

# Price Limits and Margin Requirement with Loss-Averse Investors

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## Abstract

Assuming that traders are risk-neutral, Brennan (1986) shows that price limits, in conjunction with margins, may help lower the default risk, cut down the margin requirement, and reduce the total contract cost. We show that the effectiveness of price limits is enhanced when traders are loss-averse. The effectiveness of price limits, however, is crucially affected by the precision of the information regarding the equilibrium price. When precise information is available, risk attitude only plays a limited role in ensuring the price limit performance.

Key Words: price limits, margin requirement, default risk, Loss aversion.

JEL Code: G0, G1, G2.

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# Price Limits and Margin Requirement with Loss-Averse Investors

## 1 Introduction

Although price limits have been widely known as a circuit-breaker mechanism that provides a “cool-off” period in the event of extreme market conditions, they may also serve as a partial substitute for margin requirement in derivative trading. Brennan’s (1986) seminal paper shows that price limits, in conjunction with margin requirements, provide a more “efficient” contract design by lowering the contract cost. His model, however, assumes that the trader is risk-neutral. Chou et al (2005) show that when the trader is risk-averse, as is implied by the standard utility theory, the trader has a higher probability to renege when a limit is triggered. As a result, in a risk-averse framework, price limits may become ineffective because they increase the default probability, the margin requirement, and the contract cost. But what if the trader is loss-averse, as suggested by the recent evidence on behavioral finance? In other words, would price limits be effective in improving the market performance when trader is realistically modeled as a loss-averse one?<sup>1</sup>

In contrast with the standard expected utility theory where an agent’s preference depends on the absolute level of wealth, the prospect theory of Kahneman and Tversky (1979) suggests that the preference depends on gains and losses relative to some reference point. The value function is concave in the domain of gains (implying risk aversion), and convex in the domain of losses (implying risk seeking). Additionally, the decrease in utility implied by a marginal loss is greater than the increase in utility from a marginal gain, thus suggesting a loss-averse behavior. People therefore tend to take more risks in order to break even after initial losses, but become conservative following initial gains.

A large literature has studied the implications of loss aversion (see Shiller (1998) and Shleifer (1999) for detailed surveys). For example, the disposition effect, namely the tendency for investors to sell their winners and to hold on to their losers, confirms such an asymmetric response to gains versus losses (see, e.g., Shleifer and Statman

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<sup>1</sup> For example, Coval and Shumway (2005) document that traders in the futures markets exhibit loss aversion behavior.

(1985), Odean (1998), among others). Genesove et al (2001), Heath et al. (1999), and Dhar and Zhu (2006), respectively, find that investors in housing, option and common stock markets are more likely to liquidate winner stocks than loser stocks. A direct examination of the trading behavior by professional traders in the Treasury bond futures market also supports the hypothesis that the traders with morning gains (losses) are more likely to assume below-average (above-average) risks in afternoon trading (Coval and Shumway, 2005).

But how would loss-averse behavior affect the effectiveness of price limits? Intuitively, price limits provide a mechanism that obscures the exact amount of the loss incurring to the trader when price limits are triggered. Being uncertain about the magnitude of the loss, a trader bases his decision on the expected loss, conditional on the information of a limit move. If the trader is risk-neutral, he will adhere to his position if the expected loss is less than the effective margin, and will renege otherwise. Thus, there exist conditions under which certain combinations of price limits and margin requirement may improve the operation of the market. However, the effectiveness of price limits disappears when the trader can obtain precise information derived from other markets such as the spot market.

In the presence of loss aversion, equal magnitudes of gains and losses do not have symmetric impacts on decision-making. Realized losses hurt more than realized gains satisfy, other things being equal. Hence, people tend to take more risks in order to break even the initial losses. Instead of pursuing the maximum expected utility, people may rather engage in avoiding losses (e.g., Barberis et al., 2001; Shleifer, 2000). As a result, for futures trading with price limits, there may be a lower default probability in the event of a limit move; a relatively lower margin is required in order for the loss-averse trader to abide by the contract.

Our numerical results confirm the conjecture. But additional information regarding the equilibrium price in the case of a limit move still plays a crucial role on the effectiveness of price limits. Specifically, when the information about the unobserved equilibrium price is imprecise, price limits are efficient in reducing the optimal margin requirement, the default probability and the total contract cost. However, when the information precision is high, the optimal contract cost is greater than that without price limits. In comparison with the loss-neutral case, the

effectiveness of price limits improves when loss aversion is incorporated into the utility function.

The remainder of this paper is organized as follows. The next section presents the model. Section 3 demonstrates the numerical results, and Section 4 concludes the paper.

## 2 The model

Our model follows the setting of Brennan (1986), except that the trader is assumed to be loss-averse upon facing losses. A central idea of Brennan's setting is that if a financial contract were to survive in a competitive financial market, the contract must be designed to minimize the total cost of trading for market participants. Thus, there is a potential role for price limits because they may serve as a partial substitute for margin requirement; they may also reduce the default probability because they act as an aerosol bomb that obscures the exact loss of trading when limits are triggered.

### 2.1 The basic setting

Consider a three-date, two-period model where a representative loss-averse trader with wealth endowment  $W_0$  enters into a futures contract at time 0 and deposits an initial margin  $m$  with his broker. Let  $P_t$  denote the futures price at time  $t$ , and  $r_t = P_t - P_{t-1}$  is the corresponding futures price change. For simplicity, we assume that the futures price change is normally distributed with mean zero and a constant variance  $\sigma^2$ . The futures price at time 0,  $P_0$ , is given and is not subject to price limits. At time 1, the position must be settled. The trader will have an incentive to renege if the expected utility of renegeing exceeds that of abiding by the contract.

