regimes. Panel B is the estimation results using Markov Switching EARCH model with 3 regimes. We sort the estimated volatility level γ_{s_t}'s of each firm by its life-cycle investment stages in time. We let γ_{first} denote the estimated γ_{s_t}'s in the first life-cycle investment stage in the sample period, and γ_{last} denotes the estimated γ_{s_t}'s in the last life-cycle investment stage in the sample period. We report the difference between the sample mean of the volatility level at the first stage γ_{first} and the last stage γ_{last} in the first column. The second column reports the standard deviation of the difference between γ_{first} and γ_{last}. The first column N is the number of the firms. Significance on a ten percent (*), five percent (**), or one percent level (***) based on t test is indicated.

Panel A. Markov Switching EARCH model with 2 regimes

Group	N	$\overline{\gamma}_{first} - \overline{\gamma}_{last}$	$std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$
Early-stage with 2 regimes	102	4.13***	1.07
Dead with 2 regimes	0	-	-
Mature with 2 regimes	32	1.02	5.8

Panel B. Markov Switching EARCH model with 3 regimes

Group	N	$\overline{\gamma}_{first} - \overline{\gamma}_{last}$	$std(\overline{\gamma}_{first} - \overline{\gamma}_{last})$
Early-stage with 3 regimes	20	5.53***	1.49
Dead with 3 regimes	0	-	-
Mature with 3 regimes	10	1.81	6.2

Table 6: Value of the early-stage firm ignoring jump risk

This table reports the value of the early-stage firm at each stage $V_i, i = 0, 1, ..., 8$. It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500,000$ independent simulations of firm. σ_X is the volatility of the of the established firm. The original state is $X_0 = 100$.

σ_X	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
0.3	20.82	40.70	53.34	68.77	85.68	103.26	121.61	140.42	159.62
. 0.4	22.04	48.32	61.40	78.83	98.74	119.38	141.39	164.71	189.53
0.5	23.23	52.97	66.18	85.49	108.25	132.97	158.8	185.13	212.11
0.6	38.67	64.12	77.59	100.52	128.75	160.03	193.47	228.44	266.37

Table 7: Jump size impact on the value of the early-stage firm

This table reports the jump size impact on the value of the early-stage firms considering jump. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY (In other words, the function for the change in value is $\gamma(X,t,Y) = XY - X = X(Y-1)$.) It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from 500,000 independent simulations of firm. Panel A is the value of the early-stage firm at each stage with jump frequency $\lambda = 0.3$. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals to zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. Panel A is the value of the early-stage firm V_0 with jump frequency $\lambda = 0.3$, $\sigma_X = 0.3$ and lognormal jump. Panel B is Value of the early-stage firm at each stage with jump frequency $\lambda = 0.3$, $\sigma_X = 0.3$ and lognormal jump.

Panel A: Value of the early-stage firm V_0 with jump frequency $\lambda = 0.3$ and lognormal jump

underling σ_X	σ_J= 0	σ_J= 0.1	σ_J= 0.2	σ_J= 0.3	σ_J= 0.4
0.3	20.82	12.06	8.35	6.24	6.44
0.4	22.04	19.89	18.77	11.90	10.39
0.5	23.23	22.45	19.36	16.02	14.72
0.6	41.67	40.83	27.49	26.50	23.45

Panel B: Value of the early-stage firm at each stage with jump frequency $\lambda = 0.3, \sigma_X = 0.3$ and lognormal jump

σ_J	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
0	20.82	40.70	53.34	68.77	85.68	83.26	121.61	140.42	159.62
0.1	12.06	33.66	45.93	61.23	77.72	94.79	113.03	131.73	150.17
0.2	8.35	31.04	43.24	59.28	76.34	93.52	111.22	129.78	149.16
0.3	6.24	29.45	41.48	57.82	75.03	93.42	112.21	131.56	151.62
0.4	6.44	29.71	42.03	58.71	76.62	95.07	114.41	134.29	155.51

