Explaining Residential Investment over the Business Cycle: The Importance of Information and Collateral Constraints[*]

Yufei Yuan[†]

June 24, 2009

[*]I am particularly indebted to Paul Klein, Karen Kopecky and John Whalley for their guidance and comments. I am grateful to Jim MacGee, Elizabeth Caucutt Hiroyuki Kasahara and James Davies for the discussions and seminars about this paper. Also thank Yu Ren for his support. All the errors are mine.

[†]Wang Yanan Institute of Studies in Economics (WISE), Xiamen University, Xiamen, Fujian, China, 361005, E-mail: yyuanwise@gmail.com

Abstract

In U.S. business cycles, residential investment differs from consumption and business investment in two respects. First, residential investment leads GDP while business investment lags and consumption coincides with GDP. Second, residential investment is more volatile than consumption and business investment. The literature attempts to account for these two features, but does so with limited success. This paper is an attempt to account for these two features of residential investment dynamics. To do this, I develop a DSGE model. In the model, I make two distinctive assumptions. First, agents face collateral constraints. Second, agents receive a signal about TFP one period in advance. In a partial equilibrium analysis where interest rates are fixed, when agents receive positive signals, they want to increase current consumption, including housing consumption, to inter-temporally smooth consumption. Because the income of agents does not increase, they must dissave in order to make most of their purchases. If they extend their houses, they can borrow against their house through mortgages based on the value of their house. As a result, agents purchase housing more than other consumption goods because they are bound by collateral constraints. This is the income effect of a positive signal. Through this mechanism, the model can generate the result that residential investment leads consumption and GDP. In a general equilibrium analysis where interest rates are endogenous, in the event of good news, the income effect dominates and residential investment still leads consumption and GDP, despite the expectation of a higher rate of return on business capital. Furthermore, residential investment increases more than business investment in response to a positive signal. This occurs because poorer agents borrow more and purchase more housing. Savings from wealthy agents are used to finance this increased borrowing. This process leads to a reallocation from business investment to residential investment. Through these mechanisms, the model can generate the lead-lag relationship observed in data and the relatively high volatility of residential investment.

1 Introduction

In U.S. business cycles, residential investment differs from consumption and business investment in two respects. First, residential investment leads US GDP, while business investment lags and consumption coincides with U.S. GDP. This lead-lag relationship is supported empirically by and widely studied in existing literature, e.g. in Green (1997) and Leamer (2007). Second, residential investment is more volatile than consumption and business investment. This chapter attempts to explain these two dynamic features of residential investment. This chapter defines the lead-lag relationship using the autocorrelation of residential investment, consumption and business investment with GDP. For instance, residential investment leads GDP. What I mean by this is that residential investment has a higher correlation with future GDP than it has with previous and current GDP. Please refer to Appendix 4 for a detailed descriptions of the data and these second moments.

Understanding the dynamics of residential investment and its role in the business cycle is important. The United States' aggregate housing value constitutes about half of the country's aggregate private wealth, as documented by Greenwood and Hercowitz (1991). Therefore, the housing market is an important element affecting the behavior of consumption and investment. In addition, residential investment is a good predictor of economic recessions. In the past fifty years, eight of ten recessions (including the most recent one) were preceded by a severe reduction in residential investment, as discussed in Leamer (2007). Indeed, Leamer (2007) suggested that housing is the sector most important to economic recessions, and any attempt to control the business cycle needs to focus particularly on household investment.[1]

[1]In Leamer (2007), household investment includes residential investment and consumer

In this chapter, two key assumptions help explain the dynamics of residential investment. First, collateralized consumer loans, such as mortgages, are less restricted in size and carry lower interest rates than unsecured consumer loans, such as credit card debt. This assumption is consistent with the reality of the U.S. financial market. In 2002, the 30-year mortgage rate in the U.S. was 6.40 percent, while the average interest rate on credit card loans was 16.6 percent.[2] This suggests that, even if unsecured consumer loans are available to everyone, their high cost of borrowing will keep most consumers from using them as a major financing resource. This finding is consistent with the fact that most consumer debt in the United States is collateralized.[3]

Second, agents receive a signal of TFP one period in advance, which provides more information than current TFP. There is an empirical literature that presents evidence supporting this assumption about information. For example, Beaudry and Portier (2006) show that changes in interest rates and equity prices are almost perfectly correlated with innovations in future TFP. Some recent studies incorporate this assumption in their business cycle models and match the U.S. data better than models without this assumption, which further validates the assumption. For instance, Backus, Routledge and Zin (2007) demonstrate that by adding a predictable component into TFP growth, the DSGE model can account for the fact that the interest rate leads GDP by two quarters. Jaimovich and Rebelo (2006) provide a new theory of recessions from the perspective of information shocks. Recessions can

durables. The data from National Income and Product Accounts (NIPA) shows that residential investment not only leads GDP more but also has higher volatility than consumer durables.

[2]The data on the mortgage rate is taken from International Monetary Fund (2002). The data on the credit card rate is taken from Gross and Souleles (2002).

[3]In 2001, 81.5 percent of consumer loans are collateralized by residential properties, while unsecured consumer loans take only 10 percent. The education loans constitute about 50 percent of the unsecured loans. These statistics are taken from Aizcorbe, Kennickell and Moore (2003) computing these numbers with the data of Survey of Consumer Finance (SCF, 2001).

occur when current productivity fails to reach the level that has previously been signaled.

A simple example can illustrate the main mechanism at work in this chapter. An agent receives good news about future productivity shocks and wants to increase current purchases, including housing purchases, in order to intertemporally smooth her consumption. Because her current income does not increase, she has to dissave to finance her increased expenditures. She is able to borrow at a lower rate of interest for most of her housing purchases, which is not possible for purchases of other types of consumption. As a result, the agent will buy more housing relative to other goods. In other words, the accessibility of credit through mortgages makes residential investment respond more quickly to signals of future TFP shocks. This can account for why residential investment leads consumption and GDP. If the signal turns out to be accurate, the agent will achieve a higher income and become less financially constrained. At this time, she is able to increase her consumption of other goods, which explains why consumption tends to coincide with GDP. This is the income effect of a positive signal.

However, there is also a substitution effect in the general equilibrium model where interest rates are endogenously determined by the marginal return on business capital. Because future productivity is expected to increase, the agent can also expect to obtain a higher capital income if she chooses to invest more in business capital now and consume later. The result discussed above will therefore reverse if the substitution effect dominates the income effect. The substitution effect is the reason why the existing literature gets the counterfactual result that business investment leads consumption, which leads residential investment in business cycles.[4] This question is addressed in my model by considering income and wealth heterogeneity. The agent's current consumption increases with her expected lifetime income. For a wealthy individual, financial wealth constitutes the majority of her expected lifetime income, which decreases in the event of good news due to a higher expected discount rate. Thus, wealthy agents tend to reduce housing and invest more in business capital in response to positive news. In contrast, for a poor individual, labor income constitutes the majority of her expected lifetime income, which increases because of a higher expected wage. Poor individuals borrow more and buy more housing in response to positive signals. The savings from the wealthy will be used to finance increased mortgages taken by the poor. Because most of the consumption in this economy is carried out by the agents whose wealth is below the mean, this process leads to a reallocation of wealth from business investment to residential investment in the event of positive signals. Therefore, the model can generate the lead-lag relationship and the high volatility of residential investment. This mechanism is consistent with the differences in the compositions of family finances over various wealth levels in the United States. As shown in 2001 data, households between the 10th and 90th percentiles of the wealth distribution borrowed 72.7 percent of collateralized consumer loans. Households above the 90th percentile in terms of wealth held 72.8 percent of financial assets.[5] This mechanism is also consistent with the existing literature on consumption and saving. For example, Storesletten, Telmer and Yaron (2004) document that consumption choices of less wealthy agents, such as the young, are more sensitive to changes in expected labor income.

The existing literature attempts to account for these two dynamic features of

[4]Please refer to the next paragraph for the discussions of this literature.

[5]These statistics are taken from Campbell and Hercowitz (2005), which calculates these numbers with the data of Survey of Consumer Finance (2001).

residential investment, but has done so with limited success. In standard DSGE models of homogeneous agents, when there is a positive technology shock, the representative agent tends to reduce residential investment and increase business investment because of the substitution effect explained above.[6] Modified versions of these standard models have had some success, but they need very special assumptions. Davis and Heathcote (2005), for instance, can obtain the result that both residential investment and business investment coincide with GDP. In their models, the increase of residential investment is due to the positive productivity shocks in construction sectors. Consumers buy more houses as a result of lower prices. However, this story of supply-driven cycles is inconsistent with the positive correlation between house prices and residential investment, which instead favors a demand-driven explanation. Gomme, Kydland and Rupert (2001) introduce a time-to-build component, making the assumption that business investment projects require one more period to start than does home investment. Their models generate similar results in terms of the lead-lag relationship as Davis and Heathcote (2005). This key assumption might be true for the constructions of business structures, which take more time than residential construction. However, these models do not account for the fact that business investments in equipment and software can happen almost instantly.

Fisher (2007) assumes a complementary relationship between business capital and household capital in business productivity. Fisher (2007) makes improvements in accounting for the lead-lag relationship. His model is consistent with residential investment leading business investment, though the leadership of residential investment to GDP is still absent. Moreover, this key assumption that larger houses

[6]This literature includes the papers by e.g. Greenwood and Hercowitz (1991), Benhabib, Rogerson and Wright (1991) and Gomme and Rupert (2007).

can improve labor efficiency is not that firmly convincing to me.

In contrast to these papers, my chapter stresses the demand effect. It emphasizes that, as a form of consumption, housing mostly relies on the aggregate effect of individuals with low and medium wealth. These individuals are more adversely affected by borrowing constraints. Hence, mortgages are important in that they enable poor agents to respond rapidly to expectation shocks in the form of increased housing purchases.

In section 2, I first solve the partial equilibrium model where interest rates are fixed. Following Jaimovich and Rebelo (2006), I introduce information as noisy signals about next period's productivity. The signal has the probability $p \in [0, 1]$ of being correct. The agents form their expectations of future productivity based on the current TFP and the signal. I compute three different models. By comparing the numerical results from the three models, I find that the two assumptions of information and collateral constraints both crucial in generating the result that residential investment leads consumption and GDP.

In section 3, I discuss the general equilibrium model where interest rates are determined endogenously as the marginal return on business capital. Numerical results display the difference of policy functions among agents at different wealth levels. In the event of good signals, poor agents tend to borrow more and buy larger houses while rich agents tend to save more and buy less. I compute the correlation coefficients of residential investment, consumption and business investment with GDP from the simulated data. They are consistent with the broad patterns of lead-lag relationship and the higher volatility of residential investment that we observe in the data.

Section 4 concludes this chapter. It applies the theory put forward in this chapter to explain the current U.S. financial crisis while offering some avenues for future research.