To incorporate loss aversion in the preference, we follow Lien (2001) by assuming that the trader's preference is characterized by the CARA utility:

$$U(r) = \begin{cases} 1 - \exp(-\theta \cdot r) & \text{when } r \geq 0 \\ -\lambda(1 - \exp(\theta \cdot r)) & \text{when } r < 0 \end{cases} \quad (3)$$

where  $\theta > 0$  is the risk aversion coefficient, and  $\lambda > 1$  is the loss aversion

coefficient. As noted by Lien (2001), an increase in  $\theta$  here does not imply that the individual becomes more risk-averse. Rather, it implies that the utility function becomes more concave in gains but also becomes more convex in losses. When  $r < 0$ ,  $U'(r) = \lambda\theta \exp(\theta \cdot r) > 0$  and  $U''(r) = \lambda\theta^2 \exp(\theta \cdot r) > 0$ . Thus,  $U(r)$  is convex when  $r$  is negative. Likewise, when  $r > 0$ ,  $U'(r) = \theta \exp(-\theta \cdot r) > 0$  and  $U''(r) = -\theta^2 \exp(-\theta \cdot r) < 0$ . That is,  $U(r)$  is concave when  $r$  is positive. Then, the utility function is concave in gains and convex in losses. Note that  $U'(-r) = \lambda U'(r)$  when  $r > 0$ , implying greater sensitivity to losses.<sup>2</sup>

Let  $\Pi$  be the probability that the broker will not take legal action if the trader reneges, and that even if he does take legal action, it will be unsuccessful. Also, let  $Y$  be the sum of the expected reputation and legal costs the trader must bear as a result of renegeing. If the trader holds a long position, then he will have an incentive to renege if the expected benefit of renegeing is larger than the cost associated with it, i.e.,  $\Pi[P_0 - P_1 - m] > Y$ . Hence the “effective margin” can be set as  $M = m + \Pi^{-1}Y$ .

Since each contract includes both a long position and a short position, one of the parties will have an incentive to renege whenever the absolute price change exceeds the effective margin, i.e.,

$$|P_1 - P_0| > M. \quad (1)$$

The effective margin represents the lowest margin requirement for a contract to be self-enforcing, but it may not necessarily be the optimal one that minimizes the total contract cost. Without price limits, the contract cost is composed of the cost of capital and the cost of renegeing. Let the cost of capital be  $kM$ , where  $k$  is the unit cost of margin. The cost of renegeing includes not only the legal and reputation costs borne directly by the trader, but also legal and other costs borne by the clearinghouse or the broker when attempting to enforce the contract, which is passed on to the trader in the form of higher commissions.<sup>3</sup>

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<sup>2</sup> Kahnemann and Tverskys (1992) considered the following utility function:  $V(x) = x^\alpha$ ; when  $x \geq 0$  and  $V(x) = -\lambda(-x)^\beta$  when  $x < 0$ , where  $\lambda > 0$ . Based on experimental evidence, they provided estimates of  $\alpha$ ,  $\beta$ , and  $\lambda$  as 0.88, 0.88, and 2.25, respectively.

<sup>3</sup> While setting up the effective margin, Brennan (1986) assumes that the representative trader is risk neutral. Here, the same function form applies as long as the probability  $\Pi$  or the potential cost  $Y$  is

Lacking a formal theory concerning the determinants of these costs, Brennan (1986) assumes that a fixed cost ( $\beta$ ) is incurred whenever the trader reneges. The cost of renegeing is further assumed to be proportional to the probability of renegeing, i.e.,  $\beta P_r(l r_1 \triangleright M)$ . Hence, the total cost in the absence of price limits is

$$C^{NL}(M) = kM + \beta P_r(l r_1 \triangleright M). \quad (2)$$

## 2.2 With price limits

Suppose that a limit,  $L$ , is imposed on the price change so that no trades may occur at time 1 at prices above  $P_0 + L$ , or below  $P_0 - L$ . When a limit is triggered at time 1, his consideration will focus on his expected utility at time 2 since he cannot trade at time 1. Ignoring discounting, he will have an incentive to renege if the utility of default exceeds the utility of adhering to the contract, conditional on the limit move at time 1.

The cumulative change in wealth at time 2 can be given as follows:

$$r_1 + r_2 = \begin{cases} P_2 - P_0, & \text{if the trader adheres to the contract;} \\ -M, & \text{otherwise.} \end{cases} \quad (4)$$

Suppose that the trader is able to observe a signal  $Y_t$ , a random variable that is correlated with the change in the equilibrium futures price,  $r_t$ . Such a signal may be derivable from the spot market for the underlying commodity, from the markets for other correlated futures contracts, or from other sources. If the trader reneges, then his utility at time 2 is  $-\lambda(1 - e^{-\theta M})$ . On the contrary, if the trader sticks with the contract, his expected utility at time 2 is:

$$\begin{aligned} EU(r) &= EU(r_1 + r_2 | r_1 \leq -L, Y_1) \\ &= U^+(r_1 + r_2 | r_1 \leq -L, Y_1) - \lambda U^-(r_1 + r_2 | r_1 \leq -L, Y_1), \end{aligned} \quad (5)$$

where

$$U^+(r_1 + r_2 | r_1 \leq -L, Y_1) = \int_0^\infty (1 - e^{-\theta(r_1 + r_2)}) f(r_1 + r_2 | r_1 \leq -L, Y_1) d(r_1 + r_2),$$

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properly risk-adjusted.

