# Heterogeneous Beliefs, Competition, and the Viability of Financial Innovation[1]

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**Abstract**

This paper studies a dynamic equilibrium model of financial innovation with heterogeneous beliefs and competition among sellers of a new security. We show that both volume and price of the security after it is introduced are sensitive to the differing beliefs of participating players. We present conditions for the innovator to continue issuing the new security and for the imitator to enter the market, and identify scenarios when there is a breakdown in the market with no transactions. We illustrate that the market for forward-type securities is more resilient to the underlying market movement compared to the market for option-type contracts, which is more vulnerable as trades in the security are sensitive to the underlying market condition and may disappear with a drastic market movement. We discuss the implications of our analysis for complex financial instruments, such as tranches of collateralized debt obligations (CDOs).



*Keywords*: Financial innovation, competition, heterogeneous beliefs.

*JEL Classification Codes*: G01, G12, G14, G20.

# 1 Introduction

Financial innovation has played an important role in the development of financial markets (see, e.g., Allen and Gale (1994) and Tufano (2003)). Recent turmoils in the credit market stemming from mortgage-backed securities, especially collateralized debt obligations (CDOs), however, have raised serious questions about the efficacy and viability of new financial products. In particular, some of the problems may be attributable to the uncertainty about the underlying asset process faced by issuers, investors and rating agencies alike (see, e.g., Coval, Jurek, and Stafford (2008a)). This uncertainty can lead to dispersed beliefs about the valuation of related derivative securities and, in a sharp market downturn, cause a dramatic decline in trading activities in these securities.

This paper investigates how heterogenous beliefs about the underlying fundamental process on which a new financial product is built and the competition from an imitator affect the viability of the financial innovation under different market conditions. A successful financial innovation is viable with a high level of market participation over time, while a vulnerable innovation may fail to survive, with low or no trading volume, under adverse market conditions.[1] Diversity of beliefs among both buyers and sellers of a new financial instrument and competition between the innovator and the imitator can impact the equilibrium price and the trading volume of the product. They can also affect the viability of a financial innovation under significant movements in the underlying fundamental process.

In order to analyze the viability of financial innovation, we develop a dynamic equilibrium model in which an innovator optimally designs and sells a new financial product that pays off in one period and an imitator may decide to compete by offering the same product in the second period. In both periods, the demand for the financial product is determined by a representative investor who is risk averse and optimally allocates her wealth into the new security. The uncertainty about the underlying fundamental process is represented by an unobservable mean of the unconditional distribution, and all players have their own initial beliefs for the underlying process and revise their expectations based on realized payoffs through Bayesian updating. This belief structure shares similar characteristics as those of the rational belief structure used in Garmaise (2001) and is also found in Calvert, Gonzales-Eiras, and Sodini (2004), Cao (2008), and Easley and O'Hara (2008).

[1] Black (1986) and Bettzüge and Hens (2001) define success or failure of a financial contract in the market in a similar way. In a recent address, Plosser (2009) points out that "As a general proposition, ..., not every innovation is successful." As an example, the recent collapse of the auction rate security market is examined in Han and Li (2009).

In our analysis, we start with solving for financial equilibrium in the second period with a given contract structure and the possible entry of an imitator. The competition between the innovator and the imitator is modeled in a Cournot game, similar to a feature in Rahi and Zigrand (2008) who study the role of arbitrageurs in using financial innovation to exploit mispricing across segmented markets. We identify the condition, based on the realization of the underlying variable that the security is contracted on, for different participation scenarios for the innovator and the imitator and for the case when the market for the security breaks down with no transactions. In the first period, the innovator optimally chooses appropriate parameters for the contract and determines the offer price in anticipation of the potential entry of the imitator with a different belief.

Through a comparison with a benchmark model in which the innovator and the imitator share the same belief, we find that the security price can deviate from the benchmark price significantly due to the divergence in beliefs. The price is higher if the imitator is relatively more optimistic about the future payoff from the security. This benefits the innovator as her expected profit increases because she sells more of the security at a higher price. The opposite situation holds if the imitator is relatively more pessimistic about the security payoff. While the entry of an imitator does introduce competition that reduces the innovator's monopolistic profit, the potentially better-informed belief the innovator has may still confer significant first-mover advantage. Thus our analysis yields new insights into the question about the first-mover advantage for the innovator that was raised in Tufano (1989).

We illustrate the intuition of our model through examples of basic derivative securities, which are building blocks of more complex financial innovations. We show that the market for forward-type securities is rather resilient in the sense that, for a reasonable range of parameters, there is almost always a seller of the security, especially if beliefs are diverse. The market for option-type contracts, however, appears to be more vulnerable in that the supply of the security may dry up quickly when the underlying asset value experiences a drastic shock. Even with a moderate movement in the underlying asset value, the nonlinearity of the payoff structure, coupled with heterogeneous beliefs and competition among sellers of the security, makes the availability of the security very sensitive to the market condition.

To the extent that a path-through security on an underlying collateral pool of assets may resemble a forward contract on the value of the asset pool and that tranches of the capital structure in CDOs can be viewed as various option contracts on the underlying collateral pool, our analysis has implications for their differing behavior under changing market conditions. During the boom market for the underlying assets, originators of CDO structures, both the innovator and the imitator, would expect earning profits selling the call option (the equity tranche), but they may have to take a loss to sell other option contracts (senior and mezzanine tranches) at prices investors would want to buy.[2] If the loss can be offset by the profit in selling the equity tranche, in addition to the underwriting fees we ignore here, there are incentives for theses sellers to construct and market CDOs. This may provide one potential explanation for the high yields observed for the highly rated senior and sometimes mezzanine tranches during the heyday of the CDO market. While Coval, Jurek, and Stafford (2008a) offer an explanation based on the mis-representation of ratings and the unpriced risk of economic catastrophe (see also Coval, Jurek, and Stafford (2008b)), the implication of our analysis may indicate a complementary motive from the supply side. When the market experiences a large adverse move, expectations of security payoffs from different players can shift quite dramatically, injecting a degree of instability into the market and even causing a market breakdown with no offering of new securities. This is reminiscent of the situation in the CDO market during the current credit crisis starting in 2007.

Our work builds on Allen and Gale (1991) to establish the financial equilibrium in the presence of financial innovation. We distinguish the role of an imitator by fully developing the intuition sketched in Allen and Gale (1994) in a dynamic framework that departs from the evolutionary approach of Bettzüge and Hens (2001) and allows us to examine the benefits for the innovator to be the first mover.[3] Another important feature in our model is to incorporate heterogeneous beliefs in a multi-period setting with competition which extends the static rational-belief model of security design by Garmaise (2001) and provides a rich context for analyzing the vulnerability of newly designed financial securities.

There is a vast literature on security design that bears on our work. DeMarzo and Duffie (1999) consider a security design problem in the presence of the perceived information advantage of the issuer that trades off between holding un-securitized cash flows and incurring the cost of adverse selection (or liquidity cost) associated with full securitization. DeMarzo (2004) further uses this framework to deal with the optimality of pooling and tranching in security design when there is information advantage to the issuer. Axelson (2008), in contrast, models the situation in which investors is more informed than the issuer and there is competition among investors. Our paper departs from this line of literature. Rather

[2]Franke, Herrmann, and Weber (2007) find empirically that senior tranches are not sold to investors using European CDO transaction data.

[3]In Bettzüge and Hens (2001), the market participation structure is exogenously given while in our model the participation structure is endogenously determined in equilibrium.

than explicitly addressing the issue of optimal security design from the perspective of a firm trying to raise more capital, we consider a scenario in which financial intermediaries offer new derivative instruments, possibly structured products, based on existing underlying fundamental securities. In our model, we allow all players to have different beliefs and investigate in a dynamic framework how belief heterogeneity affects market prices and volume and impacts the viability of financial innovation of various payoff structures.

Another related work is by Easley and O'Hara (2008) who investigate the implications of uncertainty for the liquidity and valuation of assets. They show that heterogeneous beliefs and incomplete preferences over portfolios among traders due to their uncertainty about the asset value distribution contribute to the absence of trading in the market. While these authors focus on the trading and valuation of existing securities, we study the issuance of new securities as part of financial innovation from financial intermediaries. The type of financial innovation we consider here is related to asset-backed securities and their derivatives in that the underlying pools are not directly tradable. It therefore departs from the asset structure in Calvert, Gonzales-Eiras, and Sodini (2004) and Cao (2008), which is set to examine the impact of the availability of derivative securities on the trading in and the price of the underlying security in the presence of heterogeneous beliefs among investors.

The rest of paper is organized as follows. In Section 2, we describe the structure of the model and present an analysis of the effect of belief heterogeneity on the equilibrium price, volume and expected profits. We analyze in Section 3 the viability of different derivative securities and discuss implications for tranched offerings of the collateralized debt obligations. We conclude in Section 4. All proofs are collected in the appendix.

# 2 The Model

There are three players in the model: an innovator, an imitator, and a representative investor. We use $i \in \{n, m, v\}$ to indicate the type of players. Specifically, "$n$" denotes the innovator, "$m$" the imitator, and "$v$" the investor, respectively.

There are three dates: $t = \{0, 1, 2\}$ in the model. At time $t = 0$, the innovator produces a new financial product that has a one-period payoff $f(x)$, which depends on the value of an underlying variable $x$ at time $t = 1$ when the contract matures, and sells it to the investor. We assume that the type of security issued is known, i.e., the functional form of the payoff,

$f(x)$, is fixed. The innovator, however, has the discretion in determining the parameters associated with the contract that affect the allocation of the final cash flow between the issuer and the investor. Hence, the payoff of the security is represented by $f(\{a\}; x)$, with $\{a\}$ being the relevant parameters chosen by the innovator.[4]

At time $t = 1$, an imitator may decide to sell the same security to the investor.[5] The innovator will also decide whether to continue offering the same product to the investor. If both decide to participate in the market, the price of the security is determined in a Cournot equilibrium.[6] The equilibrium price of the security at $t = 1$ also depends on the realization of $x$ in the first period through the conditional expectations of players as discussed later.

There are two marketable assets traded in this model. One is the new security with a one-period payoff of $f(x)$, another is the risk-free security, which is assumed to have a zero risk-free rate with no loss of generality in our analysis. Therefore, in this setting, the underlying process of $x$ is not a tradable price process. This structure of assets is therefore representative of asset-backed financial instruments whose underlying assets are not directly tradable. It departs from the asset structure in Calvert, Gonzales-Eiras, and Sodini (2004) and Cao (2008), which is set to examine the impact of the availability of derivative instruments on the trading in and the price of the underlying security.

Both the innovator and the imitator are risk neutral. The innovator is assumed to make her decisions at time $t = 0$ in order to maximize her expected profits over two periods. The investor is risk averse. For simplicity without much impact on the intuition from our model, we assume that the investor is a myopic mean-variance optimizer and makes her allocation choices at both dates, $t = 0, 1$, by maximizing her expected utility of wealth period by period. This assumption is also used in Bettzüge and Hens (2001) and similar to the CARA-normal setting in other papers, such as Cao (2008) and Easley and O'Hara (2008).[7]

[4] Therefore, the focus here is not on the optimal choice of the form of security, as in the optimal security design literature. Rather we investigate how different forms of security may prevail in the market while allowing the innovator to choose contract parameters.

[5] We do not allow for modification of contract parameters at $t = 1$ for two reasons. First, according to Gale (1992), standardization of security offerings helps mitigate the effect of information uncertainty that can cause coordination failure in the market. Second, we want to focus on the effect of competition from imitation without having the complication of differentiated products and the associated monopolistic competition.

[6] This structure of competition is also used in Rahi and Zigrand (2008).

[7] The rationality on the part of the market participants in our model may mitigate, although not completely rule out because of the heterogeneous belief structure described below, the occurrence of "the dark side of financial innovation" as discussed in Henderson and Pearson (2009).

**The Belief Structure**

We assume that the fundamental distribution for $x$ is normal, i.e., $x = \mu + \eta$, where $\mu$ is an unobservable constant and $\eta$ represents shocks that are independently and identically normally distributed with a zero mean. The variance of $\eta$ is $\sigma_\eta^2$. Each player correctly models the distribution of $\eta$, but has a different estimation of the expected value $\mu$ at time $t = 0$. In other words, different players have different priors of $\mu$, which is believed to be normally distributed as

$$\mu_i \sim \mathcal{N}(\alpha_{i0}, \sigma_{i0}^2), \tag{1}$$

where $i \in \{n, m, v\}$. We assume that $\sigma_{n0} \leq \sigma_{m0} \leq \sigma_{v0}$ to capture differential sophistication of players. In the ensuing analysis, $\mathbb{E}_i[\cdot]$ is the expectation taken over the posterior distribution of $x$ for player $i$. This belief structure captures the heterogeneity in agents' beliefs[8] and is reminiscent of the rational belief structure considered in Garmaise (2001).

At time $t = 1$, everyone observes the realization of $x_1$ and updates their expectation accordingly. The posteriors are then given by, respectively,

$$\mu_i | x_1 \sim \mathcal{N}(\alpha_{i1}, \sigma_{i1}^2), \quad i \in \{n, m, v\} \tag{2}$$

where

$$\alpha_{i1} = \alpha_{i0} + \frac{\sigma_{i0}^2}{\sigma_{i0}^2 + \sigma_\eta^2} (x_1 - \alpha_{i0}), \qquad \sigma_{i1}^2 = \frac{\sigma_{i0}^2 \sigma_\eta^2}{\sigma_{i0}^2 + \sigma_\eta^2}. \tag{3}$$

Therefore, the expectation of player $i$ of $x$ conditional on observing $x_1$ is based on the posterior distribution given by equation (2).