2 Partial Equilibrium Analysis

This section discusses the partial equilibrium model where interest rates are exogenously given and fixed over time. The purpose of this section is to display the crucial roles that the two assumptions of information and collateral constraints play in generating the result that residential investment leads GDP and consumption. To achieve this objective, I compute and compare three models that have different assumptions about collateral constraints and information shocks. The benchmark model has both collateral constraints and the assumption that agents receive a signal about TFP one period in advance. In the no-signal model, agents can borrow through mortgages but do not receive more information about future TFP other than what the current TFP provides. The no-mortgage model contains the assumption of advance information while assuming that agents have no access to mortgages but can still borrow though credit cards. The numerical results show that only the benchmark model can generate the result in which residential investment leads consumption and GDP. In order to highlight the mechanism through which this benchmark model works, I also describe the agent's policy functions over various wealth levels. By doing so, I find that the key mechanism of this chapter is as follows. It is poor people who make the largest adjustments in response to information shocks. Poor people are financially constrained and therefore adjust their residential investment more than other types of consumption.

2.1 The Partial Equilibrium Model Economy

2.1.1 Agents

The economy consists of a continuum of infinitely-lived agents maximizing their expected lifetime utility. For any agent $j \in [0,1]$, the objective function is defined by

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t^j, h_t^j) \right], \tag{1}$$

where $\beta \in [0,1)$ is the discount factor, $c_t^j \geq 0$ is the consumption and $h_t^j \geq 0$ is the quantity of owner-occupied housing. The period utility satisfies the following CES form

$$U(c,h) = (1-\kappa_h)\frac{c^{(1-\theta)}-1}{1-\theta} + \kappa_h \frac{h^{(1-\theta_h)}-1}{1-\theta_h}. \tag{2}$$

It is the service flow of the housing capital that contributes to personal utility. However, with the assumption of Cobb-Douglas home production, Equation (2) can be derived from the utility function relative to service flow. transform the utility function into the form specified in utility function.[7]

At each period, the agent receives her endowment which consists of idiosyncratic income and aggregate income. For the agent $j \in [0,1]$, her budget constraint is defined by

$$c_t^j + h_{t+1}^j + a_{t+1}^j \leq a_t^j + h_t^j(1-\delta_h) + \epsilon_t^j z_t + \mathcal{I}(a_t^j, h_t^j), \tag{3}$$

[7] For example, suppose that the utility function satisfies

$$\mathcal{U}(c, c_h) = (1-\kappa_h')\frac{c^{(1-\theta)}-1}{1-\theta} + \kappa_h' \frac{c_h^{(1-\theta)}-1}{1-\theta},$$

where c_h is the service flow from the housing capital. Assume that the household production function satisfies Cobb-Douglas form:$c_h = h^{\alpha_h}$. Then by putting this home production function back into the above equation, I can obtain the CES utility function like and the elasticity of substitution for housing satisfies $\theta_h = 1-(1-\theta)\alpha_h$.

where a_t^j is the financial asset, h_t^j is the quantity of owner-occupied housing, δ_h is the depreciation rate of the house, ϵ_t^j is the idiosyncratic component of income, z_t is the aggregate income shock and $\mathcal{I}$ denotes capital income.

The aggregate productivity shock z_t and the idiosyncratic income shock ϵ_t^j are two Markov processes independent of each other. I assume that Ω is the set of idiosyncratic income states. The transition matrix is denoted as $P^\epsilon = [p_{mn}^\epsilon]$ with $p_{mn}^\epsilon = \text{Prob}(\epsilon' = \epsilon^m | \epsilon = \epsilon^n)$. $\mathcal{Z}$ is the set of aggregate income states, and the transition matrix is $P^z = [p_{mn}^z]$ with $p_{mn}^z = \text{Prob}(z' = z^m | z = z^n)$.

2.1.2 Market Arrangements

There are two forms of consumer debts available in the financial market. The first form of consumer debt is a mortgage, for which agents pay a low interest rate r. But mortgages are collateralized by the value of housing capital, implying that the agents can only borrow up to a fraction $\gamma \in [0, 1)$ of their house values through mortgages. The second form of consumer debt is credit card debt, for which the agents need to pay a much higher interest rate $r^h > r$. Debts exceeding γh are charged credit card rates. The interest rates r and r^h are exogenously given and fixed over time. In addition to collateral constraints, agents also face a borrowing constraint. Agents cannot borrow more than the amount that their lowest possible wage income can support, given the interest payments based on the interest rate associated with consumer loans.[8]

Given the above financial market arrangements, capital income $\mathcal{I}$ in Equation

[8]Aiyagari (1994) is one example which makes this assumption.

(3) is given by

$$\mathcal{I}(a_t^j, h_t^j) = \begin{cases} ra_t^j & \text{if } a_t^j \geq -\gamma h_t^j \\ -r\gamma h_t^j + (a_t^j + \gamma h_t^j)r^h & \text{if } a_t^j < -\gamma h_t^j \end{cases},$$

where $\gamma > 0$ is the down-payment requirement, implying that the secured consumer loans must be smaller than γh_t^j. For the part of secured consumer debts (or financial assets), the agent pays or receives a low interest rate r. Otherwise, for the part of unsecured loans $a_t^j + \gamma h_t^j$, the agent pays a high interest rate r^h.

2.1.3 Information Structure

At period t, the agents receive the signal s_t about z_{t+1}, the aggregate income shocks of period t+1. As for the idiosyncratic income shocks ϵ_{t+1}, I assume that the agents cannot get more information than what ϵ_t can provide. The set of signal states is denoted by $\mathcal{S}$ and satisfies $\mathcal{S} = \mathcal{Z}$. The signal $s_t \in \mathcal{S}$ is sent with the probability function P^s, which satisfies the following conditions

$$p^s(s_t = z | z_{t+1} = z) = \begin{cases} p_h & \text{if } z_{t+1} \geq \bar{z} \\ p_l & \text{if } z_{t+1} < \bar{z} \end{cases}, \tag{4}$$

where $\bar{z}$ is the unconditional mean of z. This equation means that if the future TFP is higher than the average (as in economic booms), the probability of agents receiving the right information is p_h. Otherwise, if the future TFP is lower (as in recessions), the probability of receiving an accurate signal is p_l. Following Jaimovich and Rebelo (2006), I assume that $p_h > p_l$, which means agents are more likely to be right when the news is good.

The probability that the signal is wrong and equal to z' satisfies

$$p^s(s_t = z'|z_{t+1} = z, z_t) = (1 - p^s(s_t = z|z_{t+1} = z, z_t))\frac{p^z(z_{t+1} = z'|z_t)}{1 - p^z(z_{t+1} = z|z_t)}, \tag{5}$$

2.2 Definition of Recursive Partial Equilibrium

Given interest rates (r, r^h), the recursive partial equilibrium is composed of the state space, value function, policy functions, and distribution function. The state space is encompasses housing capital $h \in \mathcal{H}$, the wealth $w \in \mathcal{W}$, the present idiosyncratic income shock $\epsilon \in \Omega$, the aggregate income shock $z \in \mathcal{Z}$, and the signal $s\mathcal{Z}$.

The value function $\mathcal{V}$, the policy functions of housing asset g_h and financial asset g_w satisfy the following equations

$$\mathcal{V}(h, w, \epsilon, z, s) = \max_{\{c,h',w'\}} \{U(c, h) + \beta E\mathcal{V}(h', w', \epsilon', z', s'|\epsilon, z, s)\}$$

Subject to:

$$c + h' + a' \leq \epsilon z + w(1 + r)$$

$$w' = \begin{cases} a' + h'\frac{1-\delta_h}{1+r} & \text{if } a' \geq -\gamma h' \\ (a' + \gamma h')\frac{1+r^h}{1+r} - \gamma h' + h'\frac{1-\delta_h}{1+r} & \text{if } a' < -\gamma h' \end{cases} \tag{6}$$

$$w'r \geq -\underline{\epsilon z}. \tag{7}$$

Equation (6) defines the wealth of the next period w'. When $a' \geq -\gamma h'$, all the consumer loans are collateralized and the interest rate is the mortgage rate r.

When $a' < -\gamma h'$, there exist secured consumer debts $\gamma h'$ for which the agents pay the mortgage rate r and unsecured consumer debts $a' + \gamma h'$ for which agents pay the credit card rate r^h. Equation (7) defines the borrowing constraint keeping the agent's interest payment below her lowest possible endowment. The choice variables are consumption c, the future quantity of housing h', and the future wealth w'. In Appendix 4, I prove that the policy functions are independent of the state variable h. Therefore, the policy functions can be defined as follows: the current consumption $c = g_c(w, \epsilon, z, s)$; future quantity of housing $h' = g_h(w, \epsilon, z, s)$; and the future wealth $w' = g_w(w, \epsilon, z, s)$.

λ is the mass function of the agents over the state variables (w, ϵ), which is updated in every period. If the set $\mathcal{B}$ is defined as

$$\mathcal{B} = \{(w, \epsilon) \in \mathcal{W} \times \Omega | w' = g_w(w, \epsilon, z, s)\}.$$

Then, the updated distribution function satisfies

$$\lambda'(w', \epsilon') = \int_{\mathcal{B}} \lambda(w, \epsilon) P^{\epsilon}(\epsilon' | \epsilon) dw d\epsilon.$$

2.3 Computation Strategy

With separability of housing and consumption, the utility function can be transformed into $U(c, h) = U^c(c) + U^h(h)$ where $U^c(.)$ denotes the utility derived from consumption and U^h for owner-occupied housing. Given separability, I define the function $\widetilde{\mathcal{V}}(w, \epsilon, z, s)$, satisfying

$$\widetilde{\mathcal{V}}(w, \epsilon, z, s) = \max_{\{c, h', w'\}} \left\{ U^c(c) + \beta U^h(h') + \beta E \tilde{\mathcal{V}}(w', \epsilon', z', s' | \epsilon, z, s) \right\}.$$

Appendix 4 proves the relation between $\widetilde{\mathcal{V}}$ and $\mathcal{V}$ satisfying the following theorem.

Theorem 1. $\forall(h, w, \epsilon, z, s) \in \mathcal{H} \times \mathcal{W}\mathcal{Z}^2, \mathcal{V}(h, w, \epsilon, z, s) = U_h(h) + \widetilde{\mathcal{V}}(w, \epsilon, z, s)$.

With this theorem, I can reduce the number of dimensions of the individual optimization by one because the owner-occupied housing does not affect the agent's policy functions.