and

$$U^-(r_1 + r_2 | r_1 \leq -L, Y_1) = \int_{-\infty}^0 (1 - e^{\theta(r_1 + r_2)}) f(r_1 + r_2 | r_1 \leq -L, Y_1) d(r_1 + r_2).$$

Since the trader will have an incentive to renege if the expected utility of renegeing exceeds the expected utility of abiding by the contract, the condition for neither parties to renege and for the contract to be completely self-enforcing is the following:

$$\begin{aligned} EU(r_1 + r_2 | r_1 \leq -L, Y_1) &\geq -\lambda(1 - e^{-\theta M}), \\ EU(-(r_1 + r_2) | r_1 > L, Y_1) &\geq -\lambda(1 - e^{-\theta M}). \end{aligned}$$

Thus, for a negative price change, renegeing of the long position occurs if and only if

$$r_1 \leq -L,$$

and

$$EU(r_1 + r_2 | r_1 \leq -L, Y_1) \geq -\lambda(1 - e^{-\theta M}). \quad (6)$$

The expected utility of abiding by the contract with the availability of additional information is unknown because it is conditional on the level of the signal  $Y_1$ . It is possible that a certain level of the signal will cause the expected utility of abiding the contract to exceed utility of renegeing. Assuming that the distribution of  $(r_t, Y_t)$  is jointly normal with mean zero, the probability of renegeing is given by:

$$2 P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L)), \quad (7)$$

where  $Y_1^*(M, L)$  denotes the critical level of the information below which renegeing will occur. The cost of renegeing here is assumed to be proportional to the probability of renegeing and can be presented as  $2 \beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L))$ .

Since margin is costly, the RHS in (6) defines an optimal self-enforcing contract margin level as a function of the price limit,  $M(L)$ . Here that the condition  $L < M$  is required, because if the condition is violated (i.e.,  $L \geq M$ ), then a price limit rule has no effect on the incidence of renegeing since it would always be possible to observe directly whether the price change is more than  $M$ . Incidentally, if  $L < M$  but condition (6) is violated, then renegeing occurs whenever a limit is hit. Moreover, the default probability will not only be decreasing in  $L$ , but will also be higher than it would have been without price limits, since renegeing occurs whenever  $r_1 \leq -L$ ,

instead of  $r_1 < -M$ .

While price limits reduce the default risk to market participants, there is a liquidity cost associated with their use. Clearly, the tighter the daily limits, the more often the trading is interrupted, thereby causing greater losses in liquidity to traders. Hence, the exchange faces a trade off between the default risk and the liquidity cost associated with extreme price movements in setting the price limits. Assume that the cost of price limits is proportional to the probability that a limit is triggered, the cost of price limits at time 1 can then be written as  $\alpha \frac{P_r(l r_1 \geq L)}{P_r(l r_1 < L)}$ .

With price limits and additional information, the total contract cost contains three components: the margin cost, the cost of renegeing, and the liquidity cost due to trading interruptions. Hence, the total cost for the representative trader at time 1,  $C^{PL}(M, L)$ , becomes:

$$C^{PL}(M, L) = kM + \alpha \frac{P_r(l r_1 \geq L)}{P_r(l r_1 < L)} + 2\beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L)). \quad (8)$$

The cost components in (8) indicate that the default probability is determined by the outcome of the signal  $Y_1$ . If the information signals that the loss will be large (i.e.,  $Y_1 \leq Y_1^*(M, L)$ ), and thereby the expected utility of abiding the contract is lower, then the trader will renege.

It is worthwhile to note that for a self-enforcing contract, the condition  $\frac{\partial M}{\partial L} \geq 0$  in (6) is required in order for a price limit rule to reduce the effective margin. That is, a substitute relationship should exist between the trading-interfering cost and the margin cost. If the condition is not fulfilled, meaning that the imposition of price limits raises the margin requirement, then the total contract cost may also be higher.

Furthermore, price limits provide a mechanism that obscures the exact amount of the loss incurring to the trader. This in turn may alleviate the default problem and reduce the effective margin if there exist situations in which renegeing would have occurred in the absence of price limits, but is avoided in the presence of price limits. However, if the trader can receive some precise information about the true price, then the ambiguity effect might disappear. The effectiveness of futures price limits is weakened when precise information about the unobserved equilibrium price is observed. The precision of the additional information can be characterized by the

correlation coefficient between the signal  $Y_t$  and the equilibrium price change  $r_t$ . As the total cost function is functionally complicated, analytical solutions for the above optimization problem are generally unavailable (Brennan, 1986).

In addition to the precision of signals, the risky choice under price limits are also affected by two additional elements – the curvature of the utility and the degree of loss aversion (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). The curvature of the utility is represented by the coefficient of risk aversion  $\theta$ , but it has a different meaning from the traditional risk-averse framework. Here a larger risk-aversion coefficient implies that the utility function is both more concave in gains, and also more convex in losses (Lien, 2001). The degree of loss aversion is captured by the coefficient of loss aversion  $\lambda$ . In the next section we analyze the effectiveness of price limits with additional information under various scenarios by using numerical examples.

### 3 Numerical examples

In this section we conduct numerical analysis to determine the effects of price limits on contract costs and margin requirements. As in Brennan, changes in futures prices are assumed to be normally distributed. The parameter value  $\sigma_t$  is taken as 1000, and the following values are used for the parameters of the cost function:  $k = 0.02\%$ ,  $\alpha = 1$  and  $\beta = 50$ ; the parameter values are the same as in Brennan (1986). The precision of extra-market information is measured by the correlation between the signal and the equilibrium price change,  $\rho$ . We consider four different levels of extra-market signals: 0.25, 0.5, 0.75, and 0.96. In addition, the coefficient of risk aversion  $\theta$  is set to be 0.5; the coefficient of loss aversion  $\lambda$  is set to be 2.25.