**The Cost Structure**

We consider costs of issuing and marketing the security by both the innovator and the imitator. For the innovator, there is an initial fixed cost of developing the product incurred at time $t = 0$, which we denote as $D$. In addition, issuing $N$ units of the security incurs a cost of $C_i(N)$, for $i \in \{n, m\}$. We assume that both cost functions $C_n(\cdot)$ and $C_m(\cdot)$ are increasing

[8] This structure of heterogeneous beliefs is also used in Easley and O'Hara (2008), and its possible origins are discussed in Scheinkman and Xiong (2004).

and (strictly) convex.[9] We also allow that the per-unit cost $C_i(N)/N$ is decreasing with respect to the number of unit $N$, consistent with the economy of scale in issuing securities.

For simplicity and tractability in the subsequent analysis, we specialize the cost function to a quadratic form, i.e., $C_i(N) = \tau_i N^2 + \beta_i N + \gamma_i$, where $\tau_i, \beta_i, \gamma_i > 0$ for each $i \in \{n, m\}$. The quadratic cost structure satisfies the above assumptions when the volume, $N$, is bounded such that $\tau_i N^2 \leq \gamma_i$.

## 2.1 Characterization of the Equilibrium

We derive a Bayesian-Nash equilibrium in this model. At time $t = 0$, the innovator determines the payoff structure $f(x)$ of the new security and its price $p$ for the first period, taking into account the demand schedule from the investor that maximizes her expected utility in the first period as well as the potential competition from an imitator in the second period. At time $t = 1$, there are potentially two suppliers of the security. The investor updates her demand schedule to maximize her expected utility in the second period. The innovator and the imitator compete in a Cournot game in determining their supply schedules, conditional on their belief structure at time $t = 1$. The equilibrium price at time $t = 1$ is determined by the market clearing condition.

**Financial Equilibrium at Time $t = 1$**

At $t = 1$, the investor's allocation problem is

$$\max_{\Phi_1(p_1)} \left\{ \mathbb{E}_v \left[ W_{v2} | x_1 \right] - \frac{1}{2} \theta Var_v \left[ W_{v2} | x_1 \right] \right\}, \tag{4}$$

where $W_{v2} = \Phi_1(p_1) f(x) + W_{v1} - p_1 \Phi_1(p_1)$ is the investor's wealth at $t = 2$, $W_{v1}$ is her wealth at $t = 1$, $\Phi_1(p_1)$ is the unit of the new security demanded by the investor, and $p_1$ is the security price at time $t = 1$. $\theta$ is the absolute risk aversion of the investor. The optimal demand schedule for the investor at time $t = 1$ is then

$$\Phi_1(p_1) = \frac{(\mathbb{E}_v[f(x)|x_1] - p_1)^+}{\theta Var_v[f(x)|x_1]}, \tag{5}$$

[9]As shown in Froot, Scharfstein, and Stein (1993), a convex cost structure follows from a costly reverification model of Townsend (1979).

where the positivity restriction $()^+$ reflects the no-short-sale constraint on the new security. To simplify notations, we denote $\mathcal{V}[x_1] = Var_v[f(x)|x_1]$. For the security to be viable, i.e., $\Phi_1(p_1) > 0$, it is necessary that

$$p_1 < \mathbb{E}_v[f(x)|x_1].$$

The problems faced by the innovator and the imitator are similar:

$$\max_{\Phi_i} \mathbb{E}_i[\Phi_i(p_1)(p_1 - f(x)) - C_i(\Phi_i)|x_1], i \in \{n, m\}. \tag{6}$$

The financial equilibrium exists if and only if the market clears at an equilibrium price $p_1$, i.e.,

$$\Phi_n(p_1) + \Phi_m(p_1) = \Phi_1(p_1). \tag{7}$$

Hence, we have $p_1 = \Phi_1^{-1}(\Phi_n + \Phi_m) = G(\Phi_n + \Phi_m)$, whereas from (5) and (7),

$$G(u) = \mathbb{E}_v[f(x)|x_1] - \theta\mathcal{V}[x_1]u. \tag{8}$$

We model the competition between the innovator and the imitator through a Cournot equilibrium. Therefore, the first-order necessary conditions of their problems (6) with respect to $\Phi_n$ and $\Phi_m$ are, respectively,

$$\begin{aligned} p_1 - \mathbb{E}_n[f(x)|x_1] - C_n'(\Phi_n) + \Phi_n G'(\Phi_n + \Phi_m) &= 0, & (9) \\ p_1 - \mathbb{E}_m[f(x)|x_1] - C_m'(\Phi_m) + \Phi_m G'(\Phi_n + \Phi_m) &= 0, & (10) \end{aligned}$$

which jointly determine $\Phi_n$ and $\Phi_m$, given the existence of such solutions.

In fact, these solutions are given by

$$\Phi_n = \frac{\phi_n}{\Delta}, \qquad \Phi_m = \frac{\phi_m}{\Delta} \tag{11}$$

where

$$\begin{aligned} \phi_n &= 2(\tau_m + \theta\mathcal{V}[x_1])(\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n) \\ &\quad -\theta\mathcal{V}[x_1](\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m), \end{aligned}$$

$$\begin{aligned}\phi_m &= 2(\tau_n + \theta\mathcal{V}[x_1])(\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m) \\ &\quad -\theta\mathcal{V}[x_1](\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n),\end{aligned}$$

and

$$\Delta = 4(\tau_n + \theta\mathcal{V}[x_1])(\tau_m + \theta\mathcal{V}[x_1]) - (\theta\mathcal{V}[x_1])^2.$$

Therefore, the financial equilibrium at time $t = 1$ can be characterized by the following proposition:

**Proposition 1** *The financial equilibrium at time $t = 1$ depends on the realization of $x_1$ as follows.*

*(1) If $x_1 \in \{\Phi_n > 0, \Phi_m > 0\}$, then the optimal supply from the innovator and the imitator is given by equation (11), the optimal demand is determined by equation (5), and the equilibrium price $p_1$ is given by:*

$$\begin{aligned}p_1 := p[x_1] &= \frac{\theta\mathcal{V}[x_1]}{\Delta}(2\tau_m + \theta\mathcal{V}[x_1])(\mathbb{E}_n[f(x)|x_1] + \beta_n) \\ &\quad + \frac{\theta\mathcal{V}[x_1]}{\Delta}(2\tau_n + \theta\mathcal{V}[x_1])(\mathbb{E}_m[f(x)|x_1] + \beta_m) \\ &\quad + \left\{1 - \frac{\theta\mathcal{V}[x_1]}{\Delta}(2\tau_m + 2\tau_n + 2\theta\mathcal{V}[x_1])\right\}\mathbb{E}_v[f(x)|x_1].\end{aligned} \tag{12}$$

*(2) If $x_1 \in \{\Phi_n \leq 0, \Phi_m > 0\}$, then the innovator does not issue the security at $t = 1$, but the imitator does. The equilibrium price $p_1$ is given by*

$$p_1 := p_m[x_1] = q_m\mathbb{E}_v[f(x)|x_1] + (1 - q_m)(\mathbb{E}_m[f(x)|x_1] + \beta_m), \tag{13}$$

*where*

$$q_m = \frac{2\tau_m + \theta\mathcal{V}[x_1]}{2\tau_m + 2\theta\mathcal{V}[x_1]]}.$$

*The optimal demand is then*

$$\Phi_1 = \frac{\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m}{2\tau_m + 2\theta\mathcal{V}[x_1]}. \tag{14}$$

*(3) If $x_1 \in \{\Phi_n > 0, \Phi_m \leq 0\}$, then only the innovator issues the security at $t = 1$. The equilibrium price $p_1$ is given by*

$$p_1 := p_n[x_1] = q_n \mathbb{E}_v[f(x)|x_1] + (1 - q_n)(\mathbb{E}_n[f(x)|x_1] + \beta_n), \tag{15}$$

*where*

$$q_n = \frac{2\tau_n + \theta \mathcal{V}[x_1]}{2\tau_n + 2\theta \mathcal{V}[x_1]}.$$

*The optimal demand is then*

$$\Phi_1(p_1) = \frac{\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n}{2\tau_n + 2\theta \mathcal{V}[x_1]}. \tag{16}$$

*(4) In other scenarios of $x_1$, with $\Phi_n \leq 0, \Phi_m \leq 0$, there is no trade in the security at $t = 1$.*

Note that $\Phi_m \leq 0$ if and only if $\mathbb{E}_m[f(x)|x_1] + \beta_m \geq p_1$. This indicates that $\mathbb{E}_m[f(x)|x_1] + \beta_m$ is the reservation price for the imitator to participate in the market. When the market price is below that reservation price, the imitator stays out of the market. In this case, for the market to open we need to have $\mathbb{E}_v[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1] + \beta_n$. Incidentally, this equilibrium is also the one in the absence of any imitators.

Similarly, the reservation price for the innovator to stay in the market is $\mathbb{E}_n[f(x)|x_1] + \beta_n$. When the market price is smaller than this reservation price, the innovator will exit the market at time $t = 1$. In this case, we need to have $\mathbb{E}_m[f(x)|x_1] + \beta_m < \mathbb{E}_v[f(x)|x_1]$ for the market for the security to clear at the price $p_n[x_1]$.[10] Finally, if the market is above the reservation prices of both innovator and imitator, then both will issue the security in a competitive environment, and the market price is thus obtained in the first case of Proposition 1.

When the innovator and the imitator share the same belief ($\mathbb{E}[f(x)|x_1]$) and proportional issuing cost ($\beta$), at $t = 1$, there are only two separate scenarios: either both issue the security or they all stay out of the market. The necessary and sufficient condition for the issuance is

[10] Bhattacharya and Spiegel (1991) and Bhattacharya, Reny, and Spiegel (1995) characterize conditions for a market breakdown in the presence of asymmetric information between insiders and outsiders. In addition, Morris (1994) explores the structure of a no-trade theorem with heterogeneous prior beliefs and presents sufficient and necessary conditions on agents' beliefs for trading to take place.

$\mathbb{E}_n[f(x)|x_1] + \beta_n < \mathbb{E}_v[f(x)|x_1]$. When this condition fails to hold under some realization of $x_1$, the market breaks down with no trade. With heterogeneous beliefs, the condition for an active market is more complicated, and we will examine in Section 3 the issue of a possible breakdown in the market for the new security with different payoff structures of $f(x)$.

**Financial Equilibrium at Time $t = 0$**

At $t = 0$, the investor's allocation problem is

$$\max_{\Phi_0(p_0)} \left\{ \mathbb{E}_v \left[ \Phi_0(p_0) f(x) + (W_{v0} - p_0 \Phi_0(p_0)) \right] - \frac{1}{2} \theta Var_v \left[ \Phi_0(p_0) f(x) + (W_{v0} - p_0 \Phi(p_0)) \right] \right\}. \tag{17}$$

where $W_{v0}$ is the initial wealth of the investor, and the investor's demand in the security is $\Phi_0(p_0)$ with $p_0$ being the security price at $t = 0$. Because the contract parameters of $f(x)$ are determined at this time, we also write $\Phi_0(p_0) \equiv \Phi_0(p_0, f(x))$. Hence, the optimal demand schedule for the investor at time $t = 0$ is

$$\Phi_0(p_0, f(x)) = \frac{(\mathbb{E}_v[f(x)] - p_0)^+}{\theta Var_v[f(x)]}, \tag{18}$$

with the following feasibility condition on the equilibrium price, $p_0$:

$$p_0 < \mathbb{E}_v[f(x)]. \tag{19}$$

At $t = 0$, only the innovator offers the new security for sale who also determines its structural parameters $\{a\}$ in the payoff function $f(\{a\}; x)$ and its initial pricing $p_0$. The problem solved by the innovator is

$$\max_{\{p_0, \{a\}\}} \mathbb{E}_n \left[ \Phi_0(p_0, f(\{a\}; x_1))(p_0 - f(\{a\}; x_1)) - D - C_0(\Phi_0(p_0, f(\{a\}; x_1))) \right]$$
$$+ \frac{1}{1+\rho} \mathbb{E}_n \left[ \Phi_n(p_1)(p_1 - f(\{a\}; x_2)) - C_n(\Phi_n(p_1)) \right],$$

where $D$ is the innovation cost, $C_0(\Phi_0)$ is the issuing cost at time $t = 0$, and $p_1$ is the price at $t = 1$ determined earlier. $x_1$ and $x_2$ are the realizations of $x$ at $t = 1$ and $t = 2$, respectively. In addition, $\rho$ is an intertemporal discount factor for the innovator.

The following proposition characterizes the equilibrium price and volume of the security issued at $t = 0$.

