Takeover and Monitor under External Corporate Control

Tao-Hsien Dolly King, Weidong Tian and Xinde Cinder Zhang [1]

Department of Finance

9201 University City Blvd.

Charlotte, NC 28223

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[1]King, Tian and Zhang are from University of North Carolina at Charlotte and can be reached at tking3@uncc.edu, wtian1@uncc.edu and xzhang13@uncc.edu, respectively. We thank Lixin Huang for his helpful comments.

Abstract

We study how both external corporate governance factors and internal corporate governance factors jointly affect the large shareholder's takeover or monitor decision. The large shareholders monitor the management and have option to takeover the firm. External corporate governance forces impose stealing costs to the manager. In equilibrium, large shareholder's takeover or monitoring decision, takeover premium, manager's stealing strategy and the firm values are determined endogenously. We find that both the internal and external control factors have important effects on agency cost and strongly affect the takeover decision. Our results are also robust in the presence of managerial defense and when the market is dynamic.

1 Introduction

In this study, we model takeover, internal monitoring and external control in an equilibrium framework. One strand of research suggests that corporate governance structure and monitoring can reduce agency cost and improve firm value. For instance, Dow, Gorton and Krishnamurthy (2005) and Albuquerque and Wang (2008) claim that the legal system, regulatory pressure, and shareholder monitoring are important mechanisms to align the interests of the managers with those of the investors. Another strand of research (for example, Manne (2008)) strongly support that the market for corporate control is the only needed mechanism[1]. Empirical evidence is favorable to both views. Gompers, Ishii and Metrick (2003) and Cremers, Nair and John (2009) find that good corporate governance practice and the threat of takeovers improve firm value. Jensen and Ruback (1983) among others also find that takeovers, especially hostile takeovers improve firm value.

While the previous literature addresses either internal or external corporate control individually, this paper integrates both mechanisms into an equilibrium framework. In particular, we model takeover decisions in a framework in which agency problem, corporate governance structure, and free rider problem are explicitly addressed. In the model, we consider three players, large shareholder (also the raider), small shareholder, and manager. All players make interrelated decisions simultaneously. The large shareholder has an exclusive ability to improve the firm's performance and makes the decision to take over or remain as an active monitor. The large shareholder has two main reasons for taking over the firm: she can gain through a better asset allocation or a better use of the asset (improvement hypothesis) and/or reducing agency cost (inefficient management hypothesis). As one of the key elements in the model, we model the agency cost by allowing for manager stealing.

[1] See also Manne (1965), (2003), (2008), and Jensen (1993), (2000). This group of literature claim that the market of corporate control can be an effective force to improve efficiency.

Following Albuquerque and Wang (2008), we assume that manager bears the risk of stealing, and the stealing cost is the loss in efficiency that does not accrue to any other players in the model. Minority shareholders are free riders in both takeover and monitoring events. In the takeover event, large shareholder must pay a premium so that small shareholders are willing to tender their shares.

We explicitly model the strength of the internal governance structure, the pressure from the external governance forces, and the agency costs due to management stealing. In equilibrium, the optimal holding of the large shareholder, the tender premium when takeover occurs, manager's stealing and firm value are determined simultaneously.

The main results of this paper can be summarized as follows. First, we present how internal and external corporate controls affect the takeover decision in both static and dynamic frameworks. The presence of the external control decreases the takeover probability significantly. Second, we show that the optimal holding of the large shareholder can deviate from the fixed and legally binding threshold required to control the firm. In other words, in the event of a takeover, the optimal holding can be between 50% and 100%, rather than at 50% (or other fixed thresholds required for control). Third, while previous literature establishes a link between governance structures and firm value[2], we link corporate governance and control to the utility of the agents and show that, given the governance structures, firm value can be created by takeover events endogenously. The endogeneity of takeover and