Following Brennan's (1986) experiments, we compute contract costs for each of the four signal levels for margins at intervals of 100, and for limits at intervals of 100 for each margin. The specific functional forms for each of the costs and the associated cost components are presented in the Appendix. The optimization problem is solved numerically.

### 3.1 Precision of information

Table 1 reports the numerical results for different values of information precision,  $\rho = 0.25, 0.5, 0.75,$  and  $0.96$ . For comparison purposes, Panel A of Table 1 reports the results for the case of optimal margin without price limits. Panel B of Table 1 reports the results of the optimal combinations of margin and price limits. We consider three cases: risk-neutral (i.e.,  $\theta=0, \lambda=1$ ), risk-averse but no loss-aversion (i.e.,  $\theta=0.5, \lambda=1$ ), and loss-averse (i.e.,  $\theta=0.5, \lambda=2.25$ ). To give a clearer picture about the trade-off between margin and price limits, Figure 1 plots the contract costs that correspond to various levels of margins and price limits for various precision levels of signals. From Figure 1, it can be seen that the total contract is monotonically increasing in the price-limit level,  $L$ , for a given margin. But for a fix price-limit level, there is a U shape in margins; the contract cost is first decreasing in margin, and then increasing beyond a certain level of margin, depending on the precision of the signal. The results suggest that there is a potential role for price limits hat could lower the margin requirement, and meanwhile maintain an efficient contract with a lower contract cost.

(Figure 1 about here)

Panel A of Table 1 shows that the optimal margin without price limits is 3300; the corresponding default probability is 0.1932% and the total contract cost is 0.708342. For the risk-neutral case, Panel B of Table 1 indicates that when the information precision is low ( $\rho = 0.25$ ), the optimal combination of (margin, price limit) is (2400, 1900), with the corresponding default probability being 0.0061% and the contract costs being 0.543994. For the second case with only risk-aversion, the optimal combination of (margin, price limit) is (2400, 2300); the default probability for the optimal contract is 0% and the contract cost is 0.545755. The total contrast cost is higher than that of the risk-neutral case. For the loss-aversion case, the optimal combination of margin requirement and price limit is (2400, 2300), with the renegeing probability and contract cost being 0% and 0.545755, respectively. Overall, the results indicate that, irrespective of the degree of risk attitudes, price limits are effective in reducing margin and contract costs when the precision of information the trader receives is low.

(Table 1 about here)

When the information precision ( $\rho$ ) increases from 0.25 to 0.5, the efficient margins for all  $\lambda$  being examined are also smaller than those required without price limits, which is 3300. Specifically, the optimal combination of margin and price limit for the risk-neutral investor is (2600, 2000), with the corresponding default probability and contract cost being 0.0242% and 0.579788, respectively. When the representative trader is risk averse, the margin requirement increases to 3000, and the total contract cost also rises to 0.636498. The effectiveness of the price limit rule when the trader is loss-averse is evident with the decrease of contract cost from 0.708342 without price limits and 0.636498 for the loss-neutral case to 0.583847. Overall, when the trader receives medium precision of signals like  $\rho = 0.5$ , in comparison with those without price limits and loss-neutral cases, the imposition of price limits not only results in a reduction in margin requirement, but also cuts down the default probability and the contract cost.

As  $\rho$  rises from 0.5 to 0.75, the optimal combination of margin and price limit for the risk-neutral investor becomes (2800, 2100), with the corresponding default probability and contract costs being 0.0738% and 0.633948, respectively. When the investor is risk averse but loss neutral (i.e.,  $\theta=0.5$ ,  $\lambda=1$ ), the margin required increases to 3400 and the contract cost rises to 0.726230, both of which are larger than those required without price limits. However, for the loss-averse case, the optimal combination of margin requirement and price limits is (3200, 3100), with the reneging probability and the contract cost, respectively, being 0.1213% and 0.706451, both of which are still lower than those of the loss-neutral case.

When the loss-averse trader can get access to precise signals (i.e.,  $\rho = 0.96$ ), imposing price limits, though still reducing margin requirement, causes the contract cost to increase. Specifically, in the loss-neutral case, the default probability and the total cost for the optimal contract are, respectively, 0.0992% and 0.730578. As the loss-averse coefficient  $\lambda$  becomes 2.25, the total cost for the optimal contract decreases slightly to the level of 0.715228, which is still higher than that without price limits, 0.708342.

The above results indicate that, regardless of the degrees of risk aversion and loss aversion, price limits are effective in both reducing the contract cost and lowering the margin requirement when the loss-averse trader does not receive precise information ( $\rho=0.25$  and  $0.5$ ). On the contrary, when the loss-averse trader receives relatively precise information regarding the underlying equilibrium price ( $\rho=0.96$ ), price limits are not longer useful in reducing the contract cost, and therefore fail to serve as a tool for an efficient contract design.

An interesting result is that in the case of moderate information precision ( $\rho=0.75$ ), price limits are useless in reducing the contract cost for risk-averse traders, but are useful for loss-averse traders. In addition, for given values of  $\rho$  and  $\theta$ , the optimal contract cost of the loss-neutral trader is greater than that of the loss-averse trader. This reveals that the effectiveness of price limits enhances when loss aversion is incorporated into the utility function.