**Proposition 2** *The financial equilibrium at time $t = 0$ is characterized by the equilibrium price*

$$p_0 = \frac{2\tau_n + \theta Var_v[f(x)]}{2(\tau_n + \theta Var_v[f(x)])}\mathbb{E}_v[f(x)] + \frac{\theta Var_v[f(x)]}{2(\tau_n + \theta Var_v[f(x)])}(\mathbb{E}_n[f(x)] + \beta_n), \tag{20}$$

*and the volume*

$$\Phi_0(p_1) = \frac{1}{2(\tau_n + \theta Var_v[f(x)])}\left\{\mathbb{E}_v[f(x)] - (\mathbb{E}_n[f(x)] + \beta_n)\right\} \tag{21}$$

*of the security in the market.*

## 2.2 Analysis of the Model

In this subsection, we analyze the impact of competition and heterogeneous beliefs among sellers of the security on the equilibrium price, total volume and market shares. For simplicity, we assume that both the innovator and the imitator have the same cost structures, i.e., $\tau_n = \tau_m = \tau$, and $\beta_n = \beta_m = \beta$.

### A Benchmark Model

We start with a benchmark model with homogeneous beliefs in order to establish a base case for the relative advantage between the innovator and the imitator in market shares and profitability, as well as for the level of equilibrium price. In the benchmark model, we assume that the imitator has the same expectation as the innovator, i.e, $\alpha_{n0} = \alpha_{m0}$, and their priors have the same precision, too, i.e., $\sigma_{n0} = \sigma_{m0}$.

In this benchmark model, the price in the first period, $p_0$, is determined in the same way as in Proposition 2. In the second period, because both the imitator and the innovator have the same cost and belief structures, each will supply half of the total demand in equilibrium. By Proposition 1, the price in the second period is then

$$p_1^b = \frac{2\theta\mathcal{V}[x_1]}{2\tau + 3\theta\mathcal{V}[x_1]}(\mathbb{E}_n[f(x)|x_1] + \beta) + \frac{2\tau + \theta\mathcal{V}[x_1]}{2\tau + 3\theta\mathcal{V}[x_1]}\mathbb{E}_v[f(x)|x_1]. \tag{22}$$

Therefore, in the benchmark model, there is no advantage in market shares and profits in the second period for either seller, but the innovator will reap the first-mover advantage in the first period.

If there is no imitator, then the innovator will be a monopolist in both periods. The price and volume of the security in the second period are depicted in Case 3 of Proposition 1. The following corollary summarizes the effect of the presence of the imitator on the price and volume of the security and the innovator's expected profit in the second period.

**Corollary 1** *1. The presence of the imitator lowers the security price.*

2. *The total volume of the security is higher in the presence of the imitator. The volume issued by the innovator declines with the entry of the imitator.*

3. *The expected profit of the innovator is reduced in the presence of an imitator.*

**Effects of Belief Heterogeneity**

When the innovator and the imitator have different beliefs, their supplies of the security will differ, and the price of the security will be affected as well. The following corollary describes the deviation of the security price, $p_1$, from $p_1^b$, the price in the benchmark case of homogeneous beliefs.

**Corollary 2** *Holding the innovator's belief, $\mathbb{E}_n[f(x)|x_1]$, the same as in the benchmark case, and allowing the imitator's belief, $\mathbb{E}_m[f(x)|x_1]$, to differ, the deviation of the security price, $p_1$, from $p_1^b$ in (22) is*

$$p_1 - p_1^b = \frac{\theta \mathcal{V}[x_1]}{2\tau + 3\theta \mathcal{V}[x_1]} (\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]). \tag{23}$$

This result implies that the security price is higher than the benchmark price if the imitator has a higher valuation of the security payoff than the innovator, although the price increase is only a fraction of the difference in their valuations. The higher price will motivate the innovator to sell more of the security than the benchmark case, holding her expectation constant, while reducing the demand from investors. Therefore, the difference in beliefs will affect the relative advantage of the sellers, as demonstrated in the following corollary.

**Corollary 3** *Holding the innovator's belief,* $\mathbb{E}_n[f(x)|x_1]$*, the same as in the benchmark case, and allowing the imitator's belief,* $\mathbb{E}_m[f(x)|x_1]$*, to differ.*

*(1) If* $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$*, then compared to their counterparts in the benchmark model of homogeneous beliefs, (i) the volume of the security issued by the innovator is higher; (ii) the total volume of the security issued is lower; and (iii) the innovator's expected profit is higher.*

*(2) If* $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$*, then compared to their counterparts in the benchmark model of homogeneous beliefs, (i) the volume of the security issued by the innovator is lower; (ii) the total volume of the security issued is higher. However, the shift in the innovator's expected profit is ambiguous.*

Corollary 3 indicates that the divergence of beliefs between the innovator and the imitator impacts the amount of security issuance both by the innovator and in aggregate. It also affects the profits made by the innovator depending on the direction of deviation of the imitator's belief from that of the innovator. The following corollary characterizes the relative advantage between the innovator and the imitator in the issuing volume and profits due to the belief heterogeneity.

**Corollary 4** *Holding the innovator's belief,* $\mathbb{E}_n[f(x)|x_1]$*, the same as in the benchmark case, and allowing the imitator's belief,* $\mathbb{E}_m[f(x)|x_1]$*, to differ.*

*(1) If* $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$*, then the innovator issues a larger amount of the security and expects a higher profit than the imitator.*

*(2) If* $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$*, then the innovator issues a smaller amount of the security and expects a lower profit than the imitator.*

As we have seen earlier, the price is higher if the imitator is relatively more optimistic about the future payoff from the security. This benefits the innovator as her expected profit increases because she sells more of the security at a higher price. The opposite situation holds if the imitator is relatively more pessimistic about the security payoff. Therefore, although the entry of an imitator does introduce competition that reduces the innovator's profits from the monopolistic level, the potentially better-informed belief the innovator has can still confer the first-mover advantage, even before considering the first-period profits and other non-pecuniary benefits the innovator may earn, as discussed in Tufano (1989).

# 3 The Viability of Financial Innovation

By the frailty of financial innovation, we refer to the likelihood of no trade in a new derivative security, which represents a breakdown in the market for the security. In order to investigate this issue more closely, we consider several specific examples of derivative securities, which are building blocks of most financial innovations. The first one is a forward contract with a linear payoff structure $f(x) = ax$, where $a$ is a positive percentage and $x$ is the underlying variable. The second one is a standard call option with a payoff $\max\{x - L, 0\}$. In addition, we consider a capped forward with a payoff $\min\{x, K\}$, which is equivalent to a bond position and a short position in a put option, and a spread with a payoff $\max\{x-K, 0\}-\max\{x-L, 0\}$, which is a combination of longing a call option and writing a put option.

We study how competition and the market realization of $x_1$ at time one affect the market for these securities and identify conditions that lead to a market breakdown. In particular, we examine the effect of heterogenous beliefs on the viability of these securities. We then discuss the implications of our analysis for one prototypical structure of collateralized debt obligations (CDO), which may help shed light on the recent credit crisis.

For simplicity we assume that both the innovator and the imitator have the same expectation of the unobservable mean $\mu$ but their precisions of such expectations are different. Specifically, $\alpha_{n0} = \alpha_{m0}$, and $\sigma_{n0} < \sigma_{m0} < \sigma_{v0}$. The heterogeneity in beliefs among the innovator and the imitator is thus represented by the divergence between $\sigma_{m0}$ and $\sigma_{n0}$. In addition, we assume that the issuing cost is the same for both the innovator and the imitator.

## 3.1 Forward-type Contracts

We first consider a linear payoff structure $f(x) = ax$ where $a$ is a positive percentage parameter. This is similar to those pass-through securities of asset-backed pools. It is easy to see that $\mathbb{E}_i[f(x)] = a\alpha_{i0}$. We assume $a(\alpha_{v0} - \alpha_{n0}) > \beta$ such that the innovator is willing to issue the contract at $t = 0$.

We focus on the market in the second time period where competition ensues and belief heterogeneity matters. The following proposition establishes the participation boundaries for both innovator and imitator.

**Proposition 3** *Suppose the new security is a forward contract with payoff $f(x) = ax, a > 0$. Denote*

$$q = \frac{2\tau + \theta\mathcal{V}[x_1]}{2\tau + 2\theta\mathcal{V}[x_1]},\ g(x) = \frac{x^2}{x^2 + \sigma_\eta^2}, \tag{24}$$

*where*

$$\mathcal{V}[x_1] \equiv Var_v[f(x)|x_1] = a^2\sigma_\eta^2(1+\sigma^2),\ \text{and } \sigma^2 = \frac{\sigma_{v0}^2}{\sigma_{v0}^2 + \sigma_\eta^2}. \tag{25}$$

1. *The innovator will continue to issue the forward contract at $t = 1$ if and only if $x_1 > A(\sigma_{m0})$, where*

$$A(\sigma_{m0}) = \frac{qg(\sigma_{v0})\alpha_{v0} + (1-q)g(\sigma_{m0})\alpha_{n0} - g(\sigma_{n0})\alpha_{n0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a}{qg(\sigma_{v0}) + (1-q)g(\sigma_{m0}) - g(\sigma_{n0})}. \tag{26}$$

2. *If $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) > g(\sigma_{m0})$, then the imitator will enter the market and issue the forward contract to the investor, if $x_1 > B(\sigma_{m0})$, where*

$$B(\sigma_{m0}) = \frac{qg(\sigma_{v0})\alpha_{v0} + (1-q)g(\sigma_{n0})\alpha_{n0} - g(\sigma_{m0})\alpha_{n0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a}{qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) - g(\sigma_{m0})}. \tag{27}$$

   *If $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) < g(\sigma_{m0})$, then the imitator will enter the market and issue the forward contract to the investor, only if $x_1 < B(\sigma_{m0})$.*

We call the quantities, $A(\sigma_{m0})$ and $B(\sigma_{m0})$, *participation boundaries,* which are similar to the notion used in Person and Warther (1997) in their analysis of the boom and bust pattern in the adoption of financial innovation. Proposition 3 indicates that the innovator will continue to issue the forward contract at $t = 1$ if and only if the underlying variable for the forward contract is booming in the market, i.e., $x_1 > A(\sigma_{m0})$, whether the imitator enters the market or not. The innovator will stop issuing the contract at $t = 1$ if $x_1 \leq A(\sigma_{m0})$.

The participation decision of the imitator significantly depends on the precision of her prior, $\sigma_{m0}$. If the imitator's precision is close to that of the innovator, in the sense that

$$g(\sigma_{m0}) < qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}), \tag{28}$$

then the imitator enters the market if and only if $x_1 > B(\sigma_{m0})$. But when the imitator's uncertainty ($\sigma_{m0}$) is far greater than that of the innovator ($\sigma_{n0}$) such that

$$g(\sigma_{m0}) > qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}), \tag{29}$$

then the imitator's entry decision actually negatively depends on the realized value of $x_1$. In this case, the imitator issues the forward only when $x_1 \leq B(\sigma_{m0})$.

When the innovator and the imitator have the same belief, then their participation decisions are exactly the same, as described in the following corollary.

**Corollary 5** *When the innovator and the imitator share the same belief, i.e., $\sigma_{n0} = \sigma_{m0}$, their participation boundaries coincide, i.e.,*

$$A(\sigma_{n0}) = B(\sigma_{n0}) = \frac{g(\sigma_{v0})\alpha_{v0} - g(\sigma_{n0})\alpha_{n0} - (\alpha_{v0} - \alpha_{n0}) + \beta/a}{g(\sigma_{v0}) - g(\sigma_{n0})}, \tag{30}$$

*and they will issue the security together if and only if $x > A(\sigma_{n0})$. When $x \leq A(\sigma_{n0})$, the market for the security breaks down.*

From this corollary, it is also clear that when the sellers share the same belief, investors have to have a more diffused belief, i.e., $\sigma_{v0} > \sigma_{n0}$, in order for the market to stay open.

Given the benchmark established in Corollary 5, Proposition 3 helps us characterize the impact of heterogeneous beliefs among the sellers of the security. First, assume that $a$ satisfies $a(\alpha_{v0} - \alpha_{n0})(1 - g(\sigma_{v0})) > \beta$. If the precisions of all players' expectations line up such that $g(\sigma_{m0}) < qg(\sigma_{v0}) + (1-q)g(\sigma_{n0})$, then $A(\sigma_{m0}) > B(\sigma_{m0})$.[11] Both the innovator and the imitator issue the financial contract to the investor if $x_1 > A(\sigma_{m0})$, but only the imitator issues the security if $B(\sigma_{m0}) < x_1 \leq A(\sigma_{m0})$. The market breaks down, i.e., there is no issuance of the forward contract at $t = 1$, if $x_1 \leq B(\sigma_{m0})$. If the dispersion among precisions is large such that $g(\sigma_{m0}) > qg(\sigma_{v0}) + (1-q)g(\sigma_{n0})$, then the market breaks down if $B(\sigma_{m0}) < x_1 \leq A(\sigma_{m0})$.

[11] Let

$$I(x) := \frac{qg(\sigma_{v0})\alpha_{v0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a + \alpha_{n0}x}{qg(\sigma_{v0}) + x}.$$

Then $I(x)$ is increasing if and only if $a(\alpha_{v0} - \alpha_{n0})\frac{\sigma_\eta^2}{\sigma_{v0}^2 + \sigma_\eta^2} > \beta$. Moreover, $(1-q)g(\sigma_{m0}) - g(\sigma_{n0}) > (1-q)g(\sigma_{n0}) - g(\sigma_{m0}) > -qg(\sigma_{v0})$. Hence, $A(\sigma_{m0}) > B(\sigma_{m0})$. By the same derivation we also see that $A(\sigma_{m0})$ is increasing while $B(\sigma_{m0})$ is decreasing with respect to $\sigma_{m0}$.

