Time-Varying Preferences and SAD: Evidence from an Asset-Pricing Model[*]

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Abstract

We investigate a representative agent consumption-based asset pricing model with two states: low risk aversion and high risk aversion. We explore whether there is a reasonable parameterization capable of generating the empirically observed seasonally-varying equity and Treasury returns documented by Kamstra, Kramer, and Levi (2008). Calibrating the asset-pricing model to observed consumption data, we produce seasonally varying risky and risk-free asset returns that mimic the broad characteristics of market data. Specifically, risky asset returns are high during the seasons when the risk-free returns are low and vice versa, risky asset returns vary seasonally much more than do the risk-free asset returns, and equity premia are much higher in the high risk aversion state than in the low risk aversion state. These findings are produced with small values for the coefficient of relative risk aversion and with small variation in the coefficient of relative risk aversion.

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A small literature has emerged linking seasonal affective disorder (SAD) with systematic seasonal effects in financial markets. Two papers are of primary interest for our purposes: Kamstra, Kramer, and Levi (2003) find that realized stock returns are higher during the period when SAD is most influential (the fall and winter seasons), and Kamstra, Kramer, and Levi (2008) find that the returns to safe government securities are lower during such periods. These empirical regularities are consistent with SAD-influenced investors favoring safe securities over risky securities during the periods when they suffer from the condition. Additional papers have explored the influence of SAD on other facets of financial markets, finding largely supportive results.[1] Collectively, these papers imply that individual risk preferences may vary seasonally, coincident with the timing of SAD.

We investigate a representative agent consumption-based asset pricing model with two states: low risk aversion and high risk aversion. We explore whether a reasonable set of values for key model parameters are capable of generating the empirically observed seasonally-varying equity and Treasury returns documented in the SAD literature. In particular, we seek to match the following empirical regularities: high risky asset returns during the seasons when the risk-free returns are low, low risky asset returns during the seasons with the risk-free returns are high, higher seasonal variation in risky asset returns than risk-free asset returns, and higher equity premia in the high risk aversion state than in the low risk aversion state.

This paper contributes to a literature that explores time-variation in expected returns which arises due to time-varying risk aversion. See, for instance, Brandt and Wang (2003) and Bekaert, Grenadier, and Engstrom (2004). There is also a significant literature in asset pricing that studies

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[1] Kamstra, Kramer, Levi, and Wermers (2008b) investigate the flow of funds between safe and risky categories of mutual funds and find, after controlling for other factors, that there are net flows out of risky funds and into safe funds in fall, and the patterns reverse in winter, consistent with the SAD hypothesis. Garrett *et al.* (2005) explore time-varying risk aversion in an equilibrium asset pricing model which allows the price of risk to vary through the seasons, and they find evidence consistent with the SAD hypothesis. DeGennaro, Kamstra, and Kramer (2008) study bid-ask spreads, Dolvin, Pyles, and Wu (2008) and Lo and Wu (2008) study analysts stock earnings forecasts, Dolvin and Pyles (2007) study the underpricing of initial public stock offerings, Pyles (2008) studies returns to real estate investment trusts; all find evidence consistent with the influence of SAD on markets. Dowling and Lucy (2008) enlarge Kamstra *et al.*'s (2003) original study to 37 countries and find similar results.

the relationship between time-varying risk and risk aversion. One strand of the literature focuses on the empirical observation that equity premia seem to be higher in recessions than in booms (Fama and French (1989)). Campbell and Cochrane (1999) build a representative agent model and show that when the representative agent has habit formation, her risk aversion is higher at business cycle troughs than it is at peaks. As a result, equity premia are higher at business cycle troughs than they are at peaks. Basal and Yaron (2004) employ a model where the representative agent has recursive preferences (Epstein and Zin (1989) and show that when consumption growth has a small time-varying, but persistent and predictable component, the model can generate an equity premium close to what is observed in the US data. A second strand of the literature focuses on the observation that the equity premium seems to have declined in recent years, up until recently. Lettau, Ludvigson, and Wachter (2008), using a regime-switching model, show that consumption volatility seems to have moved into a low-volatility regime and that the low observed equity premium can be rationalized by such regime changes in consumption volatility. This literature is, however, based on time-varying risk at the business-cycle frequency or even longer. In contrast, our study is based on the annual frequency of the SAD effect and its implications for the cyclical pattern of equity premia.