Moreover, for given values of  $\lambda$  and  $\theta$ , the optimal contract cost increases in  $\rho$ . This confirms the argument of Brennan (1986) that the ambiguity effect of price limits gradually disappears as the trader can receive some precise information about the true price. In sum, our results reflect that the role of ensuring contract performance is optimally shifted from the margin requirement to the price limit rule only when the loss-averse trader does not receive high precision of information ( $\rho=0.75$  or  $0.96$ ). If the trader is loss neutral, price limits are still useful as long as he does not receive very high precision of information about the equilibrium price ( $\rho=0.96$ ).

### **3.2 Degree of loss aversion**

A second factor that affects a trader's risky choice under price limits is the degree of loss aversion. Table 2 presents the results for five different values of loss-aversion coefficients,  $\lambda = 1.5, 2.25, 3, 5,$  and  $10$ . Table 2 shows that, when the trader receives information of low precision like  $\rho = 0.25$ , the contract cost does not vary with the degree of loss aversion. This result reflects that changes in the coefficient of loss aversion have little effect on optimal contracts when the trader can only receive signals of low precision.

When the trader receives signals with higher level of precision ( $\rho=0.5, 0.75,$  and  $0.96$ ), the default probability and contract cost slightly increase with the coefficient of loss aversion ( $\lambda$ ). For instance, for  $\rho=0.5$ , the contract cost increases from  $0.554094$  for  $\lambda=1.5$ , to  $0.583847$  for  $\lambda=2.25$ , and to  $0.605505$  for  $\lambda=10$ . Within the range of  $\lambda$  being examined, though the contract cost slightly increases with  $\lambda$ 's, all margin requirements and contract costs are smaller than those without price limits ( $3300$  and  $0.708342$ ) and those of the loss-neutral case ( $3000$  and  $0.636498$ ).

When the precision of signals is  $0.75$ , for a low loss-aversion coefficient like  $\lambda=1.5$ , the optimal combination of margin requirement and price limits is ( $3200, 3100$ ), with the renege probability and corresponding pair of costs being  $0.1212\%$  and  $0.706448$ , respectively. An increase of  $\lambda$  from  $1.5$  to  $2.25$  slightly raises the default probability and contract cost to  $0.1213\%$  and  $0.706451$ , respectively. Nevertheless, the contract cost is still lower than that without price limits ( $0.708342$ ) and lower than that of the loss-neutral case ( $0.726230$ ). However, when the loss aversion coefficient increases to a level of as high as  $3$ , though the default probability for the optimal contract is lowered in comparison with those without price limits (i.e.,  $0.1251\% < 0.1932\%$ ), the trading interruption cost due to limit moves increases, which in turn raises the contract cost to  $0.708346$ , slightly higher than that without price limits ( $0.708342$ ). But their margin and associated contract cost ( $3200$  and  $0.708346$ ) are smaller than those of the loss-neutral case ( $3400$  and  $0.726230$ ). Likewise, for  $\lambda=5$  and  $10$ , the optimal contract costs, respectively  $0.710149$  and  $0.711272$ , are also larger than that without price limits, though they are smaller than that of the loss-neutral case ( $0.726230$ ).

(Table 2 about here)

Similarly, since the ambiguity effect of price limits vanishes for  $\rho=0.96$ , the benefit of price limits in reducing the incidence of renege also disappears. As a result, although there exists a level of price limits that could decrease the incidence of renege even when the trader receives precise signals, the increase in trading interruption cost resulting from limit move dominates the reduction in the renege cost; the total contract cost is thereby increased. As shown in Table 2, for  $\lambda=1.5$ , the

margin and the default probability of a cost-minimizing contract are 3200 and 0.1360%, respectively; the corresponding contract cost is 0.713795. In addition, the default probability and contract cost slightly increase with the coefficient of loss aversion ( $\lambda$ ).

Summarizing, the above results show that the more loss-averse the trader, the higher the contract cost. But even for an extremely high coefficient of loss aversion, price limits still perform better in the loss-aversion case than in the loss-neutral case and in the case without imposing price limits. One possible explanation for the positive relationship between the coefficient of loss aversion ( $\lambda$ ) and the contract cost is that as  $\lambda$  is small, a loss-averse trader, though sensitive to losses, feels more pains with moderate and large losses. Consequently, a loss-averse trader will take the risk while facing a limit move, hoping that the price will reverse; this therefore results in a reduction in default probability and contract cost. However, as  $\lambda$  increases, the trader's utility of a suffering from a loss also increases in magnitude. To reduce the potential additional loss, the trader has a stronger incentive to renege, thus leading to a higher default probability and a higher contract cost. Nevertheless, even for a large  $\lambda$  (e.g.,  $\lambda$  equals 10), price limits still play a better role under loss aversion than under loss neutral.

### **3.3 Degree of risk aversion**

A third factor that affects a trader's risky choice under price limits is the degree of risk aversion, which reflects the curvature of the utility function. Conceptually, a risk-averse trader's decision relies on the expected loss plus a "certainty equivalent," which is monotonically increasing in the degree of risk aversion. For a trader with low risk tolerance (high risk aversion,  $\theta$ ), his utility of abiding by the contract is relatively low. There is therefore a higher probability for the risk-averse investor to renege in the event of a limit move in order to avoid subsequent potential losses. As a result, price limits may increase the default probability, the margin requirement, and the contract cost (Chou et al., 2005). Chou et al. (2005), however, only investigate the loss-neutral case (i.e.,  $\lambda=1$ ). This section examines the effect of risk aversion (curvature of the utility function) in a loss-averse framework ( $\lambda=2.25$ ).