Table 8: Jump frequency impact on the value of the early-stage firm

This table reports the jump frequency impact on the value of the early-stage firms considering jump. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY (In other words, the function for the change in value is $\gamma(X,t,Y) = XY - X = X(Y-1)$.) It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500,000$ independent simulations of firm. Panel A is the value of the early-stage firm V_0 with jump different frequency Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals zero,i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. We let $\sigma_J = 0.3$.Panel B is the value of the early-stage firm S_0 with different jump frequency. Assume fixed size jump Y, $Y = 0.1$ means when jump happens, X changes immediately to $0.1X$. $Y = 0.5$ means when jump happens, X changes immediately to $0.5X$. σ_X is the volatility of the of the established firm. The original state is $X_0 = 100$.

Panel A: Value of the early-stage firm with $\sigma_J = 0.3$

underling σ_X			Jump	frequency		
	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$
0.3	20.82	15.27	10.30	6.24	4.73	2.87
0.4	22.04	20.21	15.99	11.90	8.22	5.44
0.5	23.23	27.24	21.90	16.02	15.78	10.33
0.6	41.67	32.04	26.12	26.50	17.18	13.48

Panel B: Value of the early-stage firm with fixed size jump

underling σ_X	No Jump	Jump Size $Y = 0.1$		Jump size $Y = 0.5$	
		Jump	frequency	Jump	frequency
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.3$	$\lambda = 0.5$
0.3	20.82	0	0	0	0
0.4	22.04	0	0	0.03	0
0.5	23.23	0.54	0.02	1.83	0.33
0.6	41.67	2.05	0.29	5.49	1.65

Table 9: Volatility of the early-stage firm ignoring jump risk

This table reports the volatility of the early-stage firm ignoring jump risk. It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500, 000$ independent simulations of firm. At each stage, we divide the whole sample into two subsample based on underlying cash flows value change If the value underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; If the value underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. We calculate the volatility of each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 9$. σ_X is the volatility of the of the established firm,conditional on no jump. The original state is $X_0 = 100$. The numbers in parentheses give the percentage change of the underlying cash flows in "Up"and "Down" subsamples.

Table 7: Volatility of the early-stage firm with flat launching cost

σ_X	X	σ_{s2}(Stage 2)	σ_{s3}(Stage 3)	σ_{s4}(Stage 4)	σ_{s5}(Stage 5)	σ_{s6}(Stage 6)	σ_{s7}(Stage 7)	σ_{s8}(Stage 8)
0.3	*Up*	0.73 (29.60%)	0.71 (29.42%)	0.64 (29.18%)	0.60 (29.53%)	0.53 (29.51%)	0.48 (29.53%)	0.44 (29.60%)
	Down	21.49 (19.85%)	25.88 (19.91%)	50.12 (19.93%)	8.90 (19.98%)	6.03 (19.90%)	16.95 (19.87%)	4.32 (19.82%)
0.4	*Up*	0.86 (40.26%)	0.84 (40.38%)	0.79 (40.18%)	0.73 (40.33%)	0.69 (40.38%)	0.70 (40.24%)	0.60 (40.27%)
	Down	45.95 (26.13%)	84.72 (26.10%)	62.99 (26.05%)	5.14 (26.03%)	31.75 (26.14%)	19.43 (25.94%)	61.04 (26.14%)
0.5	*Up*	0.95 (51.97%)	0.99 (51.60%)	0.93 (52.02%)	0.90 (51.81%)	0.82 (51.66%)	0.78 (51.58%)	0.75 (52.15%)
	Down	77.10 (31.77%)	22.13 (31.76%)	113.77 (31.76%)	48.78 (31.73%)	13.02 (31.79%)	255.45 (31.83%)	7.92 (31.82%)
0.6	*Up*	1.09 (65.19%)	1.17 (64.47%)	1.08 (65.03%)	1.03 (64.84%)	0.98 (65.05%)	0.96 (64.75%)	1.43 (65.02%)
	Down	111.71. (37.15%)	34.71 (37.19%)	38.41 (37.02%)	106.72 (37.00%)	42.79 (37.11%)	29.11 (37.04%)	26.77 (37.01%)