We show in a simple setting similar to that of Mehra and Prescott (1985) that small changes in risk aversion can lead to seasonal patterns of equity returns, risk-free rates, and equity premia that are broadly consistent with observed annual seasonal patterns in US data. In that setting, however, we are unable to reproduce the stylized fact that risk-free returns vary much more modestly than risky asset returns. We are able to match both the stylized fact of opposing safe versus risk returns and the magnitude of the observed seasonal variation in returns if we allow annual cycles in both risk aversion and the willingness to delay or accelerate consumption (that is, the intertemporal elasticity of substitution, IES). Interestingly, variation in the IES alone is not sufficient to match even the stylized patterns of opposing seasonal cycles in risky and safe asset returns, let alone the magnitude of these changes.

In Section 1 we elaborate on the hypothesis under which SAD may influence financial markets. In Section 2 we explore asset-pricing models in which the representative agent exhibits risk preferences that vary with SAD. In Section 3 we outline the data used for our calibration exercises and we present our results. We conclude in Section 4.

1 Seasonal Affective Disorder

SAD is a form of major depressive disorder which affects about ten percent of the population, with additional numbers suffering from the milder condition known as 'winter blues.' Medical research has established that among the various possible environmental factors that might cause SAD, length of daylight appears to be the primary cause.[2] Those who suffer from SAD typically begin experiencing depression in the fall and recover by spring.

Prior research in psychology has established a link between depression and reduced risk tolerance, including risk of a financial nature. With a fraction of the population becoming depressed in the fall and winter months, Kamstra *et al.* (2003) conjecture that the proportion of risk-averse investors rises. Consistent with observations in other contexts, risk-averse investors, they argue, shun risky stocks in the fall as the length of day shortens, which has an immediate negative influence on stock prices and returns. As the amount of daylight rebounds through the winter months, investors recover from their depression and become more willing to hold risky assets, which, Kamstra *et al.* posit, has a positive influence on stock prices and returns. They demonstrate economically and statistically significant seasonal patterns in international stock indices consistent with this line of reasoning. The patterns are more prominent in stock markets at extreme latitudes, such as Sweden, where the fluctuations in daylight are more extreme. Further, the seasonal patterns are six months out of sync in southern hemisphere markets such as Australia where the seasons are six months out of phase.

Kamstra *et al.* (2008) document an opposite seasonal pattern in Treasury security returns

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[2] See, for instance, Molin *et al.* (1996) and Young *et al.* (1997).

relative to stock returns, consistent with time-varying risk aversion being the underlying force behind both effects. If SAD-affected investors are shunning risky stocks in the fall, as they become more risk averse, then they should be favoring safe assets at that time, which should lead to an opposite seasonal pattern in Treasury returns relative to stock returns. Kamstra *et al.* (2008a) demonstrate that the opposing seasonal patterns in Treasury returns versus equity returns do not arise due to any of a wide range of alternative cyclical factors, including macroeconomic risks, Fama and French (1993) risk factors, Baker and Wurgler (2006) sentiment, cross-market hedging, and others.

2 Asset Prices and Returns

In this section we develop a representative agent model of asset pricing in hopes of matching the opposing seasonal cycles that are observed in risky versus safe asset returns. The model differs from the standard Lucas model in two respects. First the representative agent has recursive utility as in Epstein and Zin (1989) and second the parameters of the utility function of the agent are time-varying. The introduction of time-varying preference in a Lucas model is a simple way of capturing the SAD effect on the agent's behavior.

2.1 A Simple Mehra-Prescott-Style Asset Pricing Model

To illustrate the basic intuition of our model and to motivate the use of recursive preference, we start with the Lucas (1978) economy that Mehra and Prescott (1985) explore, modifying it to allow the representative agent to have two states of risk aversion.

We assume a two-state first-order Markov process for the growth rate of consumption. We allow the coefficient of relative risk aversion to differ across two states: high risk aversion and low risk aversion. The representative agent is endowed with the shares of all the assets. The representative agent is assumed to have the standard intertemporal expected utility:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{c^{1-\gamma_t}-1}{1-\gamma_t}\right]$$

where γ_t is the level of risk aversion of the agent. We assume that the level of risk aversion of the agent is time-varying, but in a deterministic fashion. Denote by $\bar{\gamma}$ the average level of risk aversion.