2.4 Calibration

Table 3 displays the parameters I use in simulations. I choose the weight of housing in the utility function κ such that the ratio of residential investment over personal domestic consumption is 0.074, which is the mean of the ratio of residential investment to personal consumption expenditure from 1947:I to 2008:I (quarterly data, NIPA). The after-tax yearly mortgage rate is 4.1%. With the capital tax rate at 29.21%; this number means that the before-tax mortgage rate is 5.79%. The capital tax rate is taken from Gomme and Rupert (2007). The after-tax yearly credit card rate is 12.3%, which implies that the before-tax credit card rate is 17.38%. γ is set to be 0.8, which means that the downpayment required is 20%. Fisher and Gervais (2007) compile the actual average downpayment rate of first-time house buyers as 11% from the data of 2005. They argue that other purchasers pay higher downpayment than first-time house buyers because decreasing the down-payment increases the interest rate that the borrowers have to pay. The mortgages discussed in this chapter are different from credit card loans because of their low interest rates. Therefore, it is reasonable to set the down payment rate as 20% which is higher than the value of Fisher and Gervais (2007) but closer to the definition in this chapter. The depreciation rates of the houses δ_h are chosen from Gomme and Rupert

(2007). The idiosyncratic income ϵ follows the labor income process compiled in Guvenen (2007) and is discretized according to the method specified in Rouwenhorst (1995). Appendix 4 describes in detail the stochastic process of idiosyncratic income and the method of discretization. The aggregate income process z follows the process of HP filtered real GDP from NIPA (1947:I to 2008:II). The parameters of information structure P_h and P_l are taken from Jaimovich and Rebelo (2006). Jaimovich and Rebelo (2006) choose the signals received by the agents so as to match the predictive content of the Livingston survey.[9] $p_h > p_l$ implies that the precision of the signal for good future TFP is higher than that of bad future TFP. It is easier for the agents to predict economic booms than it is for them to predict recessions. In the numerical sample of 15,000 periods, the Mean Square Prediction Error (MSPE) of this information structure is computed to be $5.1 - 10^{-5}$.[10]

2.5 Numerical Results

I compute and compare three economies. The benchmark economy has both the assumptions of information and collateral constraints. In the no-signal economy, the agents can borrow against house values but do not receive informative signals about future TFP. In the no-mortgage economy, the agents receive informative signals about future TFP but cannot borrow using mortgages. Figure 2 displays the second moments which are computed with the US NIPA data and with the data simulated through these three models. ρ_T denotes the correlation coefficients of a

[9] Please see Croushore (1997) for details of this survey data. I consider the survey providing good measurement of the information received by the agents in US economy. In Jaimovich and Rebelo (2006), signals predict TFP two periods in advance. For simplicity, this chapter assumes that signals predict TFP one period in advance.

[10] In the sample of T-periods, the prediction for the productivity z_t is denoted as $\tilde{z}_t$, then MSPE $= \left[\sum_{t=1}^{T}(\tilde{z}_t - z_t)^2\right]/T$ where $\tilde{z}_t = E(z_t|z_{t-1}, s_{t-1})$.

time series of y with the output.[11] If $T < 0$, ρ_T defines the correlation coefficient of y with future outputs.

As can be seen from Figure 2, US data shows that correlation coefficients of residential investment with GDP reaches the highest point at $T = -1$, implying that residential investment is the most correlated with the future GDP, i.e. it leads GDP. Meanwhile, the correlation coefficients of consumption with GDP reach the highest point at $T = 0$, meaning that consumption is the most correlated with current GDP and hence coincides with GDP.

In the no-signal economy, lacking the information about the future TFP, the agents can only adjust consumption and residential investment simultaneously after observing current aggregate shocks. Hence, both consumption and residential investment reach the highest correlation with GDP at $T = 0$, which means that they both coincide with GDP. Since agents can borrow against the value of their houses, they can adjust the housing purchase level more than they can adjust consumption. Therefore, it can be observed that $\rho_0 = 0.6048$ is the highest point, but ρ_1 declines so much that it even reaches a negative value at -0.1279. This implies that agents can change residential investment while observing current aggregate shocks so much that they can even make reverse adjustments during the next period.

In the no-mortgage economy, although agents receive signals about future productivity, financial constraints keep them from responding quickly to the information shocks. Most of the people have to wait to make adjustments until the next

[11] ρ_T is defined by

$$\rho_T = \frac{cov(y_{t+T}\text{GDP}_t)}{\sqrt{var(y_{t+T})var(\text{GDP}_t)}},$$

where y_{t+T} is y at $t+T$ period, GDP_t is the output.

period, when the high income is realized. Therefore, residential investment shares similar cyclical features as consumption. Both of them have the highest correlation coefficients with the current GDP. It is interesting to compare this result with the benchmark model. In the no-mortgage economy, the lack of access to mortgages makes residential investment behave like consumption in its dynamic features, i.e., it does so as though agents do not receive any informative information. As can be shown by comparing the numerical results of the three models, only the benchmark economy can replicate the relation that residential investment leads consumption while consumption coincides with GDP. This suggests that the two assumptions of information shocks and collateral constraints are crucial in generating the cyclical features of residential investment consistent with US data.

In order to highlight the mechanism of this benchmark model, it is also worthwhile to review the agents' policy functions over various wealth levels. I use the percentage difference of policy functions when the signals shift from bad to good $\Delta g_h(w)_{\{\epsilon,z,s_b,s_g\}}$ and $\Delta g_c(w)_{\{\epsilon,z,s_b,s_g\}}$ to attain this objective. Δg_h denotes the percentage difference in the housing policy. Δg_h is defined by

$$\Delta g_h(w)_{\{\epsilon,z,s_b,s_g\}} = \frac{g_h(w,\epsilon,z,s_g) - g_h(w,\epsilon,z,s_b)}{g_h(w,\epsilon,z,s_b)},$$

where w denotes the individual wealth and $s_b > s_g$ means the signals shifts from bad to good. Similarly, $\Delta g_c(w)_{\{\epsilon,z,s_b,s_g\}}$ denotes the percentage difference of the consumption policy when the signals shift from bad to good, and satisfies

$$\Delta g_c(w)_{\{\epsilon,z,s_b,s_g\}} = \frac{g_c(w,\epsilon,z,s_g) - g_c(w,\epsilon,z,s_b)}{g_c(w,\epsilon,z,s_b)}.$$

Figure 3 depicts the two functional curves of $\Delta g_h(w)_{\{\epsilon,z,s_1,s_2\}}$ and $\Delta g_c(w)_{\{\epsilon,z,s_1,s_2\}}$ over

w. I choose $(\epsilon, z) = (5, 1)$ and $(s_b, s_g) = (1, 5)$. $\epsilon = 5$ denotes that the endowment takes the fifth state which is equal to 1.5338. $z = 1$ means that the current productivity level takes the first state, which is equal to 0.9670. $s_g = 5$ signals that the signal about the future aggregate productivity takes the fifth state 1.3199, which is higher than current TFP. $s_b = 1$ signals that the future aggregate productivity is the same as today's level.

From Figure 3, three observations can be derived. First, the percentage adjustments in housing and consumption decrease with wealth. This implies that the income effect of the good signals matters more for poor agents because their expected lifetime income increases by a higher percentage from the good signal than does that of rich people. Second, as can be seen from the right-hand side of the black dotted line, for the rich people, who are not financially constrained, the percentage adjustment in consumption is roughly equal to that in housing capital. This is consistent with the log utility function, which has unity income elasticity. Finally, as shown in the left-hand side of the black dotted line, the poor agents make much larger adjustments in housing capital than in consumption in the event of good news. This is consistent with the intuition that mortgages allow faster adjustments in house purchasing in response to information shocks. To sum up, the mechanism of the benchmark economy is as the following: it is poor people who make the largest adjustments in the event of good signals. These people are financially constrained and consequently adjust their residential investment more than they adjust their consumption.

Table 5 displays the numbers of the second moments. As shown in the two tables, in both three models, residential investment has a higher volatility than do consumption and GDP. However, the models where agents have access to mortgages have more volatile residential investment than the models where agents have no access to mortgages. This suggests that the institutional features of financial markets play an important role in the volatility of residential investment.

3 General Equilibrium Analysis

In the second step, I discuss the general equilibrium model where interest rates are determined endogenously by the marginal return on business capital. There are two kinds of capital in this economy: housing capital provides agents with a service flow of housing, while business capital combines with labor to produce a general good. With simulated data, I compute moments of residential investment, consumption, business investment, and GDP. They are broadly consistent with the patterns of the lead-lag relationship and the high volatility of residential investment. I also review the agent's policy functions over various wealth levels and compare it with the policy functions of the partial equilibrium model. The most different part is that in the partial equilibrium model, wealthy individuals increase consumption and housing capital modestly, but in the general equilibrium model, wealthy individuals decrease consumption and invest more in business capital. This verifies the other mechanism working in the model. In the event of good news, wealthy individuals tend to consume less and save more, while poor individuals tend to borrow more. The savings of wealthy individuals will be used to support the borrowing of the poor, which leads to a reallocation of wealth from business capital to residential housing. This helps generate the lead-lag relationship.

3.1 The General Equilibrium Model Economy

3.1.1 Agents and Endowments

There is a continuum of agents $j \in [0,1]$ maximizing their expected lifetime utility specified by Equation (1). In contrast to Section 2.1.1, here I assume the period utility function satisfying

$$U(c,h) = (1-\kappa_h)\frac{c^{(1-\theta)}-1}{1-\theta} + \kappa_h \frac{(h+\underline{h})^{(1-\theta_h)}-1}{1-\theta_h},$$

where $\underline{h} > 0$ implies that Inada condition does not hold for housing, i.e.

$$\forall c > 0, \limsup_{h \to 0} U_h'(c,h) < \infty.$$

Each agent is endowed with 1 unit of labor time. Agents differ in their labor efficiency $\epsilon_t^j \in \Omega$ which follows Markov process. Following Chatterjee, Corbae, Nakajima and Ríos-Rull (2007), I assume stochastic aging, which means that each period agents face a constant probability p_d of dying. The agents who do not survive will be replaced by new agents who have 0 financial assets and 0 units of housing. The labor efficiency of the new agents will be drawn randomly from the stationary distribution of ϵ.

3.1.2 Market Arrangements and Information Structure

In order to make computation more tractable, I make two simplifying assumptions about the financial market structure and the information structure. First, in contrast to Section 2.1.2, I assume that the credit card interest rate is infinite.

This means that agents will only borrow through mortgages and not through credit cards. Therefore, the borrowing constraint that the agents face is now defined by $a \geq -\gamma h$. Therefore, the set of (h, a) from which the agents can choose is defined by

$$\mathcal{P} = \{(h, a) | h \in [0, \bar{h}], \text{ and } a \in [-\gamma h, \bar{a})\}.$$

Second, I assume that the signal regarding future TFP is 100% accurate. In addition, there also exists a capital income tax T_k in this economy. This chapter does not consider labor income taxes due to the assumption of inelastic labor supply.

3.1.3 Production technology

There are two kinds of capital in this economy: housing assets provide agents with a service flow of housing, while business capital combines with labor to produce a general good. Market production satisfies the Cobb-Douglas functional form defined by

$$y_t = f(z_t, K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha},$$

where z_t denotes the productivity of business production at period t; K_t denotes the aggregate business capital; L_t denotes the aggregate efficient labor $L_t = \int_0^1 \epsilon_t^j dj$. K_t and is equal to the summation of individual financial assets $K_t = \int_0^1 a_t^j dj$. This implies that savings of rich agents is used to finance the borrowing of the poor and the business production in this economy.