[2]Gompers, Ishii and Metrick (2003) find that firms adopting less anti-takeover provisions are associated with higher excess return, higher profits, higher growth, higher firm value, less corporate acquisitions and less capital expenditures. Cremers and Nair (2005) document that a portfolio consisting of buying the highest level of takeover vulnerable firms and shorting the lowest level of takeover vulnerable firms yields an annualized return of 10 to 15 percent, if the public pension fund ownership is high. Ferreira and Laux (2007) use idiosyncratic risk as an instrument and find that firms with fewer anti-takeover provisions are more efficient in term of marginal q. They suggest that the quality of corporate governance is associated with the efficiency of corporate investment. John, Litor and Yeung (2008) find that the quality of investor protection is positively related to corporate risk-taking and firm growth rate worldwide. Masulis, Wang and Xie (2007) study the effects of the anti-takeover provisions in mergers and acquisitions. See also DeMarzo and Urosevic (2006).

monitoring decision enables us to examine the effects of the internal and external corporate governance mechanisms simultaneously. Lastly, we show that the initial holding of the large shareholder and the expected improvement in firm value due to the takeover has significant impacts on the utility of the large shareholder and thus the takeover decision (i.e., the optimal holding of the large shareholder). It is the combined effects of the initial holding and improvement, rather than the effects of either one element, that is important to the large shareholder's decision.

There are several interesting implications of the model. In particular, we find that when monitoring costs, which are closely related to the internal governance structure, are high for the large shareholder, takeover is more likely to occur. In addition, when the external governance factors are strong so as to make management stealing difficult, the firm is less likely to be taken over. Furthermore, transaction costs associated with the takeover event have a negative impact on the probability of takeovers. We also find that the large shareholder is more likely to take over for the purpose of improving performance through greater profitability or better asset allocation when the gain from the initial holding is large. On the other hand, when the gain from the initial holding is low, large shareholder tends to takes over for efficiency (i.e., reducing agency cost).

Our model provides significant contributions to the literature on the effects of governance structure consisting of both internal and external mechanisms on takeover decisions and firm value. In particular, the studies on the relation between governance and firm value suggest that existence of various internal and external governance controls have a positive impact on firm value. The implicit assumption is that the pressure from the buyout market (or the market for corporate control) leads to a credible threat for the managers to perform better. We extend this literature by presenting a framework that explicitly models the complex relations among the internal and external governance structures, buyout decision, and firm value. In addition, our model is closely related to the literature on the probability and benefits of takeover.

Our model is related to Shleifer and Vishny (1986)'s. We extend the model in the following aspects. First, we introduce agency costs and the role of manager by modeling the manager's decision to steal from the firm. Large shareholder considers the tradeoff between the monitoring cost and takeover benefit. Following La Porta, Lopez-de-Silanes, Shleifer and Vishny (2002) and Albuquerque and Wang (2008), we assume that manager can steal from the firm if he is in control. Second, we assume that the information and resource of the raider (large shareholder) is exclusive. Hirshleifer and Titman (1990) also consider unsuccessful takeover and draw implications on different defensive strategies. They find that even though most anti-takeover actions reduce the probability of takeovers, certain manager's defensive actions increase the probability of a takeover success because these actions can either increase the tender premium or reduce information asymmetry. As in Shleifer and Vishny (1986) and Hirshleifer and Titman (1990), we also treat the large shareholder and raider as the same agent. We find that anti-takeover actions can be beneficial to minority shareholders only if these actions do not block out the raider.

One of the important issues in the market for corporate control is the free rider problem. In the context of corporate governance, there are two free rider problems. We address both free rider problems in the model. The first free raider problem lies in the monitoring event. Minority shareholders have little incentive to monitor the management since the benefit relative to cost is minimal. Therefore, minority shareholders are likely to free ride on the benefits of monitoring performed by large shareholders and/or outside raiders. The second free raider problem is related to tender offers or buyouts. Minority shareholders may choose not to tender their shares since they want to free ride on the profit that will be realized by the raider if the offer is accepted. The consequence is a failed tender offer or a higher tender premium. Grossman and Hart (1980) suggest that, given the free rider problem, all tender offers fail unless we give raider the right to dilute the payout of the free riders. Shleifer and Vishny (1986) show that large shareholder can overcome the free rider problem since they offer part of their gain from the successful tender offer to small shareholders in the form of a tender premium. We address the monitoring problem by assuming that small shareholders do not monitor the manager and following Shleifer and Vishny (1986), address the second free rider problem by giving small shareholders a premium in a tender offer.