Second, if the parameter $a$ satisfies $a(\alpha_{v0} - \alpha_{n0})(1 - g(\sigma_{v0})) \leq \beta$, the result is similar. When the imitator's belief is similar to that of the innovator as indicated in (28), the market breaks down if $x_1 \leq A(\sigma_{m0})$, as neither the innovator nor the imitator issues the security. Remarkably, the market for the forward contract never breaks down when the imitator has a more uncertain belief as implied in (29). Because they have different confidence on the unobservable mean of the underlying $x$, the imitator will enter the market even when the innovator stops issuing the security.

Figure 1 displays the regions for market breakdown as $\sigma_{m0}$ varies. In this figure, we set $a = 0.8$, $\sigma_{n0} = 12\%$ and $\sigma_{v0} = 17.5\%$. $\sigma_{m0}$ ranges between 12% and 17%. The cost function $C(x) = 0.01x^2 + 0.06x + 1$. Moreover, we have $\theta = 0.9$, $\alpha_{n0} = \alpha_{m0} = 1.2$, $\alpha_{v0} = 1.5$ and $\sigma_\eta = 20\%$. Hence $a(\sigma_{v0} - \sigma_{n0})(1 - g(\sigma_{v0})) = 0.1359 > \beta$. As shown, $A(\sigma_{m0})$ is increasing and $B(\sigma_{m0})$ decreasing in $\sigma_{m0}$, and the boundary $A(\sigma_{m0})$ lies above $B(\sigma_{m0})$. In this example, a critical level of $\sigma_{m0}$ is 17.47%. When $\sigma_{m0} < 17.47\%$, the condition (28) is satisfied, and the imitator's participation decision is similar to that of the innovator. In this case, the market breaks down when $x_1 \leq B(\sigma_{m0})$. When $\sigma_{m0} > 17.47\%$, the imitator decides to enter the market only when $x_1 < B(\sigma_{m0})$, and at this level of $x_1$, the innovator will not continue issuing the security in the second time period. Hence, there is no co-existence of the innovator and the imitator in the market in this case. The market breaks down when $x_1$ is between $B(\sigma_{m0})$ and $A(\sigma_{m0})$.

If the parameter $a$ is small enough, say $a \leq 0.353$, then $a(\sigma_{v0} - \sigma_{n0})(1 - g(\sigma_{v0})) < \beta$. In this case, the market never breaks when $\sigma > 17.47\%$ for the specified forward contract. Figure 2 displays the market breakdown region for a forward contract with payoff $f(x) = 0.35x$.

As we discussed before, the innovator and the imitator make their decision to enter into or stay out of the market depending on if the market price is higher than their respective reservation prices as described in Proposition 1. To better illustrate the respective participation of the innovator and the imitator in the market, we plot in Figure 3 the deviation of market price from their respective reservation prices as a function of $x_1$. A positive deviation indicates participation while a negative deviation implies absence from the market.

Panel A of Figure 3 shows that when beliefs are similar, i.e., if $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) > g(\sigma_{m0})$, the innovator and the imitator will both participate in the market in an market, and will both stay out of the market, causing a market breakdown, when the fundamental variable is poor. If beliefs are divergent, however, when $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) < g(\sigma_{m0})$, the innovator and the imitator participate in the market at different time, leading to a resilient market regardless the underlying market condition.

## 3.2 Option-type Contracts

We now consider several examples of option contracts. Let $f_1(x) = \max\{x - L, 0\}, f_2(x) = \min\{x, K\}, f_3(x) = \max\{x - K, 0\} - \max\{x - L, 0\}$ denote the payoffs of these option contracts, respectively. These contracts also are integral components of the collateralized debt obligations (CDO) we will discuss later. We examine the market for each security as a separate example and require

$$\mathbb{E}_n[f_j(x)] + \beta_n \leq \mathbb{E}_v[f_j(x)], j \in \{1, 2, 3\} \tag{31}$$

to ensure that the market for the security originates at time $t = 0$.

#### The Market for Call Option Contracts

We first consider the innovator's decision for the issuance of a call option. The following proposition characterizes the innovator's decision under different market conditions.

**Proposition 4** *If an innovator issues a call option contract at $t = 0$, she will continue issuing the same product at $t = 1$ when the underlying market movement is very positive, i.e., $x_1 >> 0$.*[12] *When the market movement is sufficiently negative, i.e., $x_1 << 0$, she will stop the issuance of the call option contract. For the moderate range of market realization, $x_1$, the innovator's decision at $t = 1$ is not monotonic in $x_1$.*

Proposition 4 says that, with a large movement in the underlying variable, $x_1$, the innovator's decision is simple and robust, independent of the belief heterogeneity and the possibility of entry of an imitator. For instance, since the payoff to a deep in-the-money call option is similar to the payoff of a forward contract when $x_1 >> 0$, the innovator's decision is similar to that in the case of a forward contract. However, the non-linearity of the payoff and belief heterogeneity makes the participation boundary very complex when $x_1$ is in a moderate range, so the innovator decision is not clear-cut.

[12] By $x >> 0$ we mean that there exists a constant $c$ such that $x > c$. It essentially indicates that $x$ is sufficiently large. Similarly, $x << 0$ denotes a sufficiently negative movement in $x$.

**Proposition 5** *When the imitator's belief is such that $\sigma_{m0}$ satisfies (28), she will enter the market for call option contracts if $x_1 >> 0$, but will not when $x_1 << 0$. The entry decision for a medium range of $x_1$ is indeterminate.*

*When the imitator has a far different belief from the innovator, in that $\sigma_{m0}$ satisfies (29), she will not enter the market when either $x_1 << 0$ or $x_1 >> 0$. Again, the entry decision for a medium range of $x_1$ is indeterminate.*

The intuition behind this proposition is straightforward. When the imitator's belief is not too different from the innovator, such that (28) holds, her decision of entry is similar to that for the innovator to continue. For $x_1 >> 0$, both the innovator and the imitator will issue the call option. On the other hand, when $x_1 << 0$, the market breaks down because there will be no seller in this market.

When the innovator and the imitator share the same belief, which in our case means $\sigma_{n0} = \sigma_{m0}$, their participation boundaries coincide, which may be characterized more specifically as described in the following corollary.

**Corollary 6** *When the innovator and the imitator share the same belief, i.e., $\sigma_{n0} = \sigma_{m0}$, there exists a boundary $v(g)$, where $g = g(\sigma_{n0})$, such that both sellers will write the call option contracts when $x_1 \geq v(g)$. The boundary $v(g)$ is determined by the following ordinary differential equation:*

$$\begin{aligned}
&\frac{1}{2\sqrt{1+g}} v'(g) \left\{ N\left(\frac{\alpha - K + g(\sigma_{v0})(v(g)-\alpha)}{\sigma_\eta\sqrt{1+v(g)}}\right) - N\left(\frac{\alpha - K + g(v(g)-\alpha)}{\sigma_\eta\sqrt{1+v(g)}}\right)\right\} \\
&= \frac{v(g)-\alpha}{\sigma_\eta} N\left(\frac{\alpha - K + g(v(g)-\alpha)}{\sigma_\eta\sqrt{1+g}}\right) + \frac{1}{2\sqrt{1+g}} n\left(\frac{\alpha - K + g(v(g)-\alpha)}{\sigma_\eta\sqrt{1+g}}\right).
\end{aligned}$$

When the imitator's belief is far different from the innovator's belief, in the sense that

$$g(\sigma_{m0}) > qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}),$$

the imitator's entry decision is different. The imitator does not enter the market even in a very strong market when $x_1 >> 0$. This is because, with a high level of uncertainty, as represented by a large $\sigma_{m0}$, the imitator has a high estimation of the payoff from the option contract, and hence of her liability, and the market price is not high enough to make it profitable in expectation for the imitator. So she shies away from the market.

Figure 4 illustrates the participation of the innovator and the imitator in the market for call options by plotting deviations of market price from respective reservation prices. Panel A is for the case when beliefs are similar, while Panel B for the case when beliefs are divergent. The figure indicates that when their beliefs are similar, both the innovator and the imitator will issue the security when the market condition is good. When the market condition is poor, as represented by the low or negative value for $x_1$, then the market is no longer viable as both suppliers will stop issuing the security. When beliefs are divergent, the innovator's participation in the market does not change, but the innovator will only be in the market very briefly and for the most of time stay on the sideline.

### The Market for Capped Forward Contracts

The payoff to a capped forward contract may be expressed as $K - \max\{K - x, 0\}$, thus the buyer of the contract acquires a bond position while writing a put option on $x$ with the face value of the bond as the strike price. Following the discussion above about call option contracts, we can characterize the frailty of the market for capped forward market contracts as stated in the next proposition.

**Proposition 6** *With an extreme movement in the underlying variable $x$ at $t = 1$, i.e., $x_1 >> 0$ or $x_1 << 0$, the market for capped forward contracts breaks down with no trade taking place.*

Intuitively, a capped forward is similar to a zero-coupon bond when $x_1 >> 0$. The risk and return profile of the zero-coupon bond is then not attractive enough for the issuers. Moreover, because of its payoff structure, the capped forward is correlated with the call option. By Proposition 4 and 5, the market for the call option breaks down for $x_1 << 0$ when both sellers have similar beliefs, such that (28). But the capped forward market breaks down for $x_1 << 0$ irrespective of the heterogeneity of beliefs.

Figure 5 depicts the participation of the innovator and the imitator in the market for cap forward contracts. It shows that when their beliefs are similar, the innovator and the imitator will issue the security only if the realized market variable $x_1$ is in the neighborhood the strike price. As indicated by the proposition above, when $x_1$ deviates further from the neighborhood in either direction, the market shuts down. However, when the beliefs are divergent, the imitator will start issuing the security when $x_1$ is poor and when the innovator stops issuing the security. In a way, the divergent belief allows the imitator to step in and buy insurance on the market variable $x$ from the investor.

### The Market for Spread Contracts

Finally, we examine the market for spread contracts at $t = 1$. The next proposition characterizes the condition for a breakdown in the market with a large movement in $x_1$.

**Proposition 7** *The market for spread contracts breaks down at $t = 1$ when there is a large shift in the underlying market movement in $x$, that is either $x_1 >> 0$, or $x_1 << 0$.*

This proposition says that with an extreme movement of $x_1$, either $x_1 >> 0$ and $x_1 << 0$, the innovator would stop issuing the contract and the imitator will not enter the market. Note that in the spread market, the qualitative pattern of the viability of the market is not affected by the belief heterogeneity, although the exact boundary, which eludes an analytic characterization, should depend on such belief heterogeneity. The proposition also implies that the spread market is only available for a moderate range of $x_1$ at $t = 1$, which is consistent with the payoff profile of the contract.

The participation boundaries for intermediate ranges of $x_1$ are illustrated in Figure 6. It shows that the viability of the spread option is limited to a range of market variable close to the two strike prices regardless of similarity among beliefs. When beliefs are similar, as shown in the upper panel, both the innovator and the imitator will participate in the market for a time until the market variable moves out of the limited range. When beliefs are dissimilar, the imitator will largely stay out of the market, leaving only the innovator to issue the security.

### Summary

The participation zones for the innovator as well as the imitator in these option markets are summarized in Table 1. The table shows that if both the innovator and the imitator have similar beliefs, their participation decisions are also similar, especially conditional on extreme realizations of $x_1$. When their beliefs diverge, the belief heterogeneity has important and varied effects on their decisions in these markets, especially those for the option-like contracts.

## 3.3 Implications for Asset-Backed Securities

Our analysis above has implications for a complex financial innovation: asset-based securities. In essence, asset-backed securitization is to pool underlying assets and issue a prioritized structure of claims, known as *tranches*, against these collateral pools. A prototype of asset-backed securities in structured finance is collateralized debt obligations (CDOs). There are three prioritized tranches in a typical CDO structure. The tranche with the least priority and bearing the first brunt of losses is called the *equity tranche*, the tranche with the highest priority is the *senior tranche*, and the tranche with an intermediate priority is the *mazzanine tranche*. In the following discussion of the implications of our analysis for the viability of CDO tranches, we abstract from practical institutional intricacies in issuing, monitoring and managing CDO securities.

In its barest form, the payoff to the equity tranche of a CDO structure is represented by a call option, $max\{x-(F-K),0\}$, where $x$ is the value of the underlying collateral pool,[13] $F$ is its face value, and $K$ is the detachment point designating the amount of losses born by the equity tranche. The payoff to the senior tranche is represented by a capped forward contract, $min\{x, F-L\}$, where $L$ is the second detachment point designating the maximum level of losses before the senior tranche will be hit. The payoff to the mezzanine tranche will have a cash flow of $max\{x-(F-L),0\}-max\{x-(F-K),0\}$, similar to that of a spread contract. The detailed correspondence between CDO tranches and option contracts is laid out in the appendix. The combination of the three tranches constitutes a forward contract (with $a=1$) and represents a path-through security on the entire collateral pool.

Because different CDO tranches are usually sold to separate groups of investors with different risk appetite and investment restrictions, it is useful to consider different tranches as being transacted in segmented markets. It is in this sense we derive implications of our analysis for the viability of CDO tranches.