3.2 Definition of Recursive General Equilibrium

The recursive general equilibrium includes the state space, the laws of motion of the distribution function, the price system, the value function and the policy functions. The state space is defined over

$$(h, a, \epsilon, \lambda, z, s) \in \mathcal{P} \times \Omega \times \Lambda \times \mathcal{Z}^2,$$

where (h, a, ϵ) denotes the individual state while (λ, z, s) denotes aggregate state. λ is the probability measure of agents over residential housing, financial asset and idiosyncratic labor efficiency. I use Λ to denote the set of all the probability measures over the $\sigma-$algebra on the set $\mathcal{P} \times \Omega$. The law of motion of the probability measure λ is denoted as $\Gamma(\lambda, z, s) : \Lambda \to \Lambda$. The price system includes wage rate w_a and interest rate r, which are decided by the marginal returns from the market production defined in Section 3.1.3 on labor and capital respectively.

Given the above conditions and the capital income tax T_k, the value function $\mathcal{V}$, the policy functions g_h and g_a are defined by the following equations.

$$\mathcal{V}(h, a, \epsilon, \lambda, z, s) = \max_{\{h', a'\}} \{U(c, h) + \beta E \mathcal{V}(h', a', \epsilon', \lambda', z', s' | \epsilon, \lambda, z, s)\}$$

subject to:

$$c + h' + a' \leq \epsilon w_a(\lambda, z)\epsilon + a[1 + r(\lambda, z)(1 - T_k)]$$

$(h', a') \in \mathcal{P}$, $c \geq 0$, and $\lambda' = \Gamma(\lambda, z, s)$

The law of motion Γ of the probability measure λ satisfies the following condition. If the sets $\mathcal{B} \subset \mathcal{P} \times \Omega$ and $\mathcal{B}' \subset \mathcal{P} \times \Omega$ satisfy

$$\mathcal{B} = \{(h, a, \epsilon) \in \mathcal{P} \times \Omega | h' = g_h(h, a, \epsilon, \lambda, z, s), a' = g_a(h, a, \epsilon, \lambda, z, s), \text{and } (h', a', \epsilon') \in \mathcal{B}'\},$$

then the updated probability measure satisfies

$$\lambda'(\mathcal{B}') = \int_{\mathcal{B}} P^{\epsilon}(\epsilon'|\epsilon) d\lambda.$$

3.3 Calibration

The labor efficiency process follows Castaneda, Diaz-Giménez and Ríos-Rull (2003) where there exists a group of super-rich people. As argued by Chatterjee et al. (2007), the existence of super-rich agents provides the opportunity and incentive for a high concentration of earnings and wealth. Castaneda et al. (2003) applies the reverse method and highlights that the existence of a high-income group is essential to the generation of the current U.S. wealth inequality. The reason for choosing the parameters of the U.S. idiosyncratic income process from Castaneda et al. (2003) is that their model can track the U.S. wealth distribution well using these parameters.

I calibrate β and κ such that the average ratio of fixed business investment to market output and the average ratio of residential investment to market output are 5.57% and 13.06% respectively. 5.57% is the share of residential investment over output. 13.06% is the share of business investment over output. Both parameters are taken from Gomme and Rupert (2007). The Markov process for z is discretized to match the market productivity process in Gomme and Rupert (2007) with the method of Rouwenhorst (1995). The capital income tax is calibrated to match the average mortgage rate (after capital income tax) in the United States.[12] Table 6 displays the parameters I use in computation.

3.4 Computation Strategy

Solving the equilibrium in section 3.2 carries extremely large computational costs because the law of motion Γ maps an ∞-dimensional space into itself. Therefore, I employ the computational strategies in Krusell and Smith (1998) and Ríos-Rull (2004) to solve for an equilibrium approximate to the one defined in Section 3.2. Γ is simplified such that agents only use the motion of aggregate business capital $K'(K,H,z,s) : \mathcal{K} \times \mathcal{H} \times \mathcal{Z}^2 \to \mathcal{K}$ and the motion of aggregate residential housing $H'(K,H,z,s) : \mathcal{K} \times \mathcal{H} \times \mathcal{Z}^2 \to \mathcal{H}$ when forecasting the future. Also, given aggregates (K,H,z), I can define the function describing aggregate wealth of the next period $W(K,H,z) : \mathcal{K} \times \mathcal{H} \times \mathcal{Z} \to R^+$ as the following

$$W(K,H,z) = K + \frac{H(1-\delta_h)}{1+[1+r(K,z)(1-T_k)]}, \tag{8}$$

where the interest rate satisfies $r(K,z) = \partial f(s,K,L)/\partial K - \delta$. As long as I know the productivity s and any two of the three variables $K, H,$ and, W, the remaining variable can be determined according to Equation (8).

I assume that the laws of motion that agents use to forecast future aggregates

[12] I compute the real before-tax mortgage rates with International Financial Statistics (1971II:2008II).

(W', K', H') satisfy log-linear functional forms defined as follows

$$\log[W'(K,H,z,s)] = \underline{W}_{\{z,s\}} + \phi^W_{\{1,z,s\}} \log(K) + \phi^W_{\{1,z,s\}} \log[W(K,H,z)]$$
$$\log[H'(K,H,z,s)] = \underline{H}_{\{z,s\}} + \phi^H_{\{2,z,s\}} \log(K) + \phi^H_{\{2,z,s\}} \log(H)$$

, where $K'(K,H,z,s)$ can be obtained by solving Equation (8). Table 9 displays the parameters of the laws of motion in this approximated equilibrium.

3.5 Numerical Results

Table 7 displays the second moments computed with the data from the general equilibrium model. Figure 4 plots the correlation of residential investment, consumption and business investment with GDP (T, ρ_T). Both Table 7 and Figure 4 show that residential investment leads consumption, which leads business investment in business cycles. Table 7 also shows that among the four variables, residential investment has the highest volatility. Although the GE model does not replicate perfectly the dynamics of residential investment, it have successfully generated the two cyclical features of residential investment: lead-lag relationship and high volatility.

In order to show the key mechanism working in the general equilibrium model, here I analyze the agents' policy functions over various wealth levels. I define the functions $\Delta c_{\{\epsilon,z_b\}}(w) : R_+ \to R$ as the following

$$\Delta c_{\{\epsilon,z_b\}}(w) = g_c(w,\epsilon,z_b,s_g) - g_c(w,\epsilon,z_b,s_b),$$

where the corresponding aggregate business capital and housing stock (K, H) are the means of simulated data. Δc denotes the magnitude difference in consumption when the signal shifts from bad to good ($s_b < s_g$). Similarly, $\Delta h_{\{\epsilon,z_b\}}$ and $\Delta a_{\{\epsilon,z_b\}}$ are the magnitude of adjustments in housing and financial asset respectively. Figure 5 plots the points $(w, \Delta a_{\{\epsilon,z_b\}})$. It shows that the poor agents reduce their financial assets while the rich agents increase their financial assets when the signal shifts from bad to good. Figure 6 plots the points $(w, \Delta h_{\{\epsilon,z_b\}})$, and shows that poor agents increase their owner-occupied house while rich agents reduce their owner-occupied house. These two figures suggest that the agents of different wealth levels have different responses to information shocks. For a wealthy agent, financial wealth constitutes the majority of his expected lifetime income, which therefore decreases in the event of good news because of a higher expected return on business capital. Thus, wealthy agents tend to reduce residential investment and invest more in business capital. In contrast, for a poor individual, labor income constitutes the majority of his expected lifetime income, which increases due to a higher expected wage. Thus, poor individuals borrow more and make higher residential investments in response to positive signals. Figure 7 plots the points $(w, \Delta a_{\{\epsilon,z_b\}})$ in blue and $(w, \Delta c_{\{\epsilon,z_b\}})$ in red. This figure shows that although the consumption of the poor agents increases in the event of good news, the magnitude of increase of consumption is much smaller than that of residential housing, which is consistent with the income effect discussed in Section 2.

Figure 5 also suggests that the agents whose wealth satisfies $w < \bar{w} = 9.3657$ choose to borrow more in the event of good news. Figure 8 depicts the cumulative density function over wealth level, which shows that 90% of the agents have wealth level lower than 5.5821. Therefore, most of the agents in this economy choose to borrow more and buy more in the event of good news. Since the savings from the wealthy will be used to finance increased consumer loans taken by the poor, this process leads to the reallocation of wealth from business investment to residential investment in the aggregate. Therefore, the model can qualitatively generate the lead-lag relationship and the high volatility of residential investment.

Table 8 complies the percentage shares of total sample across the groups of different wealth levels. The first line of Table 8 are taken from Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) which uses the data from SCF (1998) and Panel Study of Income Dynamics to report the economic inequality of the United States. It shows that the GE model of this section can approximately replicate the US wealth distribution. The biggest discrepancy exists in the poorest people who consist of the first quintile in household wealth. The US data shows that these people hold negative wealth while in this paper, they hold positive wealth. The major reason of this discrepancy is that in this section, agents are not allowed to borrow through credit cards.

4 Conclusion

This chapter establishes a dynamic stochastic general equilibrium model to explain residential investment dynamics in the United States, focusing on the two cyclical features of residential investment: lead-lag relationship and high volatility. Being different from the existing literature, this chapter applies three new modeling features into the DSGE model that help to generate the dynamics of residential investment matching the data. These three distinctive assumptions are information shocks, collateral constraint and wealth heterogeneity. The partial equilibrium analysis where interest rates are exogenously fixed shows that these two assumptions are essential to the generation of the condition where residential investment leads consumption and GDP, which we observe in data. The key mechanism for these results is that in response to good information, agents purchase housing more than other consumption goods because they are bound by collateral constraints. The general equilibrium analysis where interest rates are determined by clearing of the financial market reveals that wealth heterogeneity generates different responses to information shocks. In the event of good signals, rich agents tend to reduce housing capital and invest more business investment, while poor agents tend to increase residential investment. Since poor agents constitute most of the population, this process leads to the reallocation from business investment to residential investment. It is shown by numerical results that this model can better match the cyclical features of U.S. residential investment.

This chapter assumes that housing equity and business capital can be transferred into each other liquidly. This assumption allows for instant and costless responses to false information. But in reality, in a crashing housing market, transferring wealth currently in housing back into business capital is very costly and time-consuming. From this perspective, if we can reasonably estimate the speed and costs of transferring house equity into business capital, this chapter can provide a quantitative framework to explore how long the economy might take to recover from the financial crisis and how much the corresponding welfare losses might be. On the other hand, this chapter tells us that the housing market bubble may hurt the economy for a long time because it has distorted the behavior of individuals vis-a-vis housing investment. Housing market bubble removed capital that should have been used in business production or technological investments towards housing stocks that agents would not have purchased if there was no bubble. However, in current literature, few papers have considered the negative effects of housing market bubbles on TFP, let alone any policy study concerning this point. Especially for countries with high growth rates and high population densities, there always seems to be an extremely high uncertainty concerning the prices and trading volumes of the housing market. How much does this housing market uncertainty affect growth and social welfare in these economies? This chapter can be regarded as a start point for these topics.