Our paper is also closely related to Dow, Gorton and Krishnamurthy (2005) and Albuquerque and Wang (2008). In Dow, Gorton and Krishnamurthy (2005) large shareholder monitor the management to reduce agency cost. In our model, we follow the same approach by introducing the cost of monitoring. Albuquerque and Wang (2008) explicitly model manager's stealing cost but the internal monitoring capability is not modeled. In this paper, we analyze the tradeoff between monitoring and controlling in the presence of management stealing cost (external control).

The remaining of the paper is structured as follows. In Section 2 we present the model and equilibrium solutions. In Section 3, we discuss sensitivity analysis and implications. In Section 4 and 5 we extend the model to dynamic setting and to incorporate managerial defense, respectively. Section 6 concludes. Technical proofs are given in Appendix.

2 Model

We consider a one-period economy with two securities: One is a locally riskfree asset with risk free rate, $r = 0$, the other is the stock of a firm with a certain production technology. The firm is 100% equity funded. There are two types of shareholders: one large and active shareholder (L, hereafter) and a group of atomistic shareholders (S, hereafter). Shareholders with 50% or more share has the controlling power. Initially, no one holds more than 50% of the shares. Shareholders and manager (M, hereafter) are assumed to be risk neutral. The initial firm value is V_0 and production rate of the production technology is q. A dynamic extension of the model will be presented in Section 4.

At time $t = 0$, M is hired by the firm with the following compensation package:[3] manager receives $\theta \in (0, 1)$ of the realized firm value if manager remains employed; otherwise, the manager receives nothing. θ is fixed. The obligation of the compensation package is never violated. We assume that the manager's initial wealth is zero.

The large shareholder (L) initially owns α_0 proportion of the firm and L has and exclusive access to the technology for finding and implementing valuable improvements using the firm's current assets and her own resources (Shleifer and Vishny (1986)). The large shareholder's management quality gives L the capability to improve firm value by $(Z + q)V_0$, where Z is a nonnegative random variable to represent the improvement ability of L.

At time $t = 0$, L decides her new level of holding (α) and whether to takeover the firm. If $\alpha \geq 50\%$, it means that L decides to takeover.[4] Grossman and Hart (1980) point out the free-rider problem in takeover events. S, anticipating a higher quality of management and a positive improvement of the firm value, would not sell the shares to L. L has to pay a premium to S so that she can obtain control. Let $\pi(\alpha) > 0$ denote the takeover premium. [5] The takeover transaction cost is cV_0 in addition to the takeover premium paid to S. If

[3] The compensation package can be argued too simple in this version. Actually the model can be easily extended to the case that some cash amounts are included in the compensation package. According to Dittmann and Maug (2007), restricted stock is more beneficial to the firm owners than the employee stock option. We do not address the optimal compensation package design. Instead, our objective is to compare internal and external corporate governance and their effects to the firm as well as the manager. Hence we use employee stock as a prototype example of the compensation package in our discussion. In the compensation package we consider, the manager is fully entrenched since he receives nothing if he is fired.

[4] In this paper the large shareholder aware of the successful takeover probability and incorporate it into her takeover decision. Hence the successful takeover probability is either one or zero.

[5] Shleifer and Vishny (1986) assume existence of competitor on the takeover market while we assume that L has exclusive resource and ability. Hirshleifer and Titman (1990) also assume L has better information

$\alpha < 50\%$, it means that L want to be a monitor only during the time period with monitoring cost $I(\alpha)$ where $I'(\alpha) > 0$ and $I''(\alpha) < 0$. M is still in control. Even L need to buy more shares from S, L does not pay extra premium. The model does not examine the probability of successful takeover, but they could be incorporated into the model without loss of generality.

The monitoring cost to L is

$$I(\alpha) = \frac{1}{2}\gamma\alpha^2 V_0.$$

where γ represents the easiness level of monitoring. As an internal control parameter, γ can be viewed as the proxy of the internal corporate governance structure and firm characteristics such as board composition, firm charter, bylaws, industry sector, level of R&D and so on. S free rides in both takeover and monitoring games.