As our analysis indicates, when the market condition is poor, i.e, $x_1 << 0$, the innovator stops selling any of the option-like contracts. Hence, the innovator will not attempt to issue any tranche of the CDO. With this poor market situation, the imitator will not attempt to issue CDO tranches either, unless she has a dispersed prior with a large $\sigma_{m0}$. In that case, the imitator may issue the senior tranche, but she will have to keep the mezzanine and equity

[13] In this simple model, the assumption of a normal distribution for $x$ may admit a negative value. This assumption does not affect the intuition of our discussion, and in all likelihood the probability of a negative $x$ is very small.

tranches after constructing a CDO structure. Therefore, with a poor realization of the value of the collateral pool, $x_1$, the CDO market essentially freezes up with barely any trade.

When the market realization of the underlying collateral value is strong, i.e., $x_1 >> 0$, Proposition 4 implies that the innovator, and as well as the imitator, will keep issuing the equity tranche (the call option), while having to retain the senior tranche as well as the mezzanine tranche at prices investors are willing to pay. Although it is outside of the model itself, it is conceivable that the issuers may be tempted to sell the senior and the mezzanine tranches to investors at lower prices than they would like, hence higher yields, as long as the potential losses can be offset by the gains from selling the equity tranche and from the benefits of not holding any of these tranches in their inventories. This is reminiscent of the situation during the boom of the CDO market.

The discussion above implies that the CDO market is vulnerable to the extreme movement in the underlying asset market. However, even when the movement in the underlying asset market is moderate, the resilience of the CDO market is affected in a complicated way by the non-linear payoff structure with implicit leverage that can shift rapidly with the changing market condition and by the diversity of beliefs among market participants. For instance, as demonstrated in Corollary 6, even when sharing a homogeneous belief, the innovator and the imitator will issue the equity tranche if and only if

$$G(\mu_n(K, x_1), g(\sigma_{n0})) < G(\mu_v(K, x_1), g(\sigma_{v0})) - \frac{\beta}{\sigma_\eta} \tag{32}$$

where

$$G(\mu, \sigma^2) = \mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \sqrt{1+\sigma^2}\, n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$$

and $\mu_i(K, x_1) = \frac{\alpha_{i0} - K + g(\sigma_{i0})(x_1 - \alpha_{i0})}{\sigma_\eta}, i \in \{m, n, v\}$. Meanwhile, the necessary and sufficient condition for both the innovator and the imitator to issue the senior tranche is

$$G(-\mu_n(L, x_1), g(\sigma_{n0})) > G(-\mu_v(L, x_1), g(\sigma_{v0})) + \frac{\beta}{\sigma_\eta}. \tag{33}$$

The nonlinearity of these conditions illustrates that the participation decision of an issuer of the equity tranche and the senior tranche depends on the realization of $x_1$ in a *non-monotonic* way. The heterogeneity in beliefs further increases the complexity of the issuing decision and hence the viability of the security.

For a forward contract or a pass-through security on a collateral pool, the market equilibrium is rather straightforward and robust. Once a critical level of $x_1$ representing a participation boundary is reached, the participation of market players is stable and independent of the level of $x_1$. Therefore, the market for the pass-through security is more resilient and viable. In contrast, for the tranche securities, the market equilibrium sensitively depends on the realization of $x_1$. It is possible that even a small movement of $x_1$ could cause the market to freeze up and break down, and then resume as the underlying asset market condition improves. Moreover, either the innovator or the imitator decides to participate in or exist from the market in a disparate fashion as $x_1$ changes. Therefore, the market for the tranched securities can be rather fickle and vulnerable to the volatility in the underlying asset market.

## 4 Conclusion

We have presented a dynamic equilibrium model to analyze the effect of heterogeneous beliefs and competition among sellers of financial innovation, in a form of a new security, on the security's equilibrium pricing and viability. We show that both volume and price of the new security after it is introduced are sensitive to the differing beliefs of participating players. We identify conditions for the innovator to continue issuing the new security and for the imitator to enter the market, and discuss scenarios when there is a breakdown in the market with no transactions. We also show that with competition from an imitator, the first-mover advantage can be affected by the relative optimism in the imitator's belief about the underlying asset value with respect to that of the innovator.

Our analysis of several specific types of contracts, i.e., forward- and option-like securities that are building blocks of financial innovation, illustrates the viability of some of these contracts under varying market scenarios. For instance, we show that under an extreme and adverse market condition, the market for option-type contracts may be unstable. In general, our analysis implies that the market for forward-type contracts is more resilient to the underlying market movement compared to the market for option-type contracts. Hence, this study sheds some light on the impact of heterogeneous beliefs and competition on the pricing and trading of complex financial securities, such as trenched CDO securities, and may help us further assess the efficacy of these securities for sharing and managing risk.

# Appendix

## A. Proofs

In the following proofs, we denote $\mathcal{E}_i \equiv \mathbb{E}_i[f(x)] + \beta_i$ for $i \in \{m, n, v\}$, with $\beta_v = 0$. Moreover, $\mathcal{V}$ denotes $Var_v[f(x)]$. Similarly, we denote $\mathcal{E}_i[x_1] \equiv \mathbb{E}_i[f(x)|x_1] + \beta_i$ and $\mathcal{V}[x_1] \equiv Var_v[f(x)|x_1]$ at time $t = 1$ conditional on $x = x_1$.

### Proof of Proposition 1

After solving $\Phi_n(p_1)$ and $\Phi_m(p_1)$ in a Cournot equilibrium, and assuming both $\Phi_n(p_1)$ and $\Phi_m(p_1)$ are positive, the equilibrium price $p_1$ follows from equation (5) and (7).

If $\Phi_m \leq 0$, then there is no imitator in the market. In this case, the total demand $\Phi_1(p_1)$ is determined by equation (5), and the total supply is determined by

$$\max_{\Phi} \mathbb{E}_n \left[\Phi(p_1 - f(x)) - C_n(\Phi)|x_1\right],$$

which yields the optimal solution

$$\Phi^* = \frac{p_1 - \mathcal{E}_n[x_1]}{2\alpha_n}.$$

In equilibrium, $\Phi_1(p_1) = \Phi^*$. Then the market price $p_1$ equals to $p_n[x_1]$, and the demand $\Phi_1(p_1)$ follows easily.

Other situations are similar and omitted. □

### Proof of Proposition 2

Given a set of parameters $\{a\}$ in the functional form $f(\{a\}; x)$ of the contract, by Proposition 1, the market price $p_1$ depends on the parameter set $\{a\}$ only. Hence, the market price $p_0 = p_0(a)$ given $\{a\}$ is determined by

$$\max_{p_0} \mathbb{E}_n \left[\Phi_0(p_0, f(\{a\}; x_1))(p_0 - f(\{a\}; x_1)) - D - C_0(\Phi_0(p_0, f(\{a\}; x_1)))\right].$$

The solution to this optimization problem is similar to case (3) of Proposition 1 and leads to expressions in (20) and (21). □

**Proof of Corollary 1**

Let $p_n[x_1], \Phi_n[x_1], \mathbb{E}_n[x_1]$ denote the security price, the market shares and the expected profit of the innovator, when only innovator issues the security at time one, for short. Then

$$p_n[x_1] = \frac{\theta\mathcal{V}[x_1]}{2\tau + 2\theta\mathcal{V}[x_1]}\mathcal{E}_n[x_1] + \frac{2\tau + \theta\mathcal{V}[x_1]}{2\tau + 2\theta\mathcal{V}[x_1]}\mathcal{E}_v[x_1]. \tag{A-1}$$

Therefore, we have

$$p_n[x_1] - p_1^b \quad = \quad \frac{\theta\mathcal{V}[x_1](2\tau + 2\theta\mathcal{V}[x_1])}{(2\tau + 2\theta\mathcal{V}[x_1])(2\tau + 3\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}. \tag{A-2}$$

Since $\mathcal{E}_v[x_1] > \mathcal{E}_n[x_1]$ in the benchmark model, we have $p_1^b < p_n[x_1]$. Moreover, the total volume difference is

$$\Phi_1(p_1^b) - \Phi_n[x_1] = -\frac{1}{\theta\mathcal{V}[x_1]}(p_1^b - p_n[x_1]) > 0. \tag{A-3}$$

As for the market shares of the innovator, by tedious calculation, we have

$$\Phi_n(p_1^b) - \Phi_n[x_1] = -\frac{\theta\mathcal{V}[x_1](2\tau + \theta\mathcal{V}[x_1])}{\Delta(2\tau + 2\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}. \tag{A-4}$$

Hence we have proved that $\Phi_n(p_1^b) < \Phi_n[x_1] < \Phi_1(p_1^b)$.

Let us denote $Z = \Phi_n(p_1^b) - \Phi_n[x_1]$. Hence $Z$ is negative, and $p_1^b - p_n[x_1] = (2\tau + \theta\mathcal{V}[x_1])Z$. Therefore we have

$$\begin{aligned}
\mathbb{E}_n^b - \mathbb{E}_n[x_1] \quad &= \quad Z \times \left\{p_1^b + \Phi_n[x_1](2\tau + \theta\mathcal{V}[x_1]) - \mathcal{E}_n[x_1] - \tau(\Phi_n^b + \Phi_n[x_1])\right\} \\
&= \quad Z\frac{4\tau + 5\theta\mathcal{V}[x_1])}{2(2\tau + 3\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}.
\end{aligned}$$

where $\mathbb{E}_n^b$ is the expected profit of the innovator in the benchmark model. Therefore, $\mathbb{E}_n^b \leq \mathbb{E}_n[x_1]$. □

**Proof of Corollary 2**

It follows from Proposition 1 and equation (22). $\square$

**Proof of Corollary 3**

By Proposition 1 and Proposition 2, the total supply (and demand) volume deviation is

$$\Phi_1(p_1) - \Phi_1^b(p_1) = \frac{\mathbb{E}_n[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1]}{2\tau + 3\theta\mathcal{V}[x_1]}. \tag{A-5}$$

The volume deviation of the securities issued by the innovator is

$$\Phi_n(p_1) - \Phi_n^b(p_1) = \frac{\theta\mathcal{V}[x_1]}{\Delta} \left(\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]\right). \tag{A-6}$$

We use $\mathbb{V}_n$ and $\mathbb{V}_n^b$ to denote the expected profit of the innovator in the heterogeneous belief model and a benchmark model of homogeneous beliefs, respectively. It is straightforward to derive that

$$\begin{aligned} \mathbb{V}_n - \mathbb{V}_n^b \;&=\; \{\Phi_n(p_1) - \Phi_n^b(p_1)\} \times \{p_1 + (2\tau + \theta\mathcal{V}[x_1])\Phi_n^b(p_1) \\ &\quad -\mathcal{E}_n[x_1] - \tau(\Phi_n(p_1) + \Phi_n^b(p_1))\}. \end{aligned} \tag{A-7}$$

Tedious calculation implies that

$$\begin{aligned} M \;&:=\; p_1 + (2\tau + \theta\mathcal{V}[x_1])\Phi_n^b(p_1) - \mathcal{E}_n[x_1] - \tau(\Phi_n(p_1) + \Phi_n^b(p_1)) \\ &=\; \frac{1}{\Delta}\{\theta\mathcal{V}[x_1](\tau + \theta\mathcal{V}[x_1])\mathcal{E}_m[x_1] + 2(\tau + \theta\mathcal{V}[x_1])(2\tau + \theta\mathcal{V}[x_1])\mathcal{E}_v[x_1] \\ &\quad -(\Delta - \tau\theta\mathcal{V}[x_1])\mathcal{E}_n[x_1]\}. \end{aligned}$$

We first show that, in the presence of both innovator and imitator as sellers,

$$\mathcal{E}_n[x_1] + \mathcal{E}_m[x_1] < 2\mathcal{E}_v[x_1]. \tag{A-8}$$

In fact, by Proposition 1, with the presence of both innovator and the imitator, we have $\Phi_n > 0, \Phi_m > 0$. Hence

$$\begin{aligned}\mathcal{E}_m[x_1] &< q\mathcal{E}_v[x_1] + (1-q)\mathcal{E}_n[x_1], \\ \mathcal{E}_n[x_1] &< q\mathcal{E}_v[x_1] + (1-q)\mathcal{E}_m[x_1]. \end{aligned} \tag{A-9}$$

Summing up the last two inequalities, we obtain $\mathcal{E}_m[x_1] + \mathcal{E}_n[x_1] < 2\mathcal{E}_v[x_1]$. Then we have

$$M \geq \frac{2(\tau + \theta\mathcal{V}[x_1])^2}{\Delta}\{\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1]\}. \tag{A-10}$$

If $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$, we have $\mathbb{V}_n - \mathbb{V}_n^b \geq \frac{2\theta\mathcal{V}[x_1](\tau+\theta\mathcal{V}[x_1])^2}{\Delta^2}(\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1])^2$. If $\mathbb{E}_m[f(x)|x_1] = \mathbb{E}_n[f(x)|x_1]$, then $\mathbb{V}_n = \mathbb{V}_n^b$.