References

Aiyagari, S Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, August 1994, *109* (3), 659–684.

Aizcorbe, Ana M., Arthur B. Kennickell, and Kevin B. Moore, "Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances," Federal Reserve Bulletin 1, Federal Reserve System January 2003.

Backus, David K., Bryan R. Routledge, and Stanley E. Zin, "Leads, Lags, and Logs: Asset Prices in Business Cycle Analysis," November 2007.

Beaudry, Paul and Franck Portier, "Stock Prices, News and Economic Fluctuations," *American Economic Review*, September 2006, *96* (4), 1293–1307.

Benhabib, Jess, Richard Rogerson, and Randall Wright, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, December 1991, *99* (6), 1166–1187.

Campbell, Jeffrey R. and Zvi Hercowitz, "The Role of Collateralized Household Debt in Macroeconomic Stabilization," NBER Working Papers 11330, National Bureau of Economic Research, Inc May 2005.

Castaneda, Ana, Javier Diaz-Giménez, and José-Víctor Ríos-Rull, "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, August 2003, *111* (4), 818–857.

Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull, "A quantitative theory of unsecured consumer credit with risk of default," *Econometrica*, October 2007, *75* (6), 1525–1589.

Croushore, Dean, "The Livingston Survey: Still Useful after All These Years," Federal Reserve Bank of Philadelphia Business Review, Federal Reserve Bank of Philadelphia March 1997.

Davis, Morris A. and Jonathan Heathcote, "Housing and the Business Cycle," *International Economic Review*, August 2005, *46* (3), 751–784.

Fisher, Jonas D.M., "Why Does Household Investment Lead Business Investment over the Business Cycle?," *Journal of Political Economy*, February 2007, *115* (1), 141–168.

___ **and Martin Gervais**, "First-time home buyers and residential investment volatility," Working Paper Series WP-07-15, Federal Reserve Bank of Chicago February 2007.

Gomme, Paul and Peter Rupert, "Theory, measurement and calibration of macroeconomic models," *Journal of Monetary Economics*, March 2007, *54* (2), 460–497.

___ **, Finn Kydland, and Peter Rupert**, "Home production meets time-to-build," *Journal of Political Economy*, October 2001, *109*, 1115–1131.

Green, Richard K., "Follow the Leader: How Changes in Residential and Non-residential Investment Predict Changes in GDP," *Real Estate Economics*, Summer 1997, *25* (2), 253–270.

Greenwood, Jeremy and Zvi Hercowitz, "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy*, December 1991, *99* (6), 1188–1214.

Gross, David B. and Nicholas S. Souleles, "Do Liquidity Constraints and Interest Rates Matter for Consumer Behavior? Evidence from Credit Card Data," *Quarterly Journal of Economics*, February 2002, *117* (1), 149–185.

Guvenen, Fatih, "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?," *American Economic Review*, June 2007, *97* (3), 687–712.

Jaimovich, Nir and Sergio T. Rebelo, "Can News About the Future Drive the Business Cycle?," NBER Working Papers 12537, National Bureau of Economic Research, Inc August 2006.

Krusell, Per and Anthony A. Smith, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, October 1998, *106* (5), 867–896.

Leamer, Edward E., "Housing Is the Business Cycle," NBER Working Papers 13428, National Bureau of Economic Research September 2007.

Ríos-Rull, José-Víctor, "Computation of Equilibria in Heterogeneous Agent Models," in Ramon Marimon and Andrew Scott, eds., *Computational Methods for the Study of Dynamic Economies*, New York: Oxford University Press, 2004, chapter 11, pp. 238–264.

Rodríguez, Santiago Budría, Javier Díaz-Giménez, Vincenzo Quadrini, and José-Víctor Ríos-Rull, "Updated facts on the U.S. distributions of earnings, income, and wealth," *Quarterly Review of Federal Reserve Bank of Minneapolis*, Summer 2002, pp. 2–35.

Rouwenhorst, K Geert, "Asset Pricing Implications of Equilibrium Business Cycle Models," in Thomas F. Cooley, ed., *Frontiers of business cycle research*, Princeton University Press, 1995, pp. 294–330.

Storesletten, Kjetil, Christopher I. Telmer, and Amir Yaron, "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics*, April 2004, *51* (3), 609–633.

Appendices

Data and Statistics

This appendix describes the data with which the second moments are computed. The quarter data are from NIPA, including consumption, household investment, business investment and GDP , from 1947:I to 2008: IV. Consumption is composed of non-durable goods, durable goods and services. Residential investment includes the costs of building new houses and renovating the old ones. Business investment consists of the costs of constructing business structures, purchasing equipments and softwares. All the data are real and detrended with hp filters.

The lead-lag relationship is measured with the second moments with GDP. ρ_T denotes the correlation coefficient of the real output with the object of previous T period. Figure 1 plots (T, ρ_T). As can be seen, the correlation coefficients of residential investment and GDP reaches the highest point before $T = 0$, which displays that residential investment leads GDP. The correlation coefficients of consumption and GDP reaches the highest point at $T = 0$, showing that consumption conincides with GDP. The correlation coefficients of business investment and GDP reaches the highest point after $T = 0$, showing that business investment lags GDP.

Table 1 displays the numbers of the second moments. The computed standard deviations show that Residential investment is about 2 times more volatile than business investment and 8 times more volatile than consumption. And business investment is more volatile than consumption.

Idiosyncratic Income Process

This appendix describes the discretization method which I use to derive parameters for idiosyncratic income process ϵ of Section 2. The grid points for the income process (normalized to 1) are given by

$$\begin{pmatrix} 0.2160 & 0.3518 & 0.5730 & 0.9417 & 1.5338 & 2.4981 \end{pmatrix}$$

.

And the transition matrix is given by

$$\begin{pmatrix} 0.49551 & 0.49675 & 0.00124 & 0.00324 & 0.00325 & 0.00001 \\ 0.24838 & 0.49675 & 0.24838 & 0.00162 & 0.00325 & 0.00162 \\ 0.00124 & 0.49675 & 0.49551 & 0.00001 & 0.00325 & 0.00324 \\ 0.00324 & 0.00325 & 0.00001 & 0.49551 & 0.49675 & 0.00124 \\ 0.00162 & 0.00325 & 0.00162 & 0.24838 & 0.49675 & 0.24838 \\ 0.00001 & 0.00325 & 0.00324 & 0.00124 & 0.49675 & 0.49551 \end{pmatrix}$$

.

I derive the income process in the following way. I assume that the working expectancy of the agents is 160 periods (40 years). Then, I take the parameters of the income process from Guvenen (2007). Guvenen (2007) estimates idiosyncratic income shocks $y_t^j = log(\epsilon_t^j)$ with the formula defined by

$$y_t^j = \phi^j + \mu_t^j + \nu_t^j$$

$$\mu_t^j = \rho\mu_{t-1}^j + \eta_t^j$$

$$\phi^j \sim i.i.d.N(0, \sigma_\phi^2)$$

$$\eta_t^j \sim i.i.d.N(0, \sigma_\eta^2)$$

$$\nu_t^j \sim i.i.d.N(0, \sigma_\nu^2)$$

Table 2 lists the parameters about this income process.[13] Finally, I discretize three variables ϕ^j, μ_t^j,and ν_t^j into two states respectively with the method of Rouwenhorst (1995). With the states and transition matrixes from discretization, I obtain 8 grid points for the income process ϵ_t^i. I find that two groups of the grid points are very close in values. Hence, I combine two groups into 1 group and finally obtain 6 gird points.

[13]The values of these parameters are listed in page 692, Guvenen (2007).

Theorem Proof I

This appendix proves Theorem 2.3. The individual value function is defined by

$$\mathcal{V}(h, w, \epsilon, z, s) = \max_{\{c,h',w'\}} \{U_c(c) + U_h(h) + \beta E\mathcal{V}(h', w', \epsilon', z', s'|\epsilon, z, s)\} \tag{9}$$

Subject to

$$c + h' + a' \leq \epsilon z + w(1 + r)$$

$$w' = \begin{cases} a' + h'\frac{1-\delta_h}{1+r} & \textbf{if } a' \geq -\gamma h' \\ (a' + \gamma h')\frac{1+r^h}{1+r} - \gamma h' + h'\frac{1-\delta_h}{1+r} & \textbf{if } a' < -\gamma h' \end{cases}$$

$$a' + \gamma h' \geq -\eta\epsilon z$$

, which can be transformed into

$$\mathcal{V}(h, w, \epsilon, z, s) = U_h(h) + \max_{\{c,h',w'\}} \{U_c(c) + \beta E\mathcal{V}(h', w', \epsilon', z', s'|\epsilon, z, s)\}. \tag{10}$$

The second part of Equation (10) does not depend on h. Hence, I can define a new function $\widetilde{\mathcal{V}}$ as the following

$$\widetilde{\mathcal{V}}(w, \epsilon, z, s) = \mathcal{V}(h, w, \epsilon, z, s) - U_h(h). \tag{11}$$

If I put Equation (11) into Equation (10), I can get the following equation.

$$\widetilde{\mathcal{V}}(w, \epsilon, z, s) + U_h(h) = U_h(h) + \max_{\{c,h',w'\}} \left\{U_c(c) + \beta E[U_h(h') + \widetilde{\mathcal{V}}(w', \epsilon', z', s'|\epsilon, z, s)]\right\} \tag{12}$$

After simplifying Equation (12), I obtain the following new Bellman equation

$$\widetilde{\mathcal{V}}(w, \epsilon, z, s) = \max_{\{c, h', w'\}} \Big\{ U_c(c) + \beta U_h(h') + \beta E \widetilde{\mathcal{V}}(w', \epsilon', z', s' | \epsilon, z, s) \Big\}$$

, the optimizers of which are the same as the optimizers of Equation (9).