When the manager is in charge, he steals β fraction of the firm. Consistent with Albuquerque and Wang (2008), Johnson, La Porta, Lopez-de-Silanes and Shleifer (2000) and La Porta, Lopez-de-Silanes, Shleifer and Vishny (2002), the external controls impose a stealing cost [6] on the manager's wealth. Assume the manage steals from the firm, βV_1, during this time period. $\Phi(\alpha, \beta; \eta, V_1)$ represents the stealing cost. For simplicity throughout out the paper we assume

$$\Phi(\alpha, \beta, \eta, V_1) = \frac{1}{2}\alpha\beta^2\eta V_1.$$

The external control parameter, η, is the proxy for legal system, regulations and social norms. The higher the α, the closer L monitors the manager and thus a higher cost for the manager's stealing. On the other hand, the more the stealing the higher the stealing cost.

but L has to reveal the information to the market eventually. Other literatures such as Goldman and Qian (2005) illustrate the tender offer when the manager is involved.

[6] It is also termed as the *cost-of-theft* function in La Porta, Lopez-de-Silanes, Shleifer and Vishny (2002).

[7] In this section there is no management defensive action when L decides to takeover the firm. We extend our model to investigate managerial defenses in Section 5.

At time $t = 1$, the production technology is realized and the manager is able to cash out the compensation package.

In equilibrium, firm value at time $t = 1$, takeover decision of L, the stealing strategy of M and the takeover premium are determined simultaneously.

Table 1 displays how firm value, and final wealth of L and M are affected by takeover decision of L and stealing decision of M, and how external and internal controls are implemented in this framework.

Table 1: Affects of External and Internal Control on M's and L's Wealth

This table reports the firm value, large shareholder's wealth and the manager's wealth, at time $t = 1$, in the presence of external control and the internal control. α and β are two decision variables by L and M, respectively.

	Firm Value V_1	L's final wealth W^l	M's final wealth W^m
Takeover	$V_0(q+Z)$	$\alpha V_1 - (\alpha - \alpha_0)(V_0 + \pi(\alpha)V_0) - cV_0$	0
Monitor	$V_0 q$	$\alpha(1-\theta)(1-\beta)V_1 - I(\alpha) - (\alpha - \alpha_0)V_0$	$\theta(1-\beta)V_1 + \beta V_1 - \Phi(\alpha, \beta, \eta, V_1)$

When L takes over the firm ($\alpha \geq 50\%$), L needs to pay a premium of $(\alpha - \alpha_0)\pi(\alpha)$ to S and bears the takeover transaction cost c. Therefore, the final wealth W^l is αV_1 minus $(\alpha - \alpha_0)(V_1 + \pi(\alpha)V_1)$, which reflects the amount payout to S and the transaction cost cV_0. If L doesn't takeover the firm, the manager has opportunity to steal β fraction of the firm, the firm's value is reduced to $(1-\beta)V_1$; after the manager's compensation, $\theta(1-\beta)V_1$, the firm has $(1-\theta)(1-\beta)V_1$ to distribute to its shareholders. Consequently, the final wealth W^l

[7]The convexity assumption of the stealing cost with respect to β is standard. Our choice of the stealing cost is the same as in Albuquerque and Wang (2008) except for in our case α is involved. We argue that our choice is reasonable in a framework with both internal and external control. Albuquerque and Wang (2008) do not consider the monitoring case.

is $\alpha(1-\beta)(1-\beta)V_1$ minus $(\alpha-\alpha_0)V_1$ and $I(\alpha)$. The final wealth of the manager is derived similarly.

We start with the determination of the takeover premium $\pi(\alpha)$ in the next section.