Table 10: Jump size impact on the volatility of the early-stage firm

This table reports the jump size impact on the volatility of the early-stage firms considering jump. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY (In other words, the function for the change in value is $\gamma(X,t,Y) = XY - X = X(Y-1)$.) It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500,000$ independent simulations of firm. At each stage, we divide the whole sample into two subsample based on underlying cash flows value change If the value underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt - X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; If the value underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt - X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. We calculate the volatility of each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 8$. Panel A is the volatility of the early-stage firm at each stage with jump frequency $\lambda = 0.3$. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. The volatility of the of the established firm $\sigma_X = 0.3$. The original state is $X_0 = 100$.

Table 8: Volatility of the early-stage firm with flat launching cost

σ_J	X	σ_{s2}(Stage 2)	σ_{s3}(Stage 3)	σ_{s4}(Stage 4)	σ_{s5}(Stage 5)	σ_{s6}(Stage 6)	σ_{s7}(Stage 7)	σ_{s8}(Stage 8)
0	*Up*	0.73	0.71	0.64	0.60	0.53	0.48	0.44
		(29.60%)	(29.42%)	(29.18%)	(29.53%)	(29.51%)	(29.53%)	(29.60%)
	Down	21.49	25.88	50.12	8.90	6.03	16.95	4.32
		(19.85%)	(19.91%)	(19.93%)	(19.98%)	(19.90%)	(19.87%)	(19.82%)
0.1	*Up*	0.765	0.724	0.67	0.58	0.55	0.49	0.46
		(29.15%)	(29.37%)	(29.26%)	(29.46%)	(29.31%)	(29.44%)	(29.28%)
	Down	98.69	25.99	27.39	16.69	24.42	5.57	3.54
		(20.75%)	(20.68%)	(20.76%)	(20.77%)	(20.68%)	(20.57%)	(20.63%)
0.2	*Up*	0.84	0.80	0.69	0.62	0.57	0.52	0.48
		(30.07%)	(30.04%)	(29.99%)	(30.18%)	(30.04%)	(30.19%)	(30.15%)
	Down	23.19	46.84	42.30	22.74	16.08	21.00	2.80
		(22.10%)	(22.07%)	(22.02%)	(22.04%)	(22.01%)	(22.15%)	(22.07%)
0.3	*Up*	0.88	0.83	0.75	0.66	0.61	0.56	0.51
		(31.32%)	(31.30%)	(31.23%)	(31.28%)	(31.33%)	(31.24%)	(31.38%)
	Down	50.77	240.96	54.90	38.96	43.08	4.60	35.74
		(23.61%)	(23.55%)	(23.65%)	(23.75%)	(23.61%)	(23.59%)	(23.69%)
0.4	*Up*	0.91	0.89	0.75	0.70	0.64	0.62	0.55
		(32.56%)	(32.58%)	(33.05%)	(32.91%)	(32.73%)	(32.80%)	(32.91%)
	Down	56.17.	17.38	10.98	11.24	5.73	104.99	12.18
		(25.31%)	(25.25%)	(25.13%)	(25.18%)	(25.22%)	(25.16%)	(25.24%)
0.5	*Up*	0.97	0.90	0.81	0.78	0.69	0.64	0.59
		(34.79%)	(34.80%)	34.83%	(34.58%)	(34.83%)	(34.80%)	(34.61%)
	Down	24.28	115.57	22.76	7.89	11.87	229.95	14.784
		(26.69%)	(26.73%)	(26.70%)	(26.75%)	(26.72%)	(26.77%)	(26.59%)

Table 11: Jump frequency impact on the volatility of the early-stage firm

This table reports the jump frequency impact on the volatility of the early-stage firms considering jump. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY (In other words, the function for the change in value is $\gamma(X,t,Y) = XY - X = X(Y-1)$.) It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from $500,000$ independent simulations of firm. At each stage, we divide the whole sample into two subsample based on underlying cash flows value change If the value underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt - X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; If the value underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt - X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. We calculate the volatility of each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 8$. Panel A is the volatility of the early-stage firm at each stage with jump size $\sigma_J = 0.3$. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that g equals to zero, i.e. $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. The volatility of the of the established firm $\sigma_X = 0.3$. The original state is $X_0 = 100$.