Tables and Figures

Table 1: US Data: Second Moments

	Standard deviation	Correlation Coefficients of GDP with						
		x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}
GDP	0.016	0.326	0.602	0.842	1.000	0.842	0.601	0.326
Consumption	0.013	0.464	0.638	0.755	0.762	0.573	0.336	0.082
Residential	0.099	0.581	0.669	0.661	0.531	0.274	-0.007	-0.237
Business	0.048	0.042	0.284	0.543	0.763	0.813	0.725	0.557

This table lists the second moments of consumption, residential investment and busienss investment with GDP

Table 2: Parameters of y_t^j

ρ	σ_ϕ^2	σ_η^2	σ_ν^2
0.988	0.058	0.015	0.061

Table 3: Parameters of the Benchmark Economy In the Partial Equilibrium

Preferences and Endowments	
$\beta = 0.985$	discount factor
$\kappa = 0.18$	the weight of housing in utility function
$\theta = 1$	risk aversion for consumption
$\theta_h = 1$	risk aversion for self-owned housing
Market Arrangements	
$r = 4.1\%$	mortgage rate
$r^h = 12.3\%$	credit card rate
$\gamma = 0.9$	down payment
$\delta_h = 0.004$	depreciation rate
TFP process	
$\rho_z = 0.84$	aggregate Persistence
$\sigma_z = 0.02$	Std. of aggregate disturbance
$N_z = 5$	The number of aggregate states
Information Structure	
$p_h = 0.99$	$p^s(s = z\|z' = z)$, if $z \geq \bar{z}$
$p_l = 0.62$	$p^s(s = z\|z' = z)$, if $z < \bar{z}$

Table 4: Correlation Coefficients of Consumption with GDP

ρ_T	US Data	Benchmark Model	No-signal Model	No-mortgage Model
ρ_{-5}	0.1149	-0.3252	-0.3692	-0.2971
ρ_{-4}	0.2927	-0.1721	-0.2508	-0.1397
ρ_{-3}	0.4630	0.0356	-0.0837	0.0700
ρ_{-2}	0.6367	0.2987	0.1508	0.3330
ρ_{-1}	0.7546	0.6623	0.4646	0.6948
ρ_0	0.7613	0.7756	0.8495	0.7921
ρ_1	0.5713	0.6451	0.7064	0.6410
ρ_2	0.3334	0.4991	0.5693	0.4806
ρ_3	0.0789	0.3766	0.4395	0.3493
ρ_4	-0.1009	0.2611	0.3122	0.2302
ρ_5	-0.2290	0.1435	0.1700	0.1112

This table lists the correlation coefficients of consumption with GDP from the data simulated in the partial equilibrium model of Section 2.

Table 5: Correlation Coefficients of Residential Investment with GDP

ρ_T	US Data	Benchmark Model	No-signal Model	No-mortgage Model
ρ_{-5}	0.3435	0.1094	0.0427	-0.0002
ρ_{-4}	0.4618	0.2219	0.1609	0.1601
ρ_{-3}	0.5524	0.3161	0.2437	0.3214
ρ_{-2}	0.6186	0.4134	0.3529	0.5107
ρ_{-1}	0.6180	0.5810	0.4797	0.7698
ρ_0	0.5162	0.2559	0.6048	0.7974
ρ_1	0.3064	-0.1279	-0.1396	0.3112
ρ_2	0.0763	-0.1756	-0.1568	0.1159
ρ_3	-0.1192	-0.1630	-0.1667	-0.0145
ρ_4	-0.2574	-0.1683	-0.1776	-0.1257
ρ_5	-0.3305	-0.1837	-0.2095	-0.2293

This table lists the correlation coefficients of residential investment with GDP from the data simulated in the partial equilibrium model of Section 2.

Table 6: Parameters of the Benchmark Economy In the General Eqilibrium

Preferences	
$\beta = 0.96$	discount factor
$\kappa = 0.319$	the weight of housing in utility function
$\theta = 1$	risk aversion for consumption
$\theta_h = 1$	risk aversion for houses
Market Structure	
$\gamma = 0.9$	downpayment requirement
$\delta_h = 0.004$	depreciation rate of housing capital
$\delta = 0.025$	depreciation rate of business capital
$T_k = 0.4$	capital income tax
Production Function	
$\alpha = 0.283$	share of capital in business produciton
$\delta = 0.025$	depreciation rate of business capital
TFP process	
$\rho_z = 0.964$	aggregate Persistence
$\sigma_z = 0.008$	Std. of aggregate disturbance
$N_z = 2$	number of states of TFP
Information	
$p_h = 0.99$	$p^s(s = z \mid z' = z)$, if $z \geq \bar{z}$
$p_l = 0.62$	$p^s(s = z \mid z' = z)$, if $z < \bar{z}$

Table 7: Second Moments of the General Equilibrium

	Standard deviation	Correlation Coefficients of GDP with						
		x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}
GDP	0.01	0.16	0.31	0.69	1.00	0.73	0.31	0.16
Consumption	0.01	-0.01	0.31	0.57	0.74	0.20	0.11	0.09
Residential	1.12	0.01	0.013	0.77	0.06	-0.33	-0.03	-0.04
Business	0.06	-0.02	0.00	-0.017	0.15	0.59	0.012	0.04

This table lists the second moments of consumption, residential investment and business investment with GDP

Table 8: Wealth Distribution of the General Equilibrium

	Households in Wealth Quintile					The Wealth-Rich			
	1st	2nd	3rd	4th	5th	90-95%	95-99%	Top 1%	Total Sample
Data	-0.3	1.3	5.0	12.2	81.7	11.3	23.1	34.7	100
Model	0.5	0.6	3.4	11.2	84.4	12.4	14.1	42.6	100

This table lists the percentage shares of total sample across the groups of different wealth levels.

Table 9: Aggregate Policy Functions of the General Equilibrium

$\log[W'(K,H,z,s)] = \underline{W}_{\{z,s\}} + \phi^W_{\{1,z,s\}} \log(K) + \phi^W_{\{2,z,s\}} \log[W(K,H,z)]$					R^2
z	s	$\underline{W}_{\{z,s\}}$	$\phi^W_{\{1,z,s\}}$	$\phi^W_{\{2,z,s\}}$	
Low	Low	-0.0236	0.4114	0.7382	0.9968
High	Low	0.0003	0.4121	0.7273	0.9968
Low	High	-0.0467	0.4105	0.7369	0.9967
High	high	-0.0421	0.4155	0.7378	0.9965
$\log[H'(K,H,z,s)] = \underline{H}_{\{z,s\}} + \phi^H_{\{1,z,s\}} \log(K) + \phi^H_{\{2,z,s\}} \log(H)$					
z	s	$\underline{H}_{\{z,s\}}$	$\phi^H_{\{1,z,s\}}$	$\phi^H_{\{2,z,s\}}$	
Low	Low	-0.0235	0.4114	0.7381	0.9968
High	Low	0.0002	0.4121	0.7272	0.9968
Low	High	-0.0167	0.4204	0.7368	0.9968
High	high	0.0221	0.4154	0.7377	0.9965

This table lists numerical results of the parameters of the aggregate policy functions. z denotes the current TFP while s denotes the signal.