2.1 Takeover Premium

We follow Shleifer and Vishny (1986) to determine the takeover premium in the event of tender offer. The takeover premium $\pi(\alpha)$ is determined at the point where S's are indifferent to tender their shares. Precisely,

$$\pi(\alpha) = \mathbb{E}[Z|\alpha Z - (\alpha-\alpha_0)\pi(\alpha) - c \geq 0]. \tag{1}$$

We assume that Z is uniformly distributed on $[0, Z_{max}]$.[8] Then, based on equation (1), we obtain

$$\pi(\alpha) = \frac{Z_{max}\alpha + c}{\alpha + \alpha_0} \tag{2}$$

and

$$\frac{\partial \pi(\alpha)}{\partial \alpha} = \frac{Z_{max}\alpha_0 - c}{(\alpha+\alpha_0)^2} \tag{3}$$

The following proposition, which follows from equation (2) and (3) easily, illustrates the general behavior of the takeover premium.

Lemma 1 *If α is continuous, then:*

[8]The uniform distribution assumption is only used in this paper as illustrate purpose. Other distributions of Z can be imposed and the main findings of this paper are still the same.

- *When the maximum takeover gain from the L' s initial holding, $\alpha_0 Z_{max}$, is greater then the cost L paid, the tender premium is increasing with respect to the number of shares she wants to purchase.*

  When the maximum takeover gain from the L' s initial holding is less then the cost L paid, the tender premium is decreasing with respect to the number of shares she wants to purchase.

- *The higher the initial holding α_0, the smaller the tender premium $\pi(\alpha)$.*

- *The takeover premium is higher if the cost of takeover, c, is greater.*

- *The takeover premium is always greater than the expected takeover gain.*

It is interesting to see how the takeover premium $\pi(\alpha)$ and the tender proportion $\alpha - \alpha_0$ are correlated. The contingency depends on the improvement Z. If a tender offer is made, high initial holding of raider means less improvement opportunity, ceteris paribus. If the transition cost is high and L makes a tender offer, S's interpret this information as high expected improvement. The last thing to note from 1 is whenever a tender offer is made, L must pay premium to S. These relations will be emphasized in latter sections as well.

We will derive the Nash equilibrium in two different situations: the maximum takeover gain from L's initial holding is larger or smaller than the takeover transition cost.

2.2 Equilibrium of Large Takeover Gain from L's Initial Holding

First we consider the case where $\alpha_0 \mathbb{E}[Z] > \frac{c}{2}$ and solve the equilibrium in two stages.

First, suppose that M observes L's holding decision α, and the manager's expected utility is

$$\mathbb{E}[W^r] = \left\{\theta(1-\beta) + \beta - \frac{1}{2}\alpha\eta\beta^2\right\}\mathbb{E}[V_1] \tag{4}$$

with the stealing strategy β. Hence, the optimal stealing share β of the manager is

$$\beta^* = argmax\mathbb{E}[W^r] = \frac{1-\theta}{\alpha\eta}. \tag{5}$$

Second, by anticipating the manager's decision, L's rational decision is to find the optimal number of shares α. For this purpose, we consider two separate situations. In the first situation, L decides to takeover the firm $(i.e., \alpha \geq 50\%)$. In the second situation, L decides against the takeover and monitors the firm. The best decision is the one with the greater expected utility of these two situations. To simplify the notations the first situation is termed as "takeover" and the second situation is "monitor" in the subsequent discussion.