The standard Euler equation is then

$$0 = E_t \left[\frac{c_{t+1}^{-\gamma_{t+1}}}{c_t^{-\gamma_t}} (R_{t+1} - r_t) \right]. \tag{1}$$

This equation can be re-written as

$$0 = E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma_t} (c_{t+1})^{\gamma_t - \gamma_{t+1}} (R_{t+1} - r_t) \right]. \tag{2}$$

Let $1 + g_{t+1} = \frac{c_{t+1}}{c_t}$. Then the equation above can be written as

$$0 = E_t \left[(1 + g_{t+1})^{-\bar{\gamma}} (c_{t+1})^{\bar{\gamma} - \gamma_{t+1}} (R_{t+1} - r_t) \right]. \tag{3}$$

While it is straightforward to use this simple model to illustrate the potential impact of time-varying risk aversion on asset returns, it is difficult to extend the model to allow for time-varying risk aversion. To see it, consider the risk-free rate in this economy. By the first order condition, the risk-free rate must satisfy

$$1 = \beta E_t \left[\frac{c_{t+1}^{-\gamma_{t+1}}}{c_t^{-\gamma_t}} (1 + r_t) \right].$$

It follows that

$$\frac{1}{1 + r_t} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma_t} c_{t+1}^{\gamma_t - \gamma_{t+1}} \right] = \beta c_t^{\gamma_t - \gamma_{t+1}} E_t \left[(1 + g_{t+1})^{-\gamma_{t+1}} \right]. \tag{4}$$

Thus if consumption c_t grows, then r_t is not stationary.

2.2 Recursive Utility with SAD

We turn now to Epstein and Zin (1989) preferences, modified to allow for a representative agent with time-varying risk aversion.

We assume that the preference of the representative agent is given by

$$U_t = \left[c_t^{(1-\gamma_t)/\theta_t} + \delta (E_t U_{t+1}^{1-\gamma_t})^{1/\theta_t} \right]^{\theta_t/(1-\gamma_t)}$$

where $\theta_t = (1-\gamma_t)/(1-1/\psi_t)$. Here ψ_t is the intertemporal elasticity of substitution. For most our analysis, we assume that $\psi_t = \psi$ is a constant. However, the analysis is also valid for a time-varying intertemporal elasticity of substitution. When $\theta_t = 1$ we get the standard intertemporally additive expected utility.

The utility maximization problem of the representative agent is

$$J_t(W_t, x_t) = \max_{c_t, \pi_t} \left[c_t^{(1-\gamma_t)/\theta_t} + \delta (E_t J_{t+1}^{1-\gamma_t})^{1/\theta_t} \right]^{\theta_t/(1-\gamma_t)}$$

subject to the constraint

$$W_{t+1} = (W_t - c_t)\pi_t(1 + R_{t+1})$$

where x_t is the vector of state variables and π_t is the vector of portfolio weights.

It is readily seen that the recursive utility is homothetic so that $J_t(W_t, x_t) = f_t(x_t)W_t$. The optimal consumption c_t satisfies $c_t = C_t(x_t)W_t$ where $C(x_t)$ satisfies

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^\theta (1 - C_t(x_t))^{1-\gamma_t-\theta_t} E_t \left[f_{t+1}(x_{t+1})^{1-\gamma_t} (\pi_t(1 + R_{t+1}))^{1-\gamma_t} \right],$$

and in equilibrium the asset returns satisfy

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^\theta (1 - C_t(x_t))^{1-\gamma_t-\theta_t} E_t \left[f_{t+1}(x_{t+1})^{1-\gamma_t} (\pi_t(1 + R_{t+1}))^{-\gamma_t} (1 + R_{i,t+1}) \right].$$