Table 9: Volatility of the early-stage firm with flat launching cost,jump sigma =0.3

λ	X	σ_{s2}(Stage 2)	σ_{s3}(Stage 3)	σ_{s4}(Stage 4)	σ_{s5}(Stage 5)	σ_{s6}(Stage 6)	σ_{s7}(Stage 7)	σ_{s8}(Stage 8)
0	*Up*	0.73 (29.60%)	0.71 (29.42%)	0.64 (29.18%)	0.60 (29.53%)	0.53 (29.51%)	0.48 (29.53%)	0.44 (29.60%)
	Down	21.49 (19.85%)	25.88 (19.91%)	50.12 (19.93%)	8.90 (19.98%)	6.03 (19.90%)	16.95 (19.87%)	4.32 (19.82%)
0.1	*Up*	0.78 (30.06%)	0.76 (29.99%)	0.68 (30.20%)	0.62 (30.20%)	0.56 (30.07%)	0.51 (30.05%)	0.46 (30.28%)
	Down	19.94 (21.20%)	50.60 (21.26%)	14.13 (21.14%)	7.09 (21.21%)	56.12 (21.28%)	148.52 (21.10%)	33.91 (21.26%)
0.2	*Up*	0.83 (30.75%)	0.79 (30.64%)	0.69 (30.61%)	0.63 (30.63%)	0.57 (30.95%)	0.53 (30.64%)	0.49 (30.73%)
	Down	75.95 (22.36%)	14.56 (22.37%)	25.63 (22.48%)	75.68 (22.35%)	6.81 (22.36%)	5.77 (22.49%)	3.87 (22.48%)
0.3	*Up*	0.88 (31.32%)	0.83 (31.30%)	0.75 (31.23%)	0.66 (31.28%)	0.61 (31.33%)	0.56 (31.24%)	0.51 (31.38%)
	Down	50.77 (23.61%)	240.96 (23.55%)	54.90 (23.65%)	38.96 (23.75%)	43.08 (23.61%)	4.60 (23.59%)	35.74 (23.69%)
0.4	*Up*	0.92 (31.86%)	0.86 (31.83%)	0.74 (31.98%)	0.66 (31.85%)	0.61 (31.70%)	0.56 (31.83%)	0.52 31.86%)
	Down	35.74. (24.63%)	37.97 (24.74%)	8.85 (24.54%)	7.42 (24.74%)	4.50 (24.67%)	52.85 (24.76%)	5.25 (25.67%)

Figure 2.Maximium likelihood estimates of the SWEARCH model

Panel A: Weekly returns on firm KOSP from Feb 1997 to Dec 2005. Panel B: Conditional volatility estimates of firm returns from the student t SWEARCH(2,2) model. Panel C: Smoothed probability that firm was in regime 1 (lower volatility γ_1=1) for each indicated week, as calculated from the student t SWEARCH(2,2). Panel D: Smoothed probability that firm was in regime 2 (higher volatility γ_2=6.30) for each indicated week, as calculated from the student t SWEARCH(2,2)