(Table 3 about here)

Table 3 presents the results for four different values of risk-aversion coefficients,  $\theta = 0.1, 0.5, 1,$  and  $10,$  respectively. The results show that, when the trader receives low precision of information like  $\rho = 0.25,$  the optimal combination of margin requirement and price limits is  $(2400, 2300),$  the default probability and the contract cost are respectively  $0\%$  and  $0.545755.$  This result remains unchanged regardless of the coefficients of risk aversion, which reflects no effect of curvature of a trader's utility on the decision of optimal contract when traders can only receive low precision of signals.

When  $\rho$  increases from  $0.25$  to  $0.5,$  the results indicate that the optimal combination of margin requirement and price limits is  $(2600, 2500)$  for  $\theta = 0.1,$  with the renege probability and the contract cost being  $0.0522\%$  and  $0.583847,$  respectively. An increase of  $\theta$  from  $0.1$  to  $0.5$  ( $10$ ) causes the default probability and contract cost to slightly increase to  $0.0523\%$  ( $0.0524\%$ ) and  $0.583883$  ( $0.583915$ ). In spite of the increase in contract cost, within the range of  $\lambda$  being examined, all margin requirements and contract costs are still smaller than those without price limits. The results are similar when  $\rho$  increases to  $0.75.$

When the trader can receive precise information ( $\rho = 0.96$ ), the results in Table 3 indicate that, regardless of the degrees of risk aversion, price limits become ineffective in either reducing the default probability, cutting down the margin requirement, or lowering the contract cost. The results are qualitatively similar to the results in Table 1 and Table 2,

A closer look at Table 3 reveals that, for given values of  $\rho$  and  $\lambda,$  except for the case of  $\rho = 0.25,$  the optimal contract cost increases slowly with the coefficient of risk aversion ( $\theta$ ). The result is consistent with Chou et al. (2005) in which an increase in risk aversion reduces the effectiveness of price limits.

#### **4 Conclusions**

This study investigates whether price limits can reduce the default risk and lower the

effective margin requirement for a self-enforcing futures contract under a loss-averse framework. As in the risk-neutral case explored in Brennan (1986), our results reveal that price limits are still effective in reducing the margin requirement and the contract cost when the loss-averse trader receives only limited information regarding the equilibrium price.

In comparison with the risk-averse, loss-neutral model of Chou et al. (2005), we find that the effectiveness of price limits in reducing margins and contract costs is enhanced when loss aversion is incorporated into the utility function. Since equal-magnitude gains and losses do not have symmetric impacts on a loss-averse trader's decision, realized losses hurt more than gains satisfy, other things being equal. To avoid potential losses, the trader would tend to take more risk, which in turn results in a lower default probability and a lower contract cost.

However, our findings also indicate that the effectiveness of price limits is limited when traders can get access to precise information, regardless of the risk attitude. Summarizing, we find that risk attitudes do affect price limit performance, but only when precise information regarding the equilibrium price is not available.

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## Appendix: Cost functions under normally-distributed price changes

This appendix presents the formulae for the utility components with a representative loss-averse trader under the assumption that price changes follow a normal distribution used in the numerical examples in Section 3.

Assume that price changes are independently, normally distributed, i.e.,  $r_t \sim N(0, \sigma_t^2)$  and  $\text{cov}(r_{t+1}, r_t) = 0$ ,  $t = 1, 2$ , and traders receive additional information about the equilibrium futures price and the additional signal is assumed to be of the form  $Y_1 = r_1 + \varepsilon_1$ , where  $\text{cov}(r_1, \varepsilon_1) = 0$ ,  $\text{cov}(r_1, \varepsilon_k) = 0$ ,  $\text{cov}(\varepsilon_1, \varepsilon_k) = 0$ ,  $\varepsilon_1 \sim N(0, \sigma_\varepsilon^2)$ , and  $t, k = 1, 2, 3$ . Conditional on the signal  $Y_1$ ,  $r_1 | Y_1 \sim N(b_1 Y_1, \sigma_\eta^2)$ ,

where  $\sigma_\eta^2 = (1 - \rho_1^2) \sigma_1^2$ ,  $b_1 = \rho_1^2 = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}$ . In the presence of information

signal, the joint distribution is the following:

$$\begin{pmatrix} r_1 | Y_1 \\ r_1 + r_2 \end{pmatrix} \sim N \left( \begin{pmatrix} b_1 Y_1 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_1^2 \end{pmatrix} \right).$$

Thus, the conditional distribution of  $r_1 | Y_1, r_1 + r_2$  is the following:

$$r_1 | Y_1, r_1 + r_2 \sim N \left( b_1 Y_1 + \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_1^2} (r_1 + r_2), \frac{\sigma_\eta^2 \sigma_1^2}{\sigma_\eta^2 + \sigma_1^2} \right).$$

Note that

$$\begin{aligned} U^+(r_1 + r_2 | r_1 \leq -L, Y_1) &= \int_0^\infty (1 - e^{-\theta(r_1 + r_2)}) f(r_1 + r_2 | Y_1, r_1 \leq -L) d(r_1 + r_2) \\ &= \left( \Phi \left( \frac{-L}{\sigma_1} \right) \sqrt{2\sigma_1^2} \right)^{-1} \int_0^\infty (1 - e^{-\theta(r_1 + r_2)}) \phi \left( \frac{r_1 + r_2}{\sqrt{2\sigma_1^2}} \right) \Phi(\alpha) d(r_1 + r_2), \end{aligned}$$

where  $\alpha = \frac{\alpha_1}{\alpha_2}$ ,  $\alpha_1 = -L - b_1 Y_1 - \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_1^2} (r_1 + r_2)$ ,  $\alpha_2 = \sqrt{\frac{\sigma_\eta^2 \sigma_1^2}{\sigma_\eta^2 + \sigma_1^2}}$ ,  $\phi$  and  $\Phi$  are

standard normal density and distribution function, respectively.