In equilibrium, the representative agent holds the market portfolio. The equation derived above can be written as

$$1 \quad = \quad \delta_t^\theta \left(\frac{1}{C_t(x_t)} - 1 \right)^{1-\gamma_t-\theta_t} E_t \left[f_{t+1}(x_{t+1})^{1-\gamma_t} \left(\frac{P_{t+1} + d_{t+1}}{P_t} \right)^{1-\gamma_t} \right]$$

where P_t is the price of the market portfolio. Homogeneity suggests that $P_t = w(x_t)d_t$. Now note that in equilibrium $c_t = d_t$ and consumer wealth is equal to the stock price because there is only one share of the stock and the consumer does not have labor income. Thus, $W_t = P_t + d_t$ and $C_t(x_t) = c_t/W_t = 1/(w_t(x_t) + 1)$. Then

$$1 \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\gamma_t-\theta_t} E_t \left[(w_{t+1}(x_{t+1}) + 1)^{\frac{(\gamma_{t+1}+\theta_{t+1}-1)(1-\gamma_t)}{(1-\gamma_{t+1})}} \left(\frac{(w_{t+1}(x_{t+1}) + 1)g_{t+1}}{w_t(x_t)} \right)^{1-\gamma_t} \right].$$

Simplifying, we can write

$$w_t(x_t)^{\theta_t} = \delta^{\theta_t} E_t\left[g_{t+1}^{1-\gamma_t}(1+w_{t+1}(x_{t+1}))^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}}\right]. \tag{5}$$

Let us assume a finite state Markov world and that $P(i,d) = w_{it}d$. Assuming there are only two states, Equation (5) yields the following system of equations for price-dividend ratios, w_{ik}, where i indexes for states and k indexes for different levels of risk aversion:

$$\begin{aligned}
(w_i^1)^{\theta_1} &= \delta^{\theta_1}\sum_{j=1}^{n}\phi(i,j)g_j^{1-\gamma_1}(1+w_j^2)^{\frac{\theta_2(1-\gamma_1)}{(1-\gamma_2)}} && (6)\\
(w_i^2)^{\theta_2} &= \delta^{\theta_2}\sum_{j=1}^{n}\phi(i,j)g_j^{1-\gamma_2}(1+w_j^1)^{\frac{\theta_1(1-\gamma_2)}{(1-\gamma_1)}}. && (7)
\end{aligned}$$

Here w_i^j has the interpretation of price-dividend ratios when the risk aversion is γ_j and current state of consumption growth is i.

More generally, the price of any stock should satisfy

$$1 = \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[(w_{t+1}(x_{t+1})+1)^{\frac{(\gamma_{t+1}+\theta_{t+1}-1)(1-\gamma_t)}{(1-\gamma_{t+1})}}((w_{t+1}(x_{t+1})+1)g_{t+1})^{-\gamma_t}\left(\frac{P_{j,t+1}+d_{j,t+1}}{P_{j,t}}\right)\right].$$

Thus

$$P_{j,t} = \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[g_{t+1}^{-\gamma_t}(w_{t+1}(x_{t+1})+1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}-1}(P_{j,t+1}+d_{j,t+1})\right]. \tag{8}$$

It follows from Equation (8) that the one-period bond price satisfies

$$B_t = \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[g_{t+1}^{-\gamma_t}(w_{t+1}(x_{t+1})+1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}-1}\right]. \tag{9}$$

Let us assume a two-state Markov world and that there are only two levels of risk aversion. Then Equation (9) yields the following system of equations for states $i = 1, 2$:

$$\begin{aligned}
B_i^1 &= \delta^{\theta_1}(w_i^1)^{1-\theta_1}\sum_{j=1}^{n}\phi(i,j)g_j^{-\gamma_1}(1+w_j^2)^{\frac{\theta_2(1-\gamma_1)}{(1-\gamma_2)}-1}, \quad \text{for odd time periods, } s = 1, 3, \ldots && (10)\\
B_i^2 &= \delta^{\theta_2}(w_i^2)^{1-\theta_2}\sum_{j=1}^{n}\phi(i,j)g_j^{-\gamma_2}(1+w_j^1)^{\frac{\theta_1(1-\gamma_2)}{(1-\gamma_1)}-1}, \quad \text{for even time periods, } u = 2, 4, \ldots. && (11)
\end{aligned}$$

Having solved for w_{ik} from Equations (6) and (7) we can now solve for the bond prices and hence the risk-free rates. It is seen from these expressions that unlike the case of expected CRRA utility, risk-free rates are stationary when the representative agent has Epstein-Zin (1989) utility. It should be noted that even if $\theta_t = 1$, that is when intertemporal elasticity of substitution is equal to risk aversion, the case of Epstein-Zin preferences with SAD does not reduce to expected utility with SAD, unlike the case without SAD.