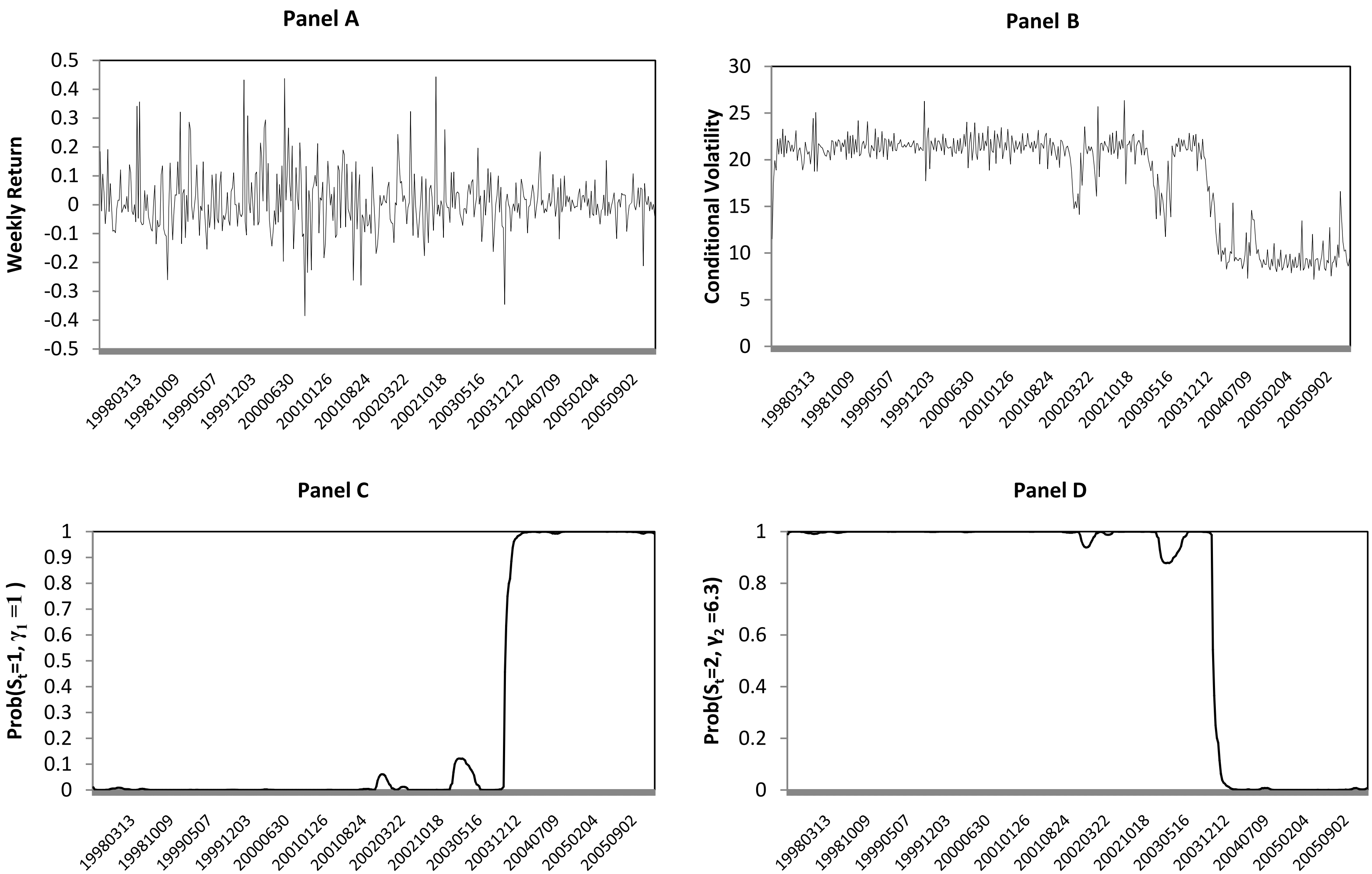

Figure 4 Jump Size impact on the volatility of the early-stage firm

This figure reports the jump size impact on the volatility of the early-stage firms considering jump risk. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY. It requires 8 stages to complete R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from 100,000 independent simulations of firm. $X_0 = 100$, σ_X is the volatility of the underlying cash flow, conditional on no arrivals of important new information (i.e. Poisson event does not occur). At each stage, we divide the whole sample into two subsamples based on the underlying cash flows value changes. If the value of the underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; If the value underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. We calculate the return volatility of early-stage firm in each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 9$. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. Figure A is the volatility of the "Up" subsample early-stage firm at each stage with jump frequency $\lambda = 0.3$. The x axis is the stage number, the y axis is the estimated volatility of the early-stage firms. "$-$" line is for the σ_J the volatility of the of the log normal jump $= 0.1$, "o" line is for the σ_J the volatility of the log normal jump $= 0.2$, "+" line is for the σ_j the volatility of the log normal jump $= 0.3$. Figure B is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$,when the underlying firm value increase. Figure C is the volatility of the "Down" subsample early-stage firm at each stage with jump frequency $\lambda = 0.3$. The x axis is the stage number; the y axis is the estimated volatility of the early-stage firms. "$-$" line is for the σ_j the volatility of the of the log normal jump $= 0.1$. "o" line is for the σ_j the volatility of the of the log normal jump $= 0.2$, "+" line is for the σ_J the volatility of the of the log normal jump $= 0.3$. Figure D is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$,when the underlying firm value decreases.