$$\begin{aligned} U^-(r_1 + r_2 | r_1 \leq -L, Y_1) &= \int_{-\infty}^0 (1 - e^{\theta(r_1 + r_2)}) f(r_1 + r_2 | Y_1, r_1 \leq -L) d(r_1 + r_2) \\ &= \left( \Phi \left( \frac{-L}{\sigma_1} \right) \sqrt{2\sigma_1^2} \right)^{-1} \int_{-\infty}^0 (1 - e^{\theta(r_1 + r_2)}) \phi \left( \frac{r_1 + r_2}{\sqrt{2\sigma_1^2}} \right) \Phi(\alpha) d(r_1 + r_2). \end{aligned}$$

Then the expected utility at date 2 given  $r_1 \leq -L$  is as follow:

$$\begin{aligned}
E(r_1 + r_2 | r_1 \leq -L, Y_1) &= U^+(r_1 + r_2 | r_1 \leq -L, Y_1) - \lambda U^-(r_1 + r_2 | r_1 \leq -L, Y_1) \\
&= \left( \Phi\left(-\frac{L}{\sigma_1}\right) \sqrt{2\sigma_1^2} \right)^{-1} \int_0^\infty (1 - e^{-\theta(r_1+r_2)}) \phi\left(\frac{r_1+r_2}{\sqrt{2\sigma_1^2}}\right) \Phi(\alpha) d(r_1+r_2) \\
&\quad - \lambda \left( \Phi\left(-\frac{L}{\sigma_1}\right) \sqrt{2\sigma_1^2} \right)^{-1} \int_{-\infty}^0 (1 - e^{\theta(r_1+r_2)}) \phi\left(\frac{r_1+r_2}{\sqrt{2\sigma_1^2}}\right) \Phi(\alpha) d(r_1+r_2).
\end{aligned}$$

The probability that renegeing will occur, given  $Y_1^*$  and  $L$ , is:

$$2 \int_{-\infty}^{-L} \Phi\left(\frac{Y_1^* - r_1}{\sigma_{\varepsilon_1}}\right) d\Phi\left(\frac{r_1}{\sigma_1}\right).$$

Hence, the contract cost at time 1,  $C_1(M, L)$ , can be written as

$$C^{PL}(M, L) = kM + \frac{2\alpha\Phi(-L/\sigma_1)}{1 - 2\alpha\Phi(-L/\sigma_1)} + 2\beta \int_{-\infty}^{-L} \Phi\left(\frac{Y_1^* - r_1}{\sigma_{\varepsilon_1}}\right) d\Phi\left(\frac{r_1}{\sigma_1}\right).$$

Table 1: Minimum Costs of a futures contract for various combinations of loss aversion coefficients under price limits

The optimal margin requirements are set to minimize the futures contract cost

$$C^{PL}(M, L) = kM + \alpha \frac{P_r(|r_1| \geq L)}{P_r(|r_1| < L)} + 2\beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L))$$

where  $C^{PL}(M, L)$  denotes the futures cost that occurs at period 1. The parameters of the cost function are:  $k=0.02\%$ ,  $\alpha=1$ ,  $\beta=50$ , and  $\sigma_1=1000$ .  $Y_1^*(M, L)$  denotes the critical level of the price change above which the utility from renegeing will exceed the utility of enforcing the contract and renegeing will occur. That is:  $r_1 \leq -L$  and  $EU(r_1 + r_2 | r_1 \leq -L, Y_1) \geq -\lambda(1 - e^{-\theta M})$  for a long position. Only the results of the cost-minimizing limit for each situation are presented.

$\theta$	$\lambda$	$\rho$	Margin	Price limit	$Y_1^*$	Default prob. (%)	Prob. of limit move (%)	Contract cost
Panel A: Without Limits								
NA	NA	NA	3300	NA	NA	0.1932	NA	0.708342
Panel B: With limits								
0	1	0.25	2400	1900	-14219.3	0.0061	5.7433	0.543994
		0.5	2600	2000	-6899.36	0.0242	4.5500	0.579788
		0.75	2800	2100	-4436.93	0.0738	3.5729	0.633948
		0.96	3200	2700	-3441.47	0.0672	0.6934	0.680558
0.5	1	0.25	2400	2300	-15223.7	0.0000	2.1448	0.545755
		0.5	3000	2900	-4823.25	0.0655	0.3732	0.636498
		0.75	3400	3300	-2334.68	0.0905	0.0967	0.726230
		0.96	3400	3300	288845	0.0992	0.0967	0.730578
0.5	2.25	0.25	2400	2300	-17147.2	0.0000	2.1448	0.545755
		0.5	2600	2500	-4746.40	0.0522	1.2419	0.583847
		0.75	3200	3100	-2865.62	0.1213	0.1935	0.706451
		0.96	3200	3100	-2574.44	0.1388	0.1935	0.715228

Table 2: Minimum Costs of a futures contract for various combinations of loss aversion coefficients under price limits

The optimal margin requirements are set to minimize the futures contract cost

$$C^{PL}(M, L) = kM + \alpha \frac{P_r(l r_1 \geq L)}{P_r(l r_1 < L)} + 2 \beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L))$$

where  $C^{PL}(M, L)$  denotes the futures cost that occurs at period 1. The parameters of the cost function are:  $k=0.02\%$ ,  $\alpha=1$ ,  $\beta=50$ , and  $\sigma_1=1000$ .  $Y_1^*(M, L)$  denotes the critical level of the price change above which the utility from renegeing will exceed the utility of enforcing the contract and renegeing will occur. That is:  $r_1 \leq -L$  and  $EU(r_1 + r_2 | r_1 \leq -L, Y_1) \geq -\lambda(1 - e^{-\theta \cdot M})$  for a long position with  $\theta=0.5$ . Only the results of the cost-minimizing limit for each situation are presented.