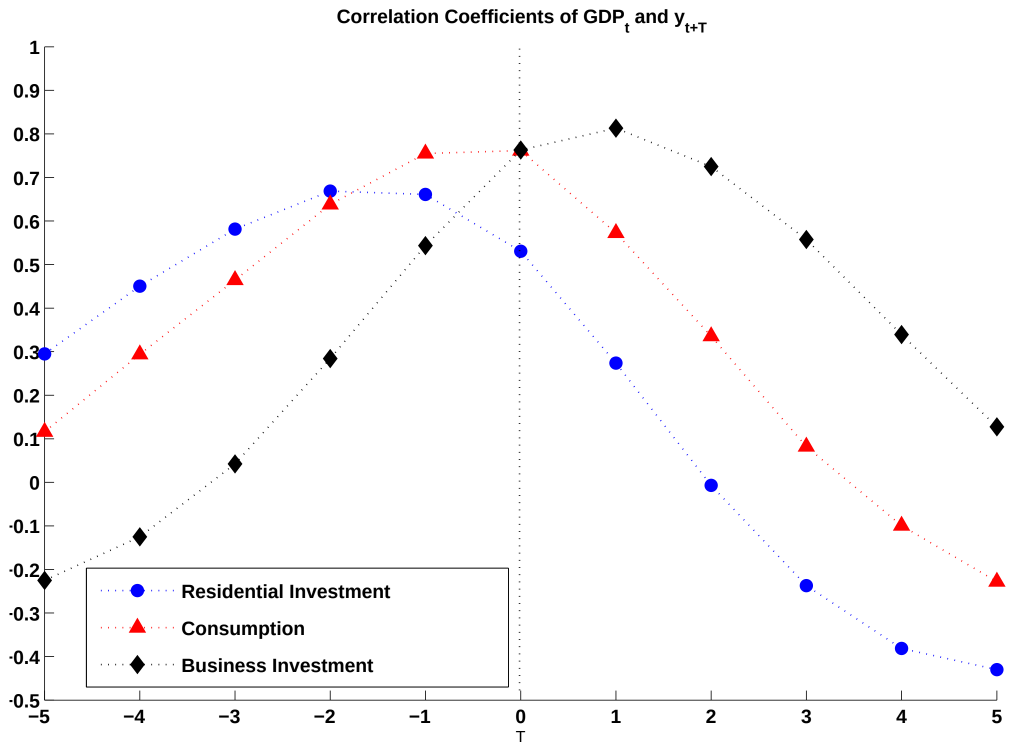

Figure 1: T vs. ρ_T

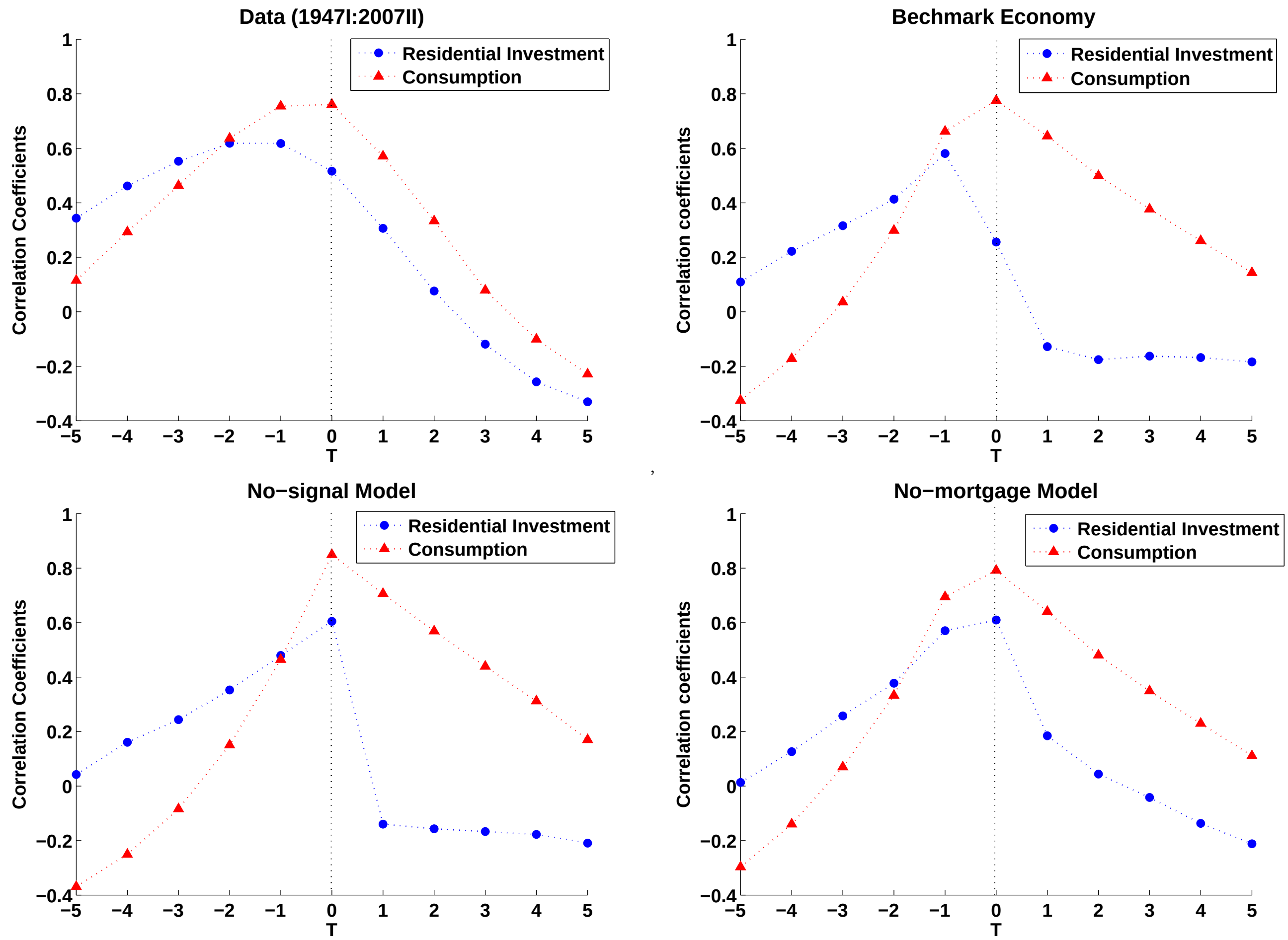

Figure 2: T v.s. ρ_T

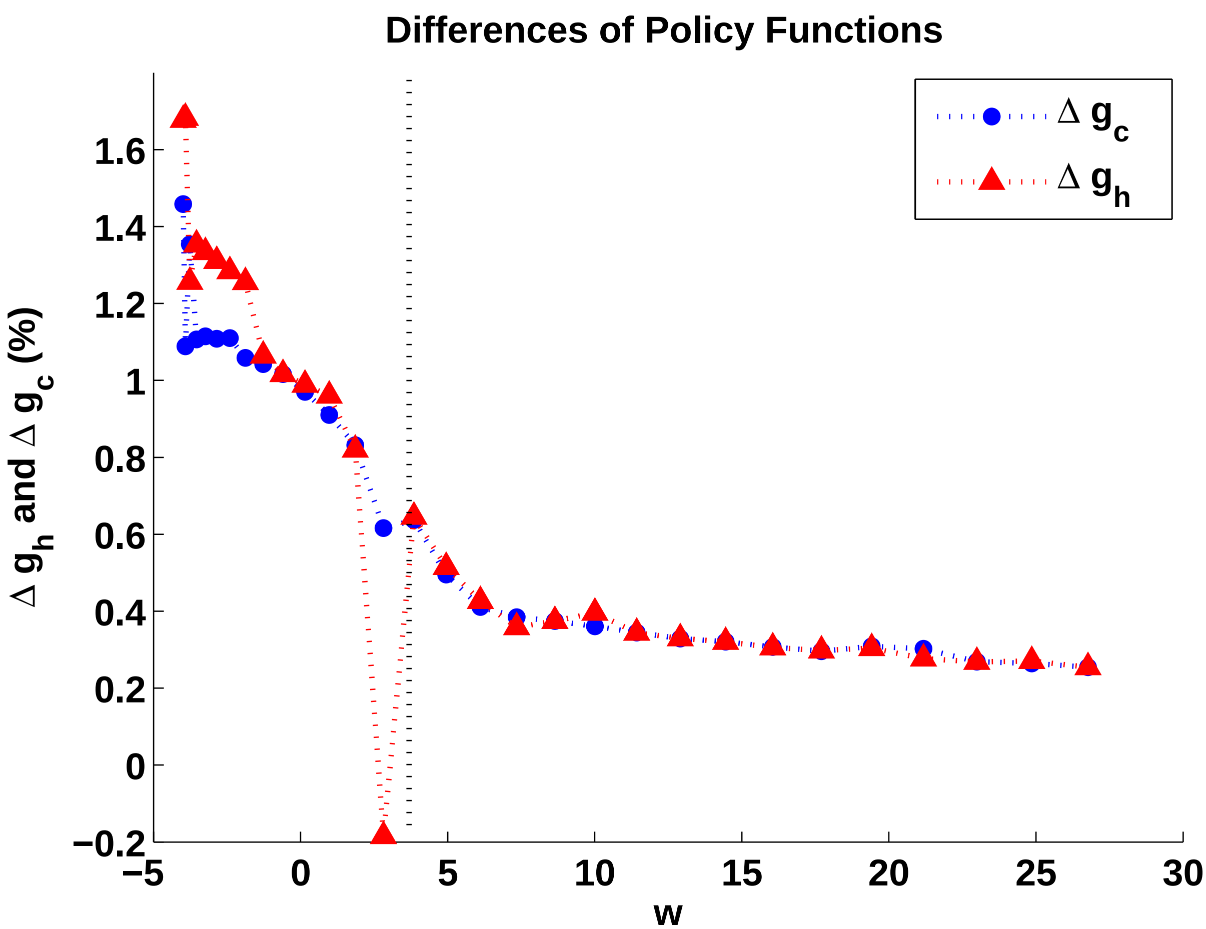

Figure 3: Δg_h and Δg_c

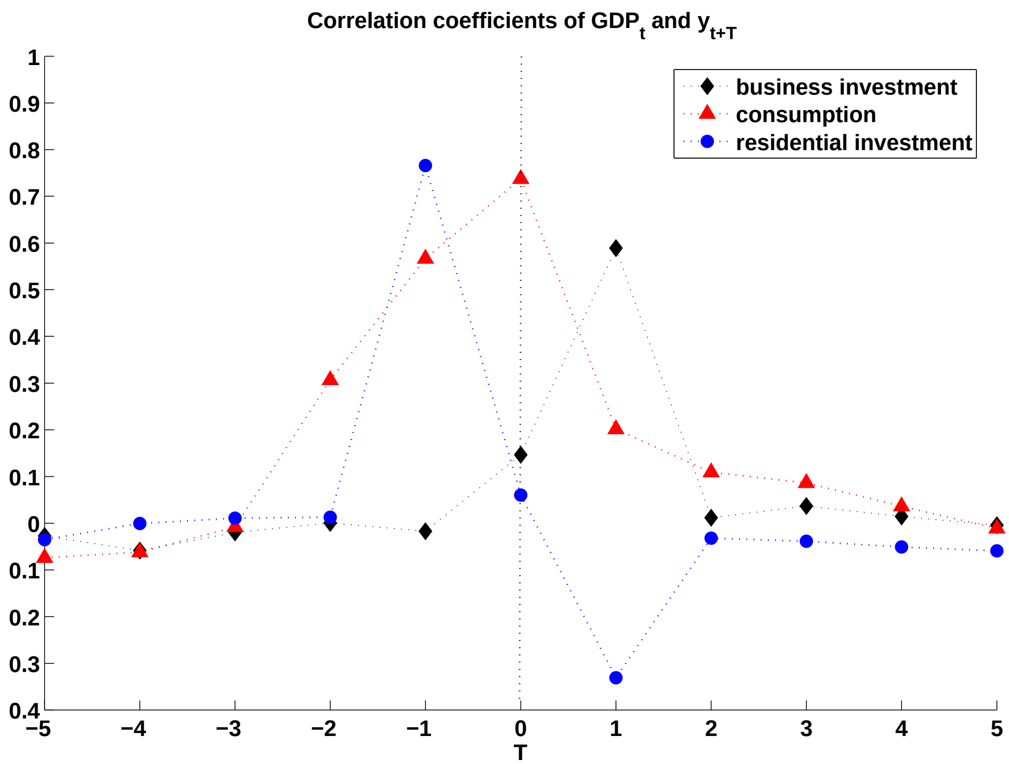

Figure 4: Δg_h and Δg_c

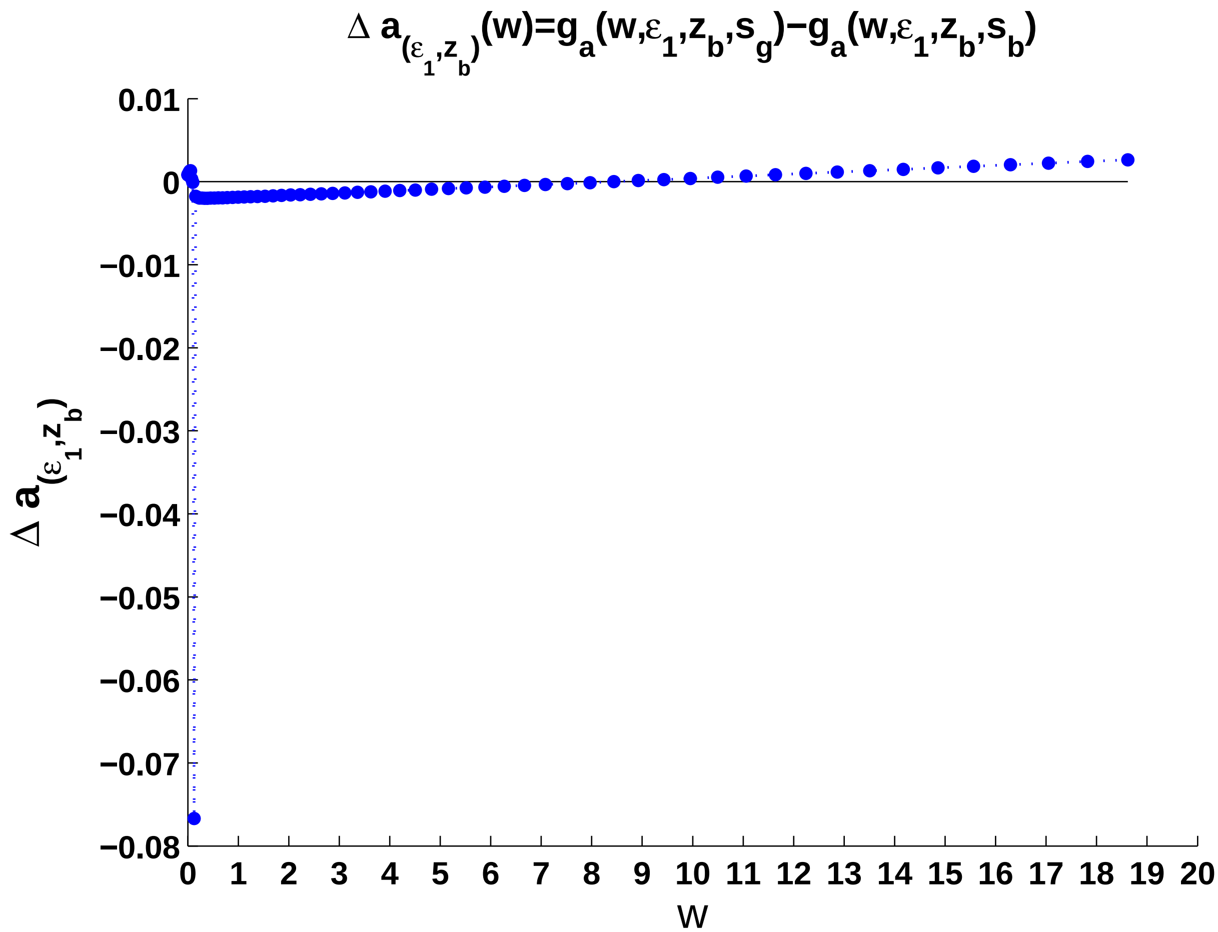