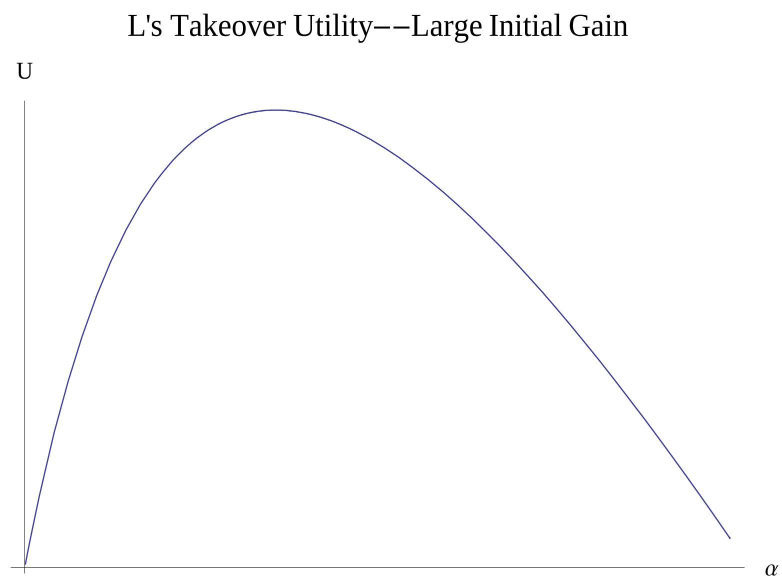

Figure 1: This figure shows L's expected utility with respect to her ex post holding if she takes over the firm in the case of $\alpha_0 E[Z] > \frac{c}{2}$.

The next lemma presents the optimal number of share α^* in the takeover situation.

Lemma 2 *If L wants to take over the firm, then the optimal number of shares α^* in the takeover region, is characterized as follows:*

1. *If the expected improvement is strictly greater than the expected return without taking over, that is, $\mathbb{E}[Z] > \mathbb{E}[q] - 1$, then*

$$\alpha^* = \min\{\max\{50\%, \alpha_1^*\}, 100\%\},$$

where

$$\alpha_1^* := \sqrt{\frac{s}{\mathbb{E}[Z] + 1 - \mathbb{E}[q]}} - \alpha_0, s := 2\alpha_0(\alpha_0 Z_{max} - c). \quad (6)$$

2. *If the expected improvement is smaller than the expected return without taking over, that is $\mathbb{E}[Z] \leq \mathbb{E}[q] - 1$, then $\alpha^* = 1$.*

Proof: See Appendix. □

This lemma follows from the convexity of L's expected utility when $\mathbb{E}[Z] > \mathbb{E}[q] - 1$, as displayed in Figure 1. By the standard portfolio choice principle, L's optimal holding of the firm is α_1^* in all possible holdings. Because $\alpha^* \in [50\%, 100\%]$ by assumption, the optimal share in the takeover region equals $\min\{\max\{50\%, \alpha_1^*\}, 100\%\}$. If $\mathbb{E}[q_2] - 1 > \mathbb{E}[Z]$, the expected utility is increasing with respect to the number of shares. Hence, if L wants to take over the firm, she purchases as many shares as possible. That is, to purchase 100% of the firm.

The intuition of this result is as follows. If the expected return without taking over $\mathbb{E}[q_2] - 1$ is greater than the expected improvement $\mathbb{E}[Z]$, L is willing to maintain 100%

control if she wants to take over. It can be the case that the takeover will never be optimal for L since the takeover premium increases with the tender fraction, α. However, if L does not takeover, she has to suffer the loss from manager's stealing on top of the monitor cost. Since both actions, taking over the firm and retaining the manager, are costly, she chooses the one with less incremental cost.

There is a remarkable implication of this lemma. In contrast to Shleifer and Vishny (1986), the optimal shares α^* of L might belong to the interior of the range $[50\%, 100\%]$ and in particular, not necessarily to be the binding shares 50%. As shown Section 5, the transformation $\alpha_0 \longrightarrow \alpha^*$ plays a key role in extending our results in this section to the dynamic model.

Lemma 3 *Assume that* $\alpha_0 \mathbb{E}[Z] > \frac{c}{2}$ *and* $\mathbb{E}[Z] > \mathbb{E}[q] - 1$. *If the initial share* α_0 *is large, or* $u := \mathbb{E}[Z] - (\mathbb{E}[q] - 1)$ *is relatively small, then* $\alpha^* > 50\%$. *Precisely,* $\alpha^* > 50\%$ *if and only if*

$$\alpha_0 > \frac{u + 2c + 2\sqrt{u(c + \mathbb{E}[Z]) + c^2}}{2(4\mathbb{E}[Z] - u)}.$$

Moreover, $\alpha^* \in (50\%, 100\%)$ *if and only if*

$$\frac{u + 2c + 2\sqrt{u(c + \mathbb{E}[Z]) + c^2}}{2(4\mathbb{E}[Z] - u)} < \alpha_0 < \frac{u + c + 2\sqrt{2u(c + \mathbb{E}[Z]) + c^2}}{4\mathbb{E}[Z] - u}. \tag{7}$$