$\rho$	$\lambda$	Margin	Price limit	$Y_1^*$	Default prob. (%)	Prob. of limit move (%)	Contract cost
0.25	1.5	2400	2300	-19904.8	0.0000	2.1448	0.545755
	2.25	2400	2300	-17147.2	0.0000	2.1448	0.545755
	3	2400	2300	-17147.2	0.0000	2.1448	0.545755
	5	2400	2300	-15334.1	0.0000	2.1448	0.545755
	10	2400	2300	-14772.2	0.0000	2.1448	0.545755
0.50	1.5	2400	2300	-5224.77	0.0167	2.1448	0.554094
	2.25	2600	2500	-4746.40	0.0522	1.2419	0.583847
	3	2400	2300	-4359.12	0.0890	2.1448	0.590265
	5	2600	2500	-4355.96	0.1106	1.2419	0.592477
	10	2500	2400	-4161.00	0.1110	1.6395	0.605505
0.75	1.5	3200	3100	-3036.78	0.1212	0.1935	0.706448
	2.25	3200	3100	-2865.62	0.1213	0.1935	0.706451
	3	3200	3100	-2806.54	0.1251	0.1935	0.708346
	5	3200	3100	-2748.65	0.1287	0.1935	0.710149
	10	3200	3100	-2711.7	0.1309	0.1935	0.711272
0.96	1.5	3200	3100	-2625.58	0.1360	0.1935	0.713795
	2.25	3200	3100	-2574.44	0.1388	0.1935	0.715228
	3	3200	3100	-2556.84	0.1398	0.1935	0.715709
	5	3200	3100	-2539.56	0.1407	0.1935	0.716176
	10	3200	3100	-2528.55	0.1413	0.1935	0.716470

Table 3: Minimum Costs of a futures contract for various combinations of risk aversion coefficients under price limits

The optimal margin requirements are set to minimize the futures contract cost

$$C^{PL}(M, L) = kM + \alpha \frac{P_r(l r_1 \geq L)}{P_r(l r_1 < L)} + 2 \beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L))$$

where  $C^{PL}(M, L)$  denotes the futures cost that occurs at period 1. The parameters of the cost function are:  $k=0.02\%$ ,  $\alpha=1$ ,  $\beta=50$ , and  $\sigma_1=1000$ .  $Y_1^*(M, L)$  denotes the critical level of the price change above which the utility from renegeing will exceed the utility of enforcing the contract and renegeing will occur. That is:  $r_1 \leq -L$  and  $EU(r_1 + r_2 | r_1 \leq -L, Y_1) \geq -\lambda(1 - e^{-\theta M})$  for a long position with  $\lambda=2.25$ . Only the results of the cost-minimizing limit for each situation are presented.

$\rho$	$\theta$	Margin	Price limit	$Y_1^*$	Default prob. (%)	Prob. of limit move (%)	Contract cost
0.25	0.1	2400	2300	-17170.3	0	2.1448	0.545755
	0.5	2400	2300	-17147.2	0	2.1448	0.545755
	1	2400	2300	-17147.2	0	2.1448	0.545755
	10	2400	2300	-17139.7	0	2.1448	0.545755
0.50	0.1	2600	2500	-4746.4	0.0522	1.2419	0.583847
	0.5	2600	2500	-4745.78	0.0523	1.2419	0.583881
	1	2600	2500	-4745.78	0.0523	1.2419	0.583883
	10	2600	2500	-4745.22	0.0524	1.2419	0.583915
0.75	0.1	3200	3100	-2867.09	0.1212	0.1935	0.706403
	0.5	3200	3100	-2865.62	0.1213	0.1935	0.706451
	1	3200	3100	-2865.44	0.1213	0.1935	0.706457
	10	3200	3100	-2865.27	0.1213	0.1935	0.706463
0.96	0.1	3200	3100	-2574.88	0.1388	0.1935	0.715216
	0.5	3200	3100	-2574.44	0.1388	0.1935	0.715228
	1	3200	3100	-2574.39	0.1388	0.1935	0.715229
	10	3200	3100	-2574.34	0.1388	0.1935	0.715231

Figure 1: Contract costs over various margin and price limit levels

This figure plots the contract costs over various margin and price limit levels for a loss-averse trader. The contact cost is as follows:

$$C^{PL}(M, L) = kM + \alpha \frac{P_r(l r_1 \geq L)}{P_r(l r_1 < L)} + 2 \beta P_r(r_1 \leq -L, Y_1 \leq Y_1^*(M, L))$$

The parameters of the cost function are:  $k=0.02\%$ ,  $\alpha=1$ ,  $\beta=50$ , and  $\sigma_1=1000$ .  $Y_1^*(M, L)$  denotes the critical level of the price change above which the utility from renegeing will exceed the utility of enforcing the contract and renegeing will occur. That is:  $r_1 \leq -L$  and  $EU(r_1 + r_2 | r_1 \leq -L, Y_1) \geq -\lambda(1 - e^{-\theta M})$  for a long position.  $\lambda$  is set to be 2.25, and  $\theta$  is 0.5.