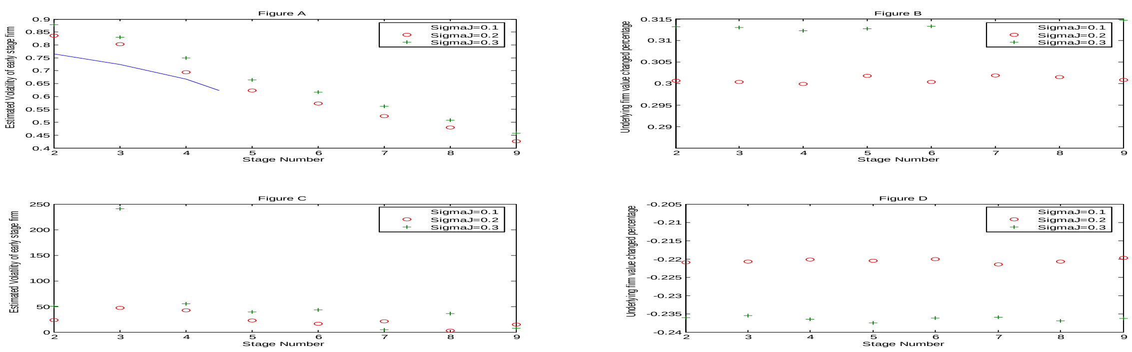

Figure 5 Jump Frequency impact on the volatility of the early-stage firm

This figure reports the jump frequency impact on the volatility of the early-stage firms considering jump risk. If the Poisson jump event occurs, then $(Y-1)$ is an impulse function which takes the project value from X_t to X_tY. It requires 8 stages to complete the R&D process and has flat launching costs at each stages, i.e. $K_t = 10$, $t = 0, 1, ..., 8$. The numbers are obtained from 500,000 independent simulations of firm. $V_0 = 100$, σ_X is the volatility of the of the established firm, conditional on no arrivals of important new information (i.e. Poisson event does not occur). At each stage, we divide the whole sample into two subsamples based on underlying cash flows value change. If the value underlying cash flows X increase, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is bigger than 0.05, it belongs to the "Up" subsample; if the value underlying cash flows X decrease, i.e. the percentage change of the value of underlying cash flows $\frac{Xt-X_{t-1}}{X_{t-1}}$ is smaller than -0.05, it belongs to the "Down" subsample. We calculate the return volatility of the early-stage firms in each subsample at each stage. σ_{si} is the volatility of the early-stage firms at stage i for $i = 2, 3, ..., 9$. Assume that the logarithmic jump amplitude, $\ln Y$, is distributed Normal (μ_J, σ_J^2). To simplify notation, we also assume that $\mu_J = -\frac{1}{2}\sigma_J^2$. The logarithmic return $r_t = \log(X_t/X_0)$ has a normal distribution. Figure A is the volatility of the "Up" subsample early-stage firm at each stage with different jump frequency and $\sigma_J = 0.3$. Figure B is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$,when the underlying firm value increase. Figure C is the volatility of the "Down" subsample early-stage firm at each stage with different jump frequency $\sigma_J = 0.3$. Figure D is the percentage change of value of the underlying firm, $\frac{Xt}{X_{t-1}}$,when the underlying firm value decrease.