Proof: See Appendix. □

This lemma is intuitively appealing. When the expected improvement is close to the expected return without taking over, indicating that the return of the firm is close before or after the takeover L's decision is similar to a standard portfolio choice decision. Therefore, the optimal number of shares is not necessarily 50%. If the expected improvement is relatively large, or α_0 is relatively small (because of the assumption that $\alpha_0 \mathbb{E}[Z]$ is greater than $\frac{c}{2}$), then L's optimal holding is 50%, as predicted in the previous literature (e.g. Shleifer and Vishny (1986) and Goldman and Qian (2005)). Indeed, in a static model, one can argue that the initial share α_0 is often small, or the expected improvement should be high, therefore the optimal shares must be binding as assumed in the previous research. However, it is not necessarily true in a *dynamic model*, which we will discuss later.

Figure 2 displays the region of α_0 when the optimal share α^* is 50%, 100% or some interior points in $[50\%, 100\%]$. There are three boundaries; the lower boundary is given by the lower bound in formula (7) while the upper boundary is given by the upper bound in the formula (7). We see that, if α_0 is small, $\alpha^* = 50\%$. However, when α_0 is in a reasonable range, it is possible for L to take an interior solution due to the concavity of the utility function.

In Shleifer and Vishny (1986) where a competitive takeover market exists, all the potential raiders have to give S the best tender price. Therefore, all raiders have to offer the same price and acquire the least number of shares to take control, which is 0.5 fraction of outstanding shares. After we relax the competitive market constraint, raider (L) is in a more flexible position. In fact, the takeover market is far from fully competitive.

The transition cost, c, is another important factor. If the cost is too high, then L needs to pay a high tender premium on top of the high cost of taking over. Then, it is difficult for her to cover both costs. The consequence is that the takeover will be unlikely to take place. So the market of corporate control cannot perform its external monitor function. Putting it to an extreme case, if the cost is close to 0, then it is always possible to attract potential raiders to collect information.

Let $A = f(\alpha^*)$ represent the maximum expected utility where α^* is the optimal holding for L in the takeover region. We now consider the case that L does not take control the firm.