If $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$, we obtain $\mathbb{V}_n - \mathbb{V}_n^b \leq \frac{2\theta\mathcal{V}[x_1](\tau+\theta\mathcal{V}[x_1])^2}{\Delta^2}(\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1])^2$. However, $\mathbb{V}_n - \mathbb{V}_n^b$ can be positive or negative, hence the innovator's advantage in the expected profit is ambiguous. □

**Proof of Corollary 4**

First, we have

$$\Phi_n(p_1) - \Phi_m(p_1) = \frac{1}{2\tau + \theta\mathcal{V}[x_1]}\{\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]\}. \tag{A-11}$$

From the proof of Corollary 3, it is easy to see that the difference in expected profits $\mathbb{V}_n - \mathbb{V}_m$ is $\Phi_n(p_1) - \Phi_m(p_1)$ times

$$N := p_1 - \mathcal{E}_m + (\tau + \theta\mathcal{V}[x_1])\Phi_n(p_1) - \tau\Phi_m(p_1).$$

After tedious calculations, we arrive at

$$N = \frac{\tau + 2\theta\mathcal{V}[x_1]}{2\tau + 3\theta\mathcal{V}[x_1]}\{2\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1] - \mathcal{E}_m[x_1]\}. \tag{A-12}$$

Then $N > 0$ by equation (A-8). □

**Proof of Proposition 3**

We first note that for $f(x) = ax, i \in \{m, n, v\}$,

(1) $\mathbb{E}_i[f(x)|x_1] = a\alpha_{i1}, \mathbb{E}_i[f(x)] = a\alpha_{i0}$,

(2) $\mathcal{V}[x_1] = a^2\sigma_{i1}^2, \mathcal{V} = a^2\sigma_{i0}^2$.

Now consider the case when the innovator will continue issue the security. By using the above formula (1) - (2), $\mathcal{E}_n[x_1] \geq p_m[x_1]$ if and only if

$$\alpha_{n1} \geq q\alpha_{v1} + (1-q)\alpha_{m1} - q\beta/a.$$

Note that $g(\cdot)$ is increasing, and $\sigma_{n0} \leq \sigma_{m0} < \sigma_{v0}$. Then $g(\sigma_{n0}) < qg(\sigma_{v0}) + (1-q)g(\sigma_{m0})$. Hence $\mathcal{E}_n[x_1] \geq p_m[x_1]$ holds only if $x_1 \leq A(\sigma_{m0})$. Similarly, $\mathcal{E}_m[x_1] \geq p_n[x_1]$ if and only if

$$\begin{aligned} \alpha_{m0} + g(\sigma_{m0})(x_1 - \alpha_{m0}) \;\; &\geq \;\; q\{\alpha_{v0} + g(\sigma_{v0})(x_1 - \alpha_{v0})\} \\ &\quad +(1-q)\{\alpha_{n0} + g(\sigma_{n0})(x_1 - \alpha_{n0})\} - q\frac{\beta}{a} \end{aligned} \qquad \text{(A-13)}$$

If $\sigma_{m0}$ satisfies formula (28), the above inequality (A-13) holds if and only if $x_1 \leq B(\sigma_{m0})$. Hence the imitator does not entre the market only if $x_1 \leq B(\sigma_{m0})$. Because $A(\sigma_{m0}) > B(\sigma_{m0})$, the first part of this proposition is proved.

If $\sigma_{m0}$ satisfies formula (29), by the same derivation, we see that the imitator does not issue the security if and only if $x_1 \geq B(\sigma_{m0})$. Therefore the second part of this proposition is proved. □

**Lemmas**

To prove our results for option-like contracts, we need the following lemmas.

**Lemma 1** *Given a normally distributed random variable $\zeta$ with mean $\mu$ and variance $\sigma^2$, we have*

*a.* $\mathbb{E}[n(\zeta)] = \frac{1}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$,

*b.* $\mathbb{E}[\zeta n(\zeta)] = \frac{\mu}{(1+\sigma^2)^{3/2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$, *c.* $\mathbb{E}[N(\zeta)] = N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$

*d.* $\mathbb{E}[\zeta N(\zeta)] = \mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \frac{\sigma^2}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$,

*e.* $\mathbb{E}[\zeta^2 N(\zeta)] = (\mu^2+\sigma^2)N(\frac{\mu}{\sqrt{1+\sigma^2}}) + \frac{\mu\sigma^2}{\sqrt{1+\sigma^2}}\frac{2+\sigma^2}{1+\sigma^2} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$.

*f.* $\mathbb{E}[\zeta^2 n(\zeta)] = \frac{\mu^2+\sigma^2(1+\sigma^2)}{(1+\sigma^2)^{3/2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$.

*where $n(\cdot)$ and $N(\cdot)$ is the normal density function and the cumulative normal distribution of a standard normal variable.*

**Proof:** The proof is standard and available upon request. □

**Lemma 2** *Given a normally distributed random variable $\zeta$ with mean $\mu$ and variance $\sigma^2$. Then*

$$\frac{\partial\{\mathbb{E}[\zeta N(\zeta)] + \mathbb{E}[n(\zeta)]\}}{\partial \mu} = N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) \tag{A-14}$$

*and*

$$\frac{\partial\{\mathbb{E}[\zeta N(\zeta)] + \mathbb{E}[n(\zeta)]\}}{\partial \sigma} = \frac{\sigma}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right). \tag{A-15}$$

**Proof:** Lemma 2 follows from Lemma 1 directly. □

The subsequent lemmas present conditional variances of non-linear payoffs from option contracts and their *asymptotic* properties under the extreme market movement. The proofs are omitted here and available upon request.

**Lemma 3** *Assume that $f(x) = (x-L)^+, L > 0$. Then*

$$\mathbb{E}_i[f(x)|x_1] = \sigma_\eta \left\{\mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \sqrt{1+\sigma^2} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)\right\} \tag{A-16}$$

*where $\mu := \frac{\alpha_{i1}-L}{\sigma_\eta}$ and $\sigma^2 := \frac{\sigma_{i0}^2}{\sigma_{i0}^2+\sigma_\eta^2}$. Moreover, $\mathbb{E}_i[f(x)]$ is calculated similarly with $\mu$ and $\sigma$ being replaced by $\frac{\alpha_{i0}-L}{\sigma_\eta}$ and $\sigma_{i0}^2$, respectively. In particular, we have*

$$\lim_{x_1\to\infty} \mathbb{E}_i[f(x)|x_1] = \infty; \lim_{x_1\to-\infty} \mathbb{E}_i[f(x)|x_1] = 0.$$

The next lemma provides a formula for the conditional variance of the payoff from the call option.

**Lemma 4** *Assume that* $f(x) = (x - L)^+, L > 0$. *Then*

$$\mathbb{E}_i[f(x)^2|x_1] = \sigma_\eta^2 \left\{(1 + \mu^2 + \sigma^2)N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \mu\sqrt{1+\sigma^2}n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)\right\} \quad \text{(A-17)}$$

*where* $\mu := \frac{\alpha_{i1}-L}{\sigma_\eta}$ *and* $\sigma^2 := \frac{\sigma_{i0}^2}{\sigma_{i0}^2+\sigma_\eta^2}$. *Moreover,* $\mathbb{E}_i[f(x)^2]$ *is calculated similarly with* $\mu$ *and* $\sigma$ *being replaced by* $\frac{\alpha_{i0}-L}{\sigma_\eta}$ *and* $\sigma_{i0}^2$, *respectively. Hence, the conditional variance of the call option is*

$$\begin{aligned} Var_i[f(x)|x_1] &= \mathbb{E}[f(x)^2|x_1] - \mathbb{E}[f(x)|x_1]^2 \\ &= \sigma_\eta^2\left\{(1+\mu^2+\sigma^2)N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \mu\sqrt{1+\sigma^2}n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)\right\} \\ &\quad -\sigma_\eta^2\left\{\mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \sqrt{1+\sigma^2}n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)\right\}^2. \end{aligned}$$

For the use in the proofs of propositions later, we denote

$$\begin{aligned} f_1(x) &= (x-L)^+; \\ f_2(x) &= K - (K-x)^+; \\ f_3(x) &= (x-K)^+ - (x-L)^+. \end{aligned}$$

and their conditional variances under the belief of the investors are denoted by $Var_v[f_j(x)|x_1]$ with $j \in \{1,2,3\}$. Consequently, we have

$$q_j(x_1) = \frac{2\tau + \theta Var_v[f_j(x_1)]}{2\tau + 2\theta Var_v}, j \in \{1,2,3\}, \quad \text{(A-18)}$$

where $q_j(x_1)$ represents the contribution of the investor to the equilibrium security price when there is only one seller of the security. Note that we assume that the cost structure of the innovator and the imitator are the same, i.e., $\tau_n = \tau_m = \tau$.

The following lemma provides asymptotic properties of conditional expectations and variances and of the weight $q_j(x_1)$ under the extreme market movement.

**Lemma 5** *Asymptotic properties:*

$$\lim_{x_1\to-\infty} Var_v[f_1(x)|x_1] = 0; \qquad \lim_{x_1\to\infty} Var_v[f_1(x)|x_1] = \sigma_\eta^2(1+\sigma^2);$$
$$\lim_{x_1\to-\infty} Var_v[f_2(x)|x_1] = \sigma_\eta^2(1+\sigma^2); \qquad \lim_{x_1\to\infty} Var_v[f_2(x)|x_1] = 0;$$
$$\lim_{x_1\to-\infty} Var_v[f_3(x)|x_1] = 0; \qquad \lim_{x_1\to\infty} Var_v[f_3(x)|x_1] = 0.$$

*where* $\sigma^2 = \frac{\sigma_{v0}^2}{\sigma_{v0}^2+\sigma_\eta^2}$.

$$\lim_{x_1\to-\infty} q_1(x_1) = 1; \qquad \lim_{x_1\to\infty} q_1(x_1) = q_0;$$
$$\lim_{x_1\to-\infty} q_2(x_1) = q_0; \qquad \lim_{x_1\to\infty} q_2(x_1) = 1;$$
$$\lim_{x_1\to-\infty} q_3(x_1) = 1; \qquad \lim_{x_1\to\infty} q_3(x_1) = 1;$$

*where* $q_0 = \frac{2\tau+\theta\sigma_\eta^2(1+\sigma^2)}{2\tau+2\theta\sigma_\eta^2(1+\sigma^2)}$.

For comparison, the conditional variance of the payoff to the forward contract is a constant that is invariant to the market realization of $x_1$, and hence so is the weight $q = q_0$.

Finally, the following lemmas will be used in later proofs.

**Lemma 6** *Let* $F(x) = x\sum_{i=1}^m a_i N(b_i + c_i x), c_i > 0$. *Then*

$$\lim_{x\to-\infty} F(x) = 0, \tag{A-19}$$

*and*

$$\lim_{x\to+\infty} F(x) = \begin{cases} \infty, & \sum_{i=1}^m a_i > 0 \\ -\infty, & \sum_{i=1}^m a_i < 0 \\ 0, & \sum_{i=1}^m a_i = 0 \end{cases} \tag{A-20}$$

**Proof of Proposition 4**

By Proposition 1, the innovator continue the issuance if and only if $\mathcal{E}_n(x_1) < p_m[x_1]$, or equivalently,

$$\begin{aligned} H(x_1) \quad := \quad & q(x_1)G(\mu_v(L, x_1), g(\sigma_{v0})) + (1 - q(x_1))G(\mu_m(L, x_1), g(\sigma_{m0})) \\ & -G(\mu_n(L, x_1), g(\sigma_{n0})) - q(x_1)\frac{\beta}{\sigma_\eta} \end{aligned}$$

is strictly positive, where

$$q(x_1) = \frac{2\tau + \theta Var_v[f(x)|x_1]}{2\tau + 2\theta Var_v[f(x)|x_1]}, f(x) = (x - L)^+.$$

By Lemma 6, and using the fact that $q(x_1)g(\sigma_{v0}) + (1 - q(x_1))g(\sigma_{m0}) > g(\sigma_{n0})$, and $\lim_{x_1 \to \infty} q(x_1) = q_1$, we have

$$\lim_{x_1 \to \infty} H(x_1) = \infty. \tag{A-21}$$

Hence $H(x_1) > 0$ for $x_1 >> 0$. Since $\beta > 0$, then

$$\lim_{x_1 \to -\infty} H(x_1) = -\frac{\beta}{\sigma_\eta} < 0. \tag{A-22}$$

Therefore, $H(x_1) < 0$ for $x_1 << 0$. □

**Proof of Proposition 5**

By Proposition 4, the imitator will enter the market in the second time period if and only if

$$\begin{aligned} L(x_1) \quad := \quad & q_1(x_1)G(\mu_v(L, x_1), g(\sigma_{v0})) + (1 - q_1(x_1))G(\mu_n(L, x_1), g(\sigma_{n0})) \\ & -G(\mu_m(L, x_1), g(\sigma_{m0})) - q_1(x_1)\frac{\beta}{\sigma_\eta} \end{aligned}$$

is strictly positive.

*Case 1.* $x_1 << 0$. Note that $\lim_{x_1 \to -\infty} q_1(x_1) = 1$, and $\lim_{x \to -\infty} L(x_1) = -\frac{\beta}{\sigma_\eta} < 0$, then there exists no imitator when $x_1 << 0$.