3 Data and Model Calibration

We need consumption growth parameters for our model before we can calibrate the preference parameters to match model-predicted expected returns to observed return patterns. We use two alternative calibrations for our modeling of consumption growth. The first is the classic parameterization of Mehra and Prescott (1985) with negative autocorrelation of consumption growth, mean consumption growth equal to 1.8%, and standard deviation of growth equal to 3.6%. The second is based on updated consumption data from January 1959 to December 2007. We restrict our attention to data starting in 1959 due to availability of the risk-free Treasury series we employ, described below. The consumption data we collect for this exercise is real nondurables and services consumption (BEA), identical to Mehra and Prescott (1985).

Our model predicts rates of return for a risky asset and for a risk-free asset. For our calibration exercise, we use CRSP value-weighted returns including dividends for the risky asset, and for the risk-free asset we use the 6-Month Treasury bill secondary market rate, from Board of Governors of the Federal Reserve System. We deflate the nominal series following Mehra and Prescott (1985) using a deflator series produced by dividing real consumption of non-durables and services by the nominal consumption on non-durables and services. For this calibration, we look to semi-annual data, with the semi-annual periods beginning in September and March of each year, matching the seasonal peaks of the onset of and recovery from SAD.[3]

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[3] Our choice to start the six-month periods in September and March is based on clinical studies by Young *et al.* (1997) and Lam (1998) who document the precise timing of the clinical onset of and recovery from SAD symptoms

We find positively autocorrelated consumption growth, and assuming a two-state Markov process for consumption growth, we derive $\phi(i,i) = 0.75$ (in contrast to Mehra and Prescott's estimate of 0.43). The mean (annualized) real growth rate in consumption we find is 0.0337 together with a standard deviation of 0.0137. We will also follow Lettau, Ludvigson, and Wachter (2008) and employ a scaling parameter (denoted λ by Lettau *et al.* (2008) and others) that increases the volatility of consumption growth match dividends, setting $\lambda = 4.5$.

The return patterns we seek to reproduce with our model indicate a remarkable seasonality in rates of return and the equity premium. The (annualized) real returns are listed in Table 1 for the period 1959-2007 and various sub-periods thereof.

We see a small decline in yield for risk-free assets in the high risk aversion period, roughly .043% and a large increase in equity returns as well as risk premia, roughly 5%. Over the past 2 decades, from 1990 to 2007, we saw a larger decline in risk-free rates in the September-February period of 15 bps, together with larger fluctuations in the equity returns and premium, with the equity return (premium) at roughly 5.3% (3.8%) in the Mar-Aug period jumping to 14.2% (12.9%) in the Sep-Feb period. If we use the yield of a 3 month T-bill (averaged over the 6 month periods) we find yields drop as much as 50 bps in the Sep-Feb period over 1990-2007.

Every sub-period we consider, also including 1970-2007, exhibits the same pattern of returns, and the higher risk-free returns in the spring/summer were also evident if we look at 1 year T-bills. This phenomenon is very robust.

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among North Americans known to be affected.

Table 1

Period	Equity Return	Risk-free Rate	Equity Premium
1959-2007			
March-August	0.05510	0.018218	0.03688
September-February	0.10354	0.017789	0.08575
1990-2007			
March-August	0.05318	0.014803	0.03838
September-February	0.14243	0.013458	0.12897
1959-1979			
March-August	0.01310	0.011237	0.00749
September-February	0.08453	0.011151	0.07897
1980-2007			
March-August	0.08715	0.023289	0.06387
September-February	0.11914	0.022782	0.09636

Notes to Table 1: The T-bill series used for the risk-free rate is deflated with the predicted inflation rate, where we used an ARMA(1,1) time series model to form our predicted inflation series. This regression, estimated on semi-annual data, had an R-squared of 68% and removed evidence of autocorrelation to 5 lags (2 1/2 years). Coefficient estimates (with standard errors in parentheses) are as follows. Intercept: 0.0025 (0.0015), AR(1): 0.89 (0.071), MA(1): -0.081 (0.145).

We calibrate to the 1959-2007 period and use a deviation in risk-free rates of 20 bps, which is in the lower-middle portion of the range that we documented above. We now explore what preference parameters of our model can best match the observed data, calibrating to the following moments:

Table 2

Period	Equity Return	Risk-free Rate	Equity Premium
March-August	0.055	0.020	0.035
September-February	0.103	0.018	0.085

We search over a grid of preference parameters as follows. The parameter of relative risk aversion, γ, has been argued to be possibly as high as 30 by Lettau *et al.* (2008) (to match observed mean equity premia, dividend yield and risk-free rates on post-war data). Although we consider high values of γ, we find values between 2 and 10 suffice for our exercise. We explore changes in γ moving into the high risk aversion case as small as 1 and as large as 30. The IES parameter (denoted ψ) has been argued by Lettau *et al.* (2008) and Vissing-Jorgensen *et al.* (2003) to be greater than 1. Vissing-Jorgensen *et al.* (2003) suggest values close to 1.5 but report values as high as 17.6 (see their Table 1). We will focus our attention on values less than 10, as small as 0.5, and we will also consider allowing the IES to drop (become less elastic) when risk aversion is rising. IES can be thought of as the payment required to persuade somebody to defer consumption to the next period. In a simple Mehra and Prescott model, the IES is the inverse of the risk aversion parameter. In the the recursive utility model that we exploit, the one-to-one relationship between risk aversion and IES is not maintained; we anticipate that if both were to change, IES would decline as the coefficient of relative risk aversion increases. Very small changes in the IES can lead to remarkable changes in expected returns, so we consider varying the value of the IES across risk aversion states.

Representative results are presented in Tables 3 (using Mehra and Prescott consumption parameters) and 4 (using using our 1959-2007 consumption parameters).

Table 3
(using Mehra and Prescott consumption parameters)

γ	ψ	Period	Equity Return	Risk-free Rate	Equity Premium
1.5	2	March-August	0.0685	0.034	0.035
2.5	2	September-February	0.0738	0.018	0.056
2	4	March-August	0.072	0.028	0.043
3	4	September-February	0.080	0.016	0.064
2	10	March-August	0.072	0.030	0.042
3	10	September-February	0.081	0.019	0.062

Table 4
(using using our 1959-2007 consumption parameters)

γ	ψ	Period	Equity Return	Risk-free Rate	Equity Premium
1.5	2	March-August	0.0727	0.062	0.011
2.5	2	September-February	0.0756	0.056	0.019
2	1.5	March-August	0.078	0.066	0.013
3	1.5	September-February	0.080	0.060	0.020
2	8	March-August	0.064	0.047	0.017
12	8	September-February	0.104	0.008	0.096
2	10	March-August	0.072	0.030	0.042
3	10	September-February	0.081	0.019	0.062

With these preference parameters, we see the expected comparative statics results, with equity premia and returns higher, and risk-free rates lower, in the high risk aversion state relative to the low risk aversion state. If ψ is below 1 we find that all rates of return increase in the high risk aversion state.

Hence, a core result is that we can easily match the sign of changes in returns and premia across risk aversion states, provided IES is greater than 1. That is, the comparative statics are easy to match with small changes in the coefficient of relative risk aversion and reasonable values of the IES preference parameter.

There are two issues that require further exploration. First, can we produce the correct comparative static results if the IES changes and the coefficient of relative risk aversion does not? Second, are we able to match the magnitude of return changes observed in the data, that is large equity and equity premium changes versus very small changes in the risk-free rate?

3.1 Changing IES

The notion that the preference parameter IES may change over time is not controversial. See, for instance, Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), and Atkeson and Ogaki (1996). Might a seasonally varying IES explain the return patterns we see?

We search over a very large grid, with the coefficient of relative risk aversion ranging from 1.5 to 30 (but the same in both IES states of the world), the IES ranging from 0.5 to 20, the low elasticity state equal to the high elasticity state minus .05, .1 or .2, and consumption growth parameters calibrated to Mehra and Prescott values or our values. We find that we are able to replicate the comparative statics results for only a single case, $\gamma = 1.5$ $\psi_1 = 0.5$ and $\psi_2 = 0.3$; however we also find that the magnitudes of returns are implausible for this case. Representative results as well as this single case are presented in Table 5 (using using our 1959-2007 consumption parameters).

Table 5
(using our 1959-2007 consumption parameters)

γ	ψ	Period	Equity Return	Risk-free Rate	Equity Premium
1.5	0.5	March-August	1.482	1.4719	0.010
1.5	0.3	September-February	4.431	1.4269	3.004
1.5	4	March-August	0.0284	0.015	0.013
1.5	3.95	September-February	0.1057	0.091	0.014
2	4	March-August	0.0310	0.013	0.018
2	3.95	September-February	0.1070	0.088	0.029

3.2 Matching Return Magnitudes

We saw above that we are unable to match return magnitudes when only the coefficient of relative risk aversion can vary over time, though it was straightforward to match the comparative statics results. We demonstrate in this section that if we allow both the coefficient of relative risk aversion and the IES to change together, with the IES preference parameter falling (substitution over states of the world becoming more costly) as the coefficient of relative risk aversion increases, we can match the magnitude as well as the changes in returns in our semi-annual data.

Representative results are presented in Tables 6 (using Mehra and Prescott consumption parameters) and 7 (using using our 1959-2007 consumption parameters).

Table 6
(using Mehra and Prescott consumption parameters)

γ	ψ	Period	Equity Return	Risk-free Rate	Equity Premium
2	8	March-August	0.059	0.017	0.042
5	7.95	September-February	0.113	0.012	0.101
2	6	March-August	0.060	0.017	0.043
5	5.95	September-February	0.110	0.009	0.102

These results capture the primary features of the data: the opposing changes in equity returns and risk-free returns, the symmetric movements in equity returns and risk premia, and the large swings in equity returns and risk premia in contrast to the small swing in risk-free returns. These effects are captured with modest changes in the coefficient of relative risk aversion and a very small change in the IES. Even if only 10% of market participants experience seasonal changes in risk tolerance (a conservative estimate), implying that the change in the coefficient of relative risk aversion understates the magnitude of the change in these individuals by a factor of 10, multiplying the change in the coefficient of relative risk aversion by 10 still leaves us in the range considered by Lettau *et al.* (2008).

Using our consumption parameters, Table 7, leads to very similar results.

Table 7
(using our 1959-2007 consumption parameters)

γ	ψ	Period	Equity Return	Risk-free Rate	Equity Premium
2	10	March-August	0.054	0.036	0.018
7	9.9	September-February	0.096	0.032	0.064
2	5	March-August	0.046	0.0292	0.017
12	4.95	September-February	0.125	0.0291	0.096

4 Conclusions

Any model of asset returns used to bridge the empirical-conceptual gap must ultimately be based on preferences for consumption over time, on risk, or on attitudes toward that risk. We explore the ability of conventional asset-pricing models to match a remarkable empirical regularity in risk-free and risky asset returns, argued by KKL (2003, 2008) to be evidence of the influence of an annual cycle of depression and risk aversion induced by the annual incidence of SAD.

We find, first, that a model incorporating seasonality in the willingness to bear risk matches the observed opposing seasonal cycles in risky versus safe asset returns, but is unable to match the magnitude of the seasonal changes in returns, notably failing to produce more modest variation in risk-free returns relative to risky asset returns. Second, we find that the magnitude of the observed seasonal variation in returns can be well matched if we allow annual cycles in both risk aversion and the willingness to delay or accelerate consumption, the intertemporal elasticity of substitution, IES. Interestingly, variation in the IES alone is not sufficient to match even the stylized patterns of opposing seasonal cycles in risky and safe asset returns, let alone the magnitude of these changes.

The empirical understanding of the SAD phenomenon has moved far ahead of our theoretical understanding of how specific preferences could generate the observed consequences in returns and asset flows and other behaviors mentioned earlier. This paper represents an attempt to bring our understanding of the theory closer to what we know from wider and wider empirical investigations of financial markets. The compatibility of theory and observation is a critical scientific step in this process. We believe that we have considerably closed the theoretical-observational gap in this exercise. The convergence of theoretical implications of a state representative agent model with reasonable parameterization to empirical observations should provide a greater degree of confidence in the relevance of SAD for the study of financial markets.

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