Figure 5: $\Delta a_{(\epsilon_1, z_b)}$

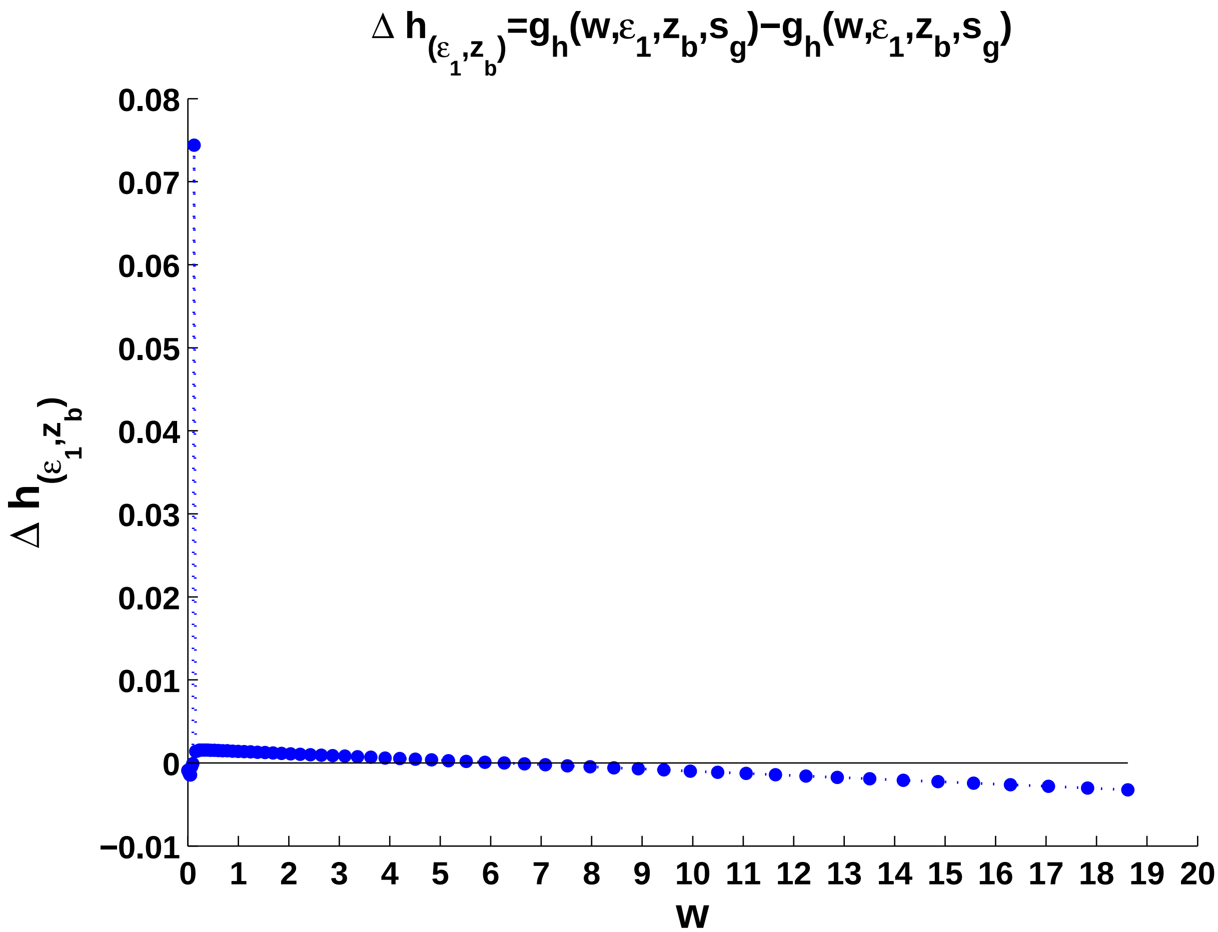

Figure 6: $\Delta h_{(\epsilon_1, z_b)}$

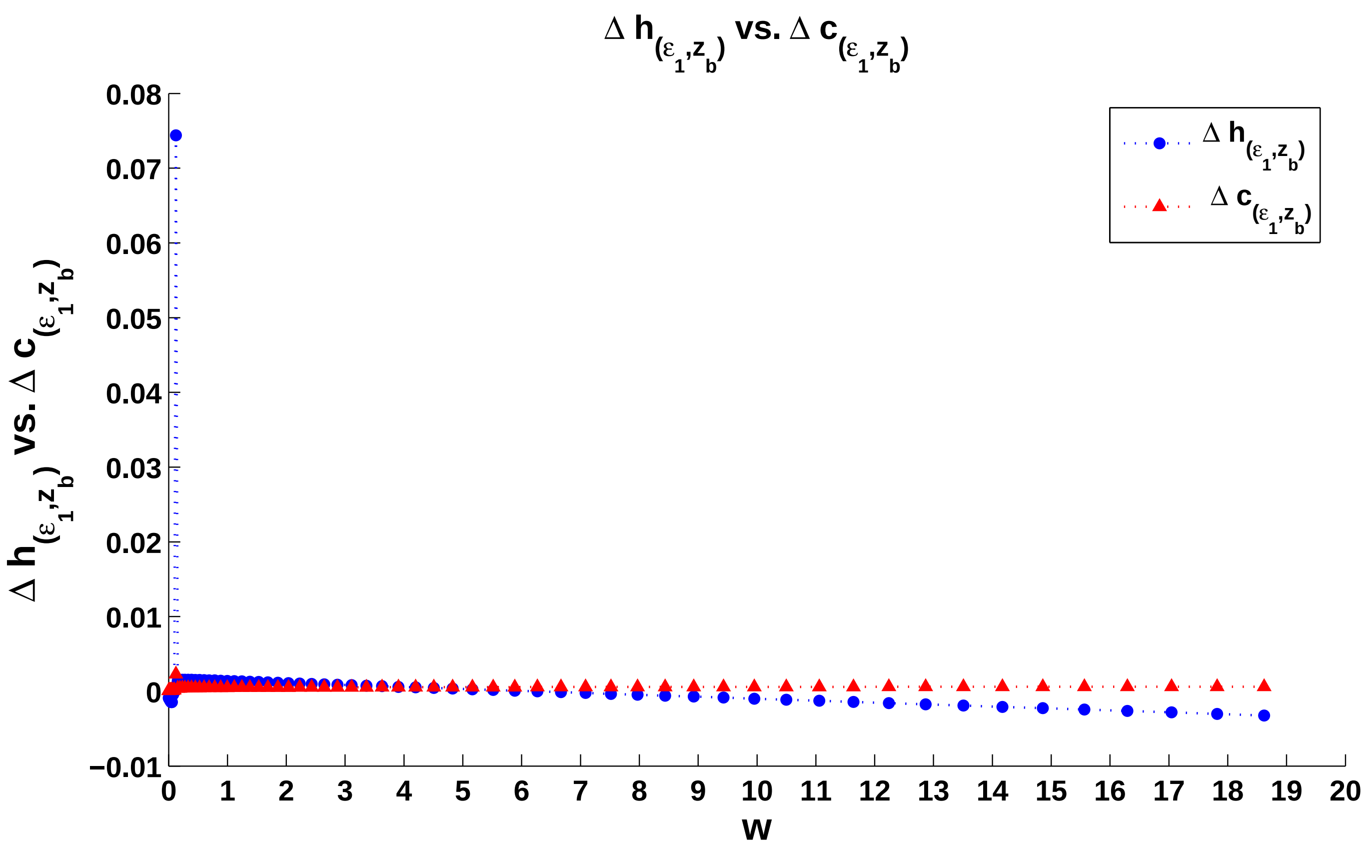

Figure 7: $\Delta h_{(\epsilon_1,z_b)}$ and $\Delta c_{(\epsilon_1,z_b)}$

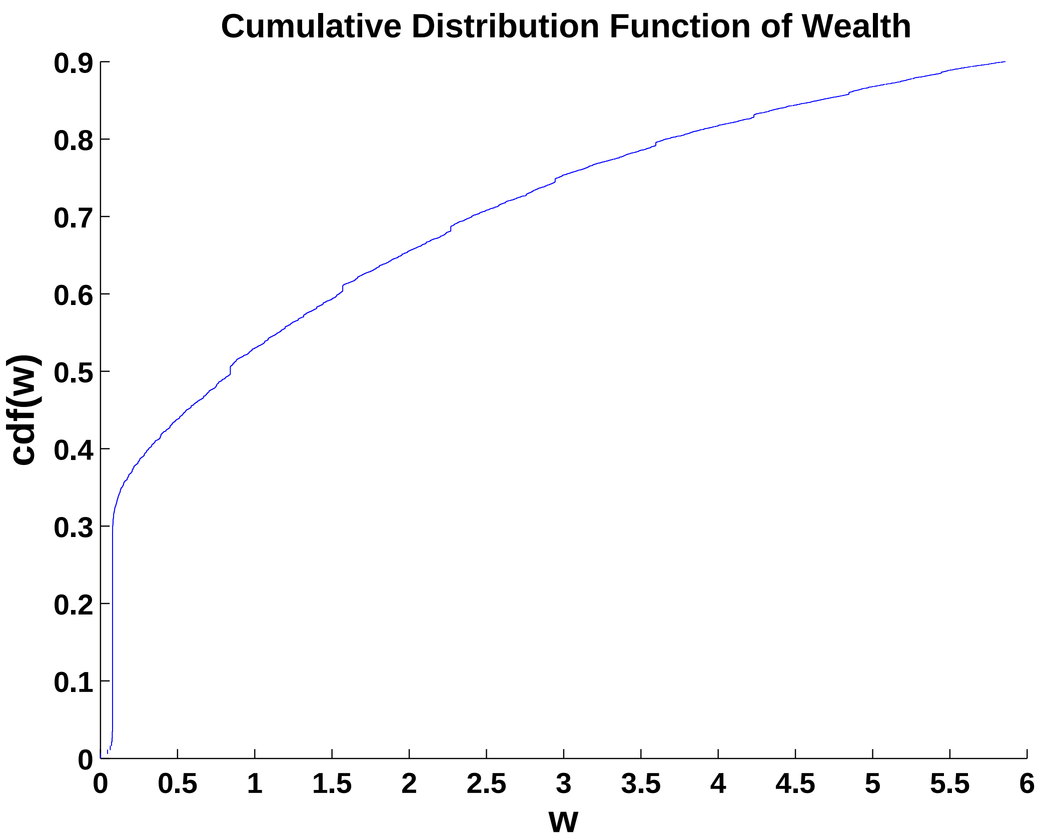

Figure 8: Cumulated Density Function of Wealth