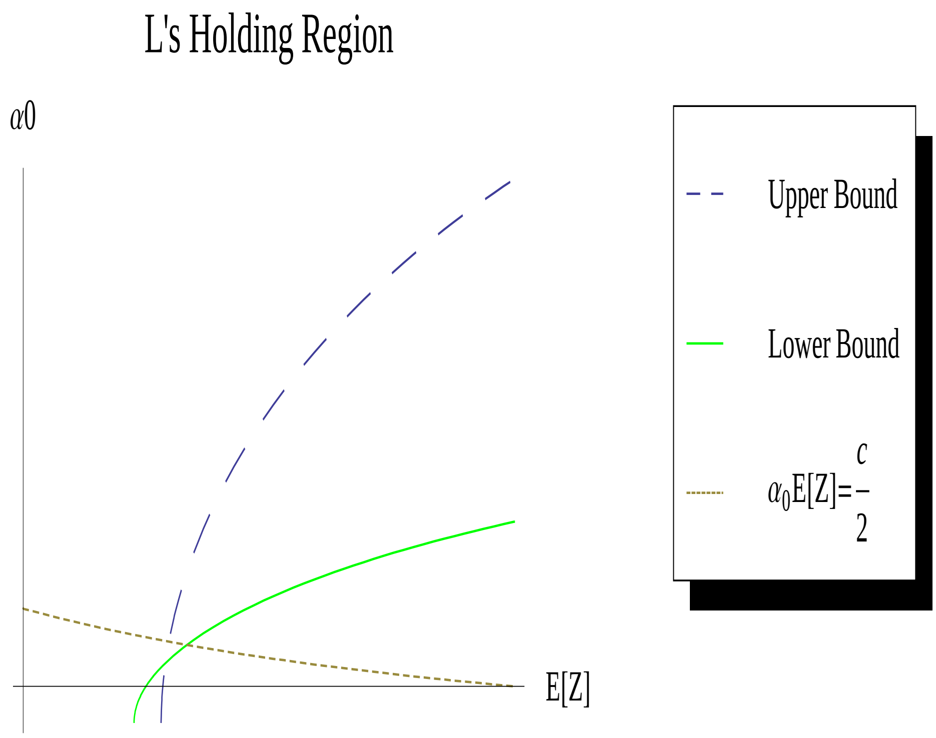

Figure 2: This Figure shows L's optimal holding if she takes over the firm and her initial holding gain is greater than $c/2$. In this figure, the dish line is the line of $\alpha_0 E[Z] = \frac{c}{2}$, the upper bound line is the upper boundary of inequality (7) and the lower bound line is the lower boundary of inequality (7). In the area above the dish line $\alpha_0 E[Z] = \frac{c}{2}$, L's optimal holding is 100%, between 50% and 100%, and 50%, in the area of above the upper bound line, between the upper bound line and the lower bound line, and under the lower bound line, respectively.

In the monitoring region, L's expected utility, $g(\alpha) = \mathbb{E}[W^l]$, can be written as

$$g(\alpha) \quad = \quad -\frac{1}{2}\gamma\alpha^2 V_0 - (\alpha - \alpha_0)V_0 + \{\alpha(1-\theta) - \frac{1}{\eta}(1-\theta)^2\}V_0\mathbb{E}[q].$$

The following proposition states the general Nash equilibrium between L and M. Hence the takeover region and the monitor region are characterized explicitly.

Proposition 2.1 *Assume* $(1-\theta)\mathbb{E}[q] < \frac{1}{2}\gamma + 1$. *Then*

1. *If* $A \geq V_0\left\{\frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q]-1]^2 - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] + \alpha_0\right\}$, *then L takes over with the optimal number of shares* α^*.
2. *If* $A < V_0\left\{\frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q]-1]^2 - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] + \alpha_0\right\}$, *then the best decision for L is to aquire shares up to* α_2^* *without paying a tender premium to take over the company.*

Assume $(1-\theta)\mathbb{E}[q] \geq \frac{1}{2}\gamma + 1$. *Then*

1. *If* $A \geq V_0\left\{-\frac{1}{8}\gamma - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\mathbb{E}[q]}{2} - (\frac{1}{2} - \alpha_0)\right\}$, *then L takes over the firm with the optimal number of shares* α^*.
2. *If* $A < V_0\left\{-\frac{1}{8}\gamma - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\mathbb{E}[q]}{2} - (\frac{1}{2} - \alpha_0)\right\}$, *then L decides against the takeover. However, L purchases shares as close to* 50% *as possible.*

Proof: Let

$$\alpha_2^* \equiv \frac{(1-\theta)\mathbb{E}[q]-1}{\gamma}. \tag{8}$$

For simplicity of notation let $B \equiv g(\alpha_2^*)$ and $C \equiv \lim_{\alpha\uparrow 50\%} g(\alpha)$. If $(1-\theta)\mathbb{E}[q] < \frac{1}{2}\gamma + 1$, then α_2^* is strictly smaller than 50%, by the above expression of $g(\alpha)$, L's optimal share in the monitor region is α_2^*. Hence, L's maximum expected utility in the monitor region is $B = g(\alpha_2^*)$. By simple calculation, we see

$$B = V_0 \left\{ \frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q] - 1]^2 - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] + \alpha_0 \right\}.$$

If $(1-\theta)\mathbb{E}[q] \geq \frac{1}{2}\gamma + 1$, the expected utility $g(\alpha)$ is increasing with respect to α in the monitor region $\{\alpha < 50\%\}$. Consequently, there exists no interior maximum point in the monitor region and the expected utility is less than but can be close enough to C. By simple calculation we get

$$C = V_0 \left\{ -\frac{1}{8}\gamma - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\mathbb{E}[q]}{2} - (\frac{1}{2} - \alpha_0) \right\}.$$

Other proofs of this proposition follows