*Case 2.* $x_1 >> 0$. Note that $\lim_{x_1\to\infty} q_1(x_1) = q_1 < 1$. Then $L(x_1)$ is close enough to a function

$$
\begin{aligned}
\mathcal{L}(x_1) \quad := \quad & q_1 G(\mu_v(L, x_1), g(\sigma_{v0})) + (1 - q_1)G(\mu_n(L, x_1), g(\sigma_{n0})) \\
& - G(\mu_m(L, x_1), g(\sigma_{m0})) - q_1 \frac{\beta}{\sigma_\eta}
\end{aligned}
$$

when $x_1 >> 0$.

For low volatility such that $g(\sigma_{m0}) < q_1 g(\sigma_{v0}) + (1 - q_1)g(\sigma_{n0})$ , by Lemma 6, we have $\mathcal{L}(x_1) > 0$ for $x_1 >> 0$. Therefore, $L(x_1) > 0$ for $x_1 >> 0$. Hence the imitator will enter the market in the second time period.

For high volatility such that $g(\sigma_{m0}) > q_1 g(\sigma_{v0}) + (1 - q_1)g(\sigma_{n0})$ , , by the same reason we have $\mathcal{L}(x_1) < 0$ for $x_1 >> 0$. Therefore $L(x_1) < 0$ for $x_1 >> 0$. □

**Proof of Proposition 6**

We consider the innovator first. By using the formula

$$
G(\mu, \sigma^2) - G(-\mu, \sigma^2) = \mu,
$$

$\mathbb{E}_i[f_2(x)|x_1]$ can be written as $K - \sigma_\eta G(-\mu_i(K, x_1), g(\sigma_{i0}))$. Therefore, the innovator will continue the issuance in the second time period if and only if

$$
\begin{aligned}
K(x_1) \quad := \quad & G(-\mu_n(K, x_1), g(\sigma_{n0})) - q_2(x_1)\frac{\beta}{\sigma_\eta} \\
& - q_2(x_1)G(-\mu_v(K, x_1), g(\sigma_{v0})) - (1 - q_2(x_1))G(-\mu_m(K, x_1), g(\sigma_{m0}))
\end{aligned}
$$

is positive.

First, $\lim_{x_1\to\infty} q_2(x_1) = \frac{1}{2}$ and $\lim_{x_1\to\infty} K(x_1) = -\frac{1}{2}\frac{\beta}{\sigma_\eta} < 0$, then no issuance from the innovator when $x_1 >> 0$.

Second, $\lim_{x_1\to-\infty} q_2(x_1) = 1$. Therefore, $K(x_1)$ is close to the function

$$
G(-\mu_n(K, x_1), g(\sigma_{n0})) - \frac{\beta}{\sigma_\eta} - G(-\mu_v(K, x_1), g(\sigma_{v0}))
$$

when $x_1 \rightarrow -\infty$. Since $\sigma_{n0} < \sigma_{v0}$, then $g(\sigma_{n0}) < g(\sigma_{v0})$. By Lemma 6, we see that $\lim_{x_1 \rightarrow -\infty} K(x_1) = -\infty$. Hence the innovator will exits from the market too when $x_1 << 0$. The proof for the imitator is similar (because $g(\sigma_{m0}) < g(\sigma_{v0})$ by assumption). □

**Proof of Proposition 7**

Note that

$$\mathbb{E}_i[f_3(x)|x_1] \;=\; \sigma_\eta\{G(\mu_i(K,x_1),g(\sigma_{i0})) - G(\mu_i(L,x_1),g(\sigma_{i0}))\}.$$

Hence, the innovator will continue to issue the mezzanine tranche if and only if

$$\begin{aligned} M(x_1) \;:=\; & q\{G(\mu_v(K,x_1),g(\sigma_{v0})) - G(\mu_v(L,x_1),g(\sigma_{v0}))\} \\ & +(1-q)\{G(\mu_m(K,x_1),g(\sigma_{m0})) - G(\mu_m(L,x_1),g(\sigma_{m0}))\} \\ & -\{G(\mu_n(K,x_1),g(\sigma_{n0})) - G(\mu_n(L,x_1),g(\sigma_{n0}))\} - q\frac{\beta}{\sigma_\eta} \end{aligned}$$

is positive. Clearly $\lim_{x_1 \rightarrow -\infty} M(x_1) = -q\frac{\beta}{\sigma_\eta} < 0$. Then there is no innovator for $x_1 << 0$. By Lemma 4, we see that

$$\lim_{x_1 \rightarrow \infty} M(x_1) = -q\frac{\beta}{\sigma_\eta}. \tag{A-23}$$

Hence there is no innovator neither for $x_1 << 0$. The proof for the imitator is similar. □

## B. Correspondence between CDO Tranches and Option Contracts

Suppose the loss of the collateral pool is $y$ and the face value of the pool is $F$, then the value of the pool is $x = F - y$.

The loss of the equity tranche is $min\{y, K\}$, and its payoff is thus $K - min\{y, K\} = max\{x - (F - K), 0\}$.

The loss of the mezzanine tranche is $max\{y - K, 0\} - max\{y - L, 0\}$, the payoff is then $L - K - max\{y - K, 0\} + max\{y - L, 0\}$, which is $max\{x - (F - L), 0\} - max\{x - (F - K), 0\}$.

The loss of the senior tranche is $max\{y - L, 0\}$, so the payoff is $F - L - max\{y - L, 0\} = min\{x, F - L\}$.

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**Table 1: Participation Zones for Sellers of Financial Innovation**

This table reports zones of $x_1$ for the innovator or the imitator to participate in the market at $t = 1$. "m" denotes a moderate level of $x_1$. We consider following examples of financial innovation: forward, call option, capped forward and spread option.

| Security | Innovator | Imitator | |
|---|---|---|---|
| | | similar beliefs per (28) | dissimilar beliefs per (29) |
| Forward | $x_1 > A(\sigma_{m0})$ | $x_1 > B(\sigma_{m0})$ | $x_1 < B(\sigma_{m0})$ |
| Call Option | $x_1 >> 0$ | $x_1 >> 0$ | m |
| Capped Forward | m | m | m |
| Spread Option | m | m | m |

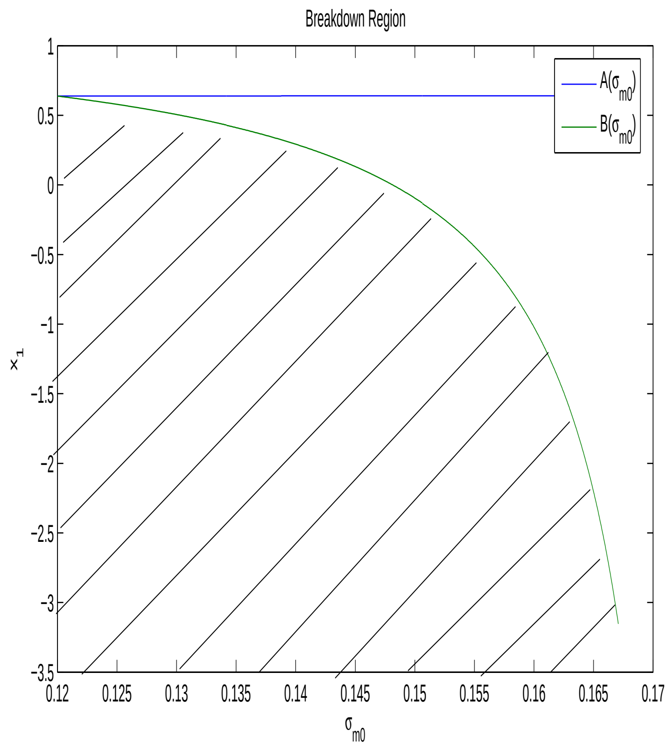


**Figure 1: This figure displays the boundaries for the breakdown of the market for a forward contract with $f(x) = 0.8x$, when $\sigma_{m0}$ changes. In this graph, $\sigma_{n0} = 12\%, \sigma_{v0} = 17.5\%$. Other parameters are: $\alpha_{n0} = 1.2, \alpha_{v0} = 1.5, \theta = 0.9, C(x) = 0.01x^2 + 0.06x + 1$. The shaded area represents the region for market breakdown.**

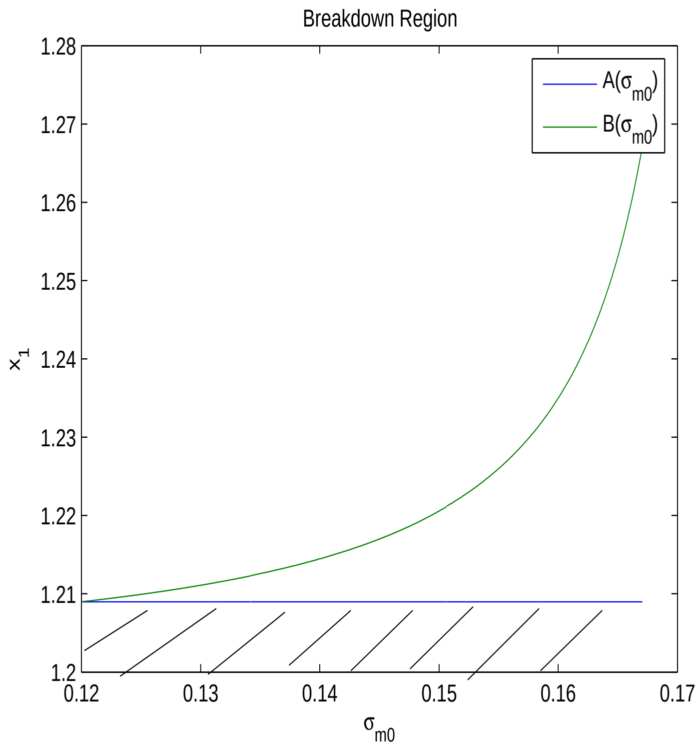


**Figure 2: This figure displays the boundaries for the breakdown of the market for a forward contract with** $f(x) = 0.35x$, **when** $\sigma_{m0}$ **changes. In this graph,** $\sigma_{n0} = 12\%, \sigma_{v0} = 17.5\%$. **Other parameters are:** $\alpha_{n0} = 1.2, \alpha_{v0} = 1.5, \theta = 0.9, C(x) = 0.01x^2 + 0.06x + 1$. **The shaded area represents the region for market breakdown.**

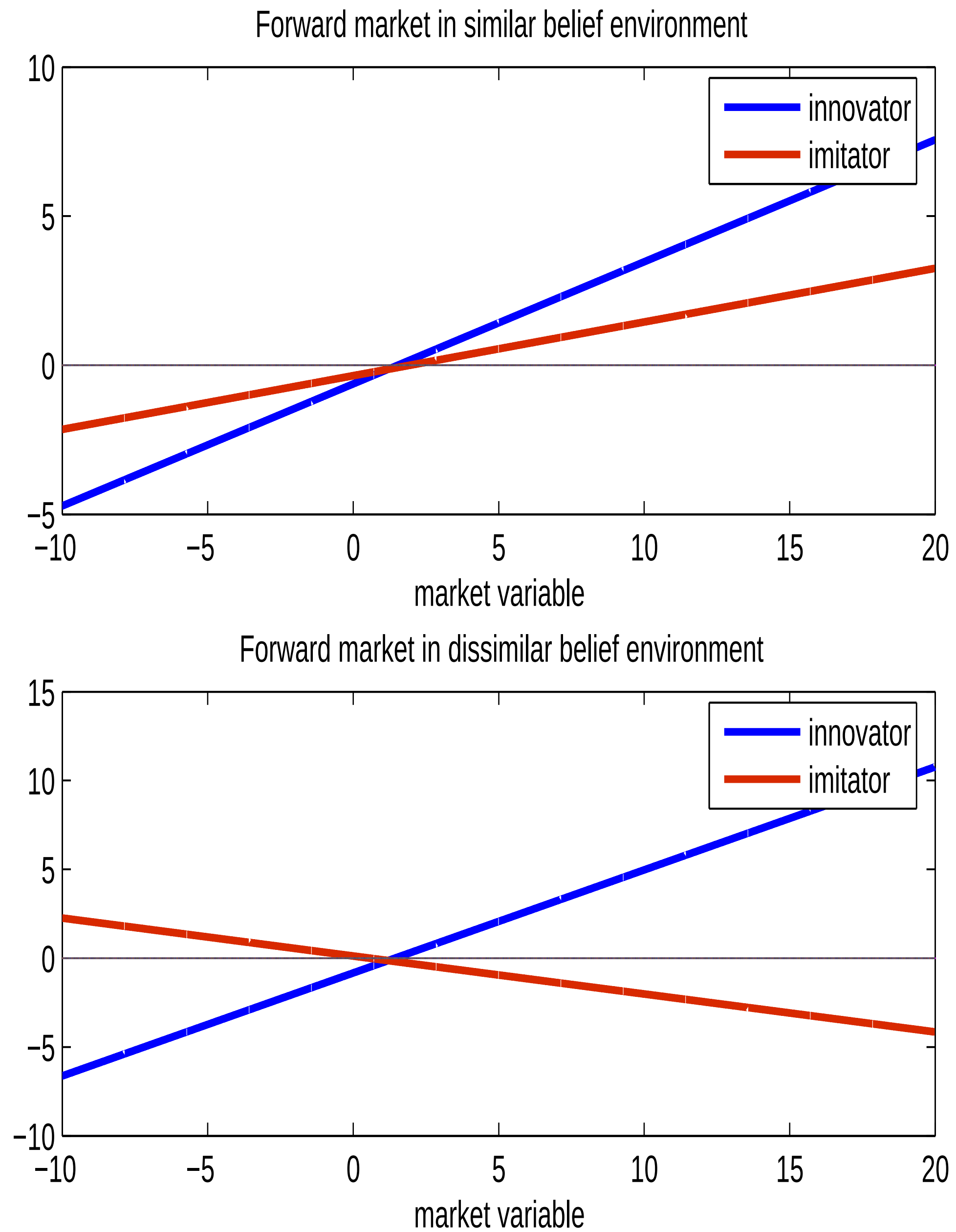


**Figure 3: This figure displays the pattern of deviations of market price of the pass-through security from reservation prices for the innovator and the imitator. Panel A illustrates the case when beliefs are similar, i.e., if** $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) > g(\sigma_{m0})$**. Panel B depicts the case when beliefs are divergent, i.e., if** $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) < g(\sigma_{m0})$**.**

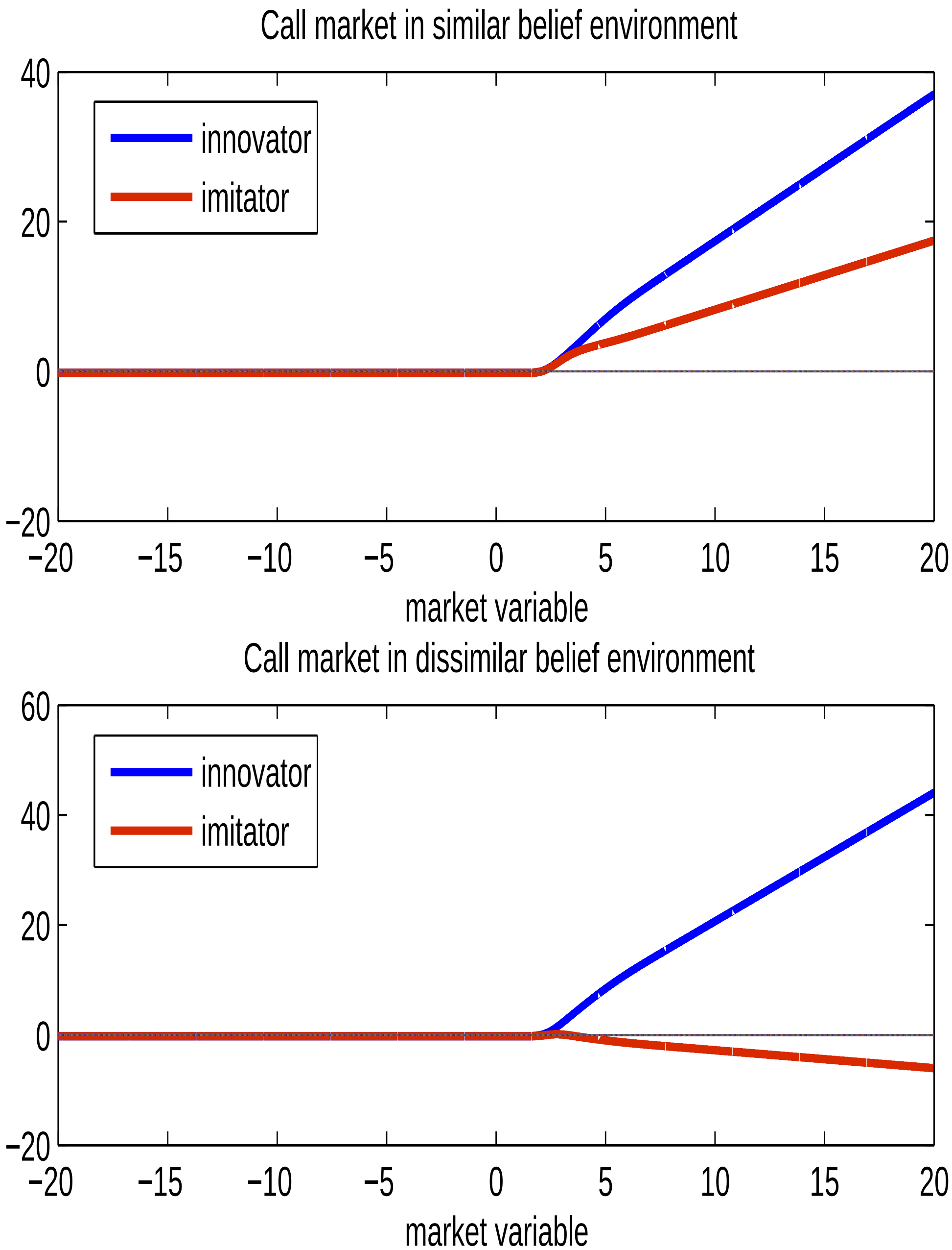


**Figure 4: This figure displays the pattern of deviations of market price from reservation prices of a call option for the innovator and the imitator. Panel A illustrates the case when beliefs are similar, i.e., if $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) > g(\sigma_{m0})$. Panel B depicts the case when beliefs are divergent, i.e., if $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) < g(\sigma_{m0})$.**

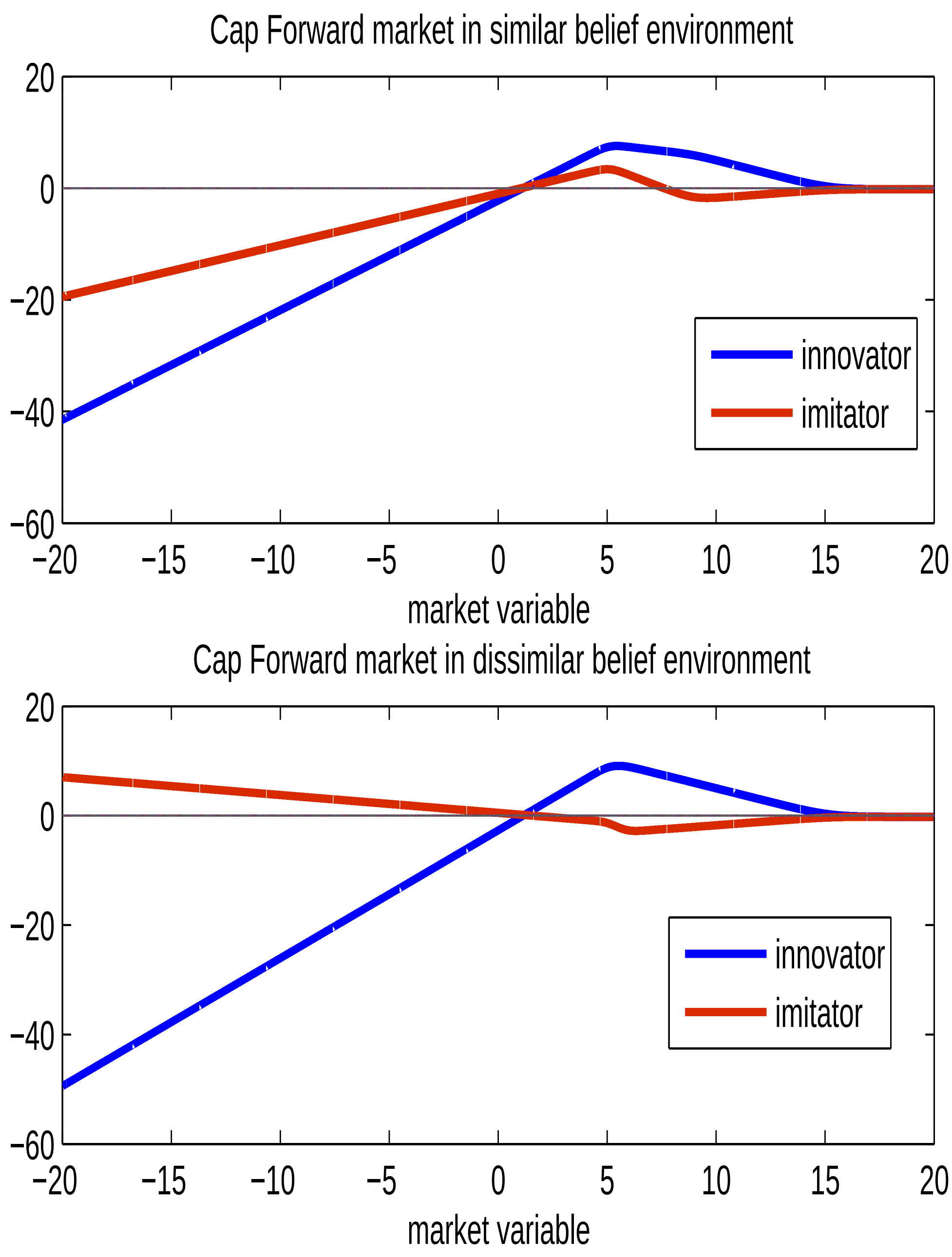


**Figure 5: This figure displays the pattern of deviations of market price from reservation prices of a cap forward contract for the innovator and the imitator. Panel A illustrates the case when beliefs are similar, i.e., if** $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) > g(\sigma_{m0})$**. Panel B depicts the case when beliefs are divergent, i.e., if** $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) < g(\sigma_{m0})$**.**

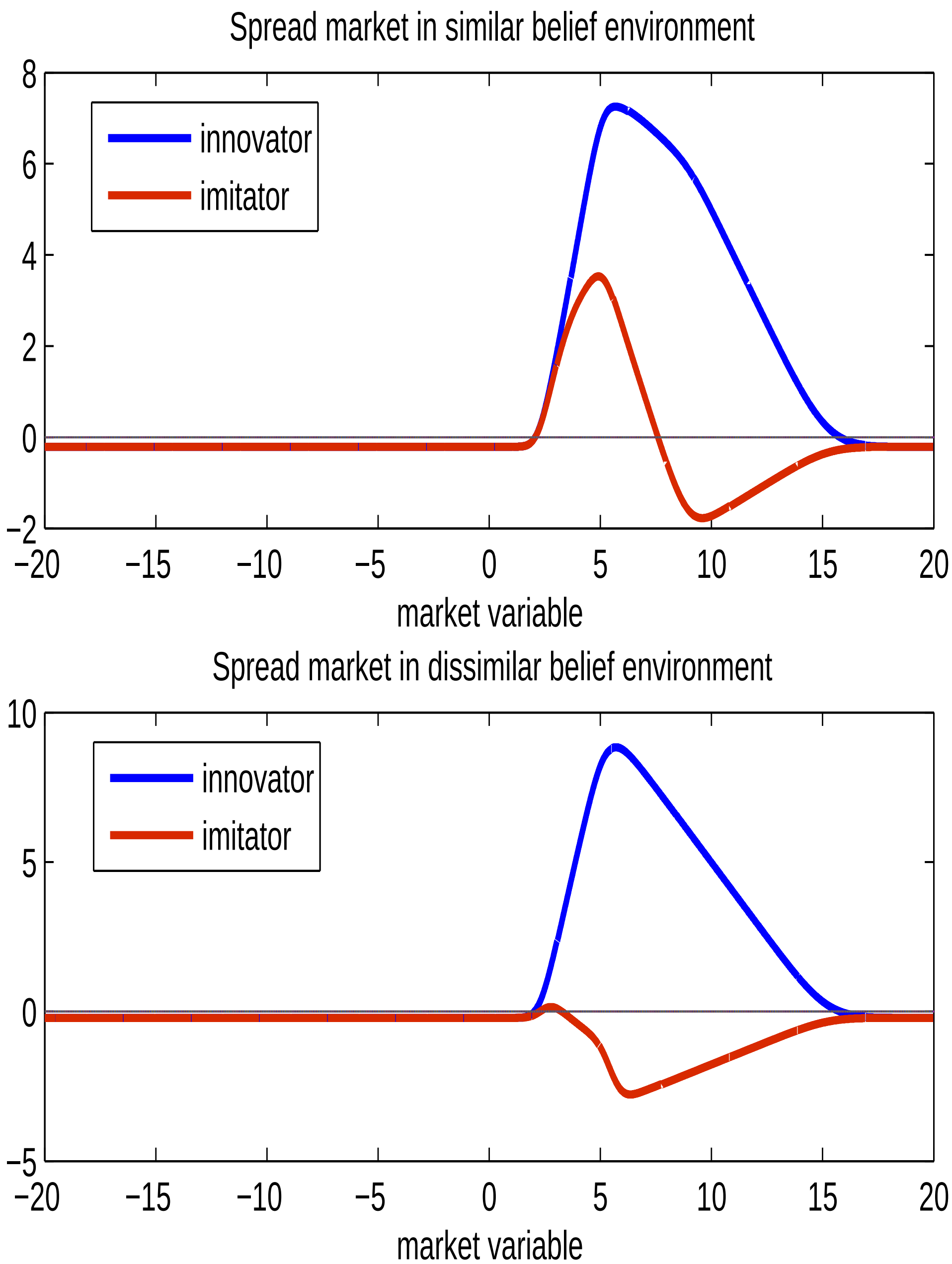


**Figure 6:** **This figure displays the pattern of deviations of market price from reservation prices of a spread option for the innovator and the imitator. Panel A illustrates the case when beliefs are similar, i.e., if** $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) > g(\sigma_{m0})$**. Panel B depicts the case when beliefs are divergent, i.e., if** $qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0}) < g(\sigma_{m0})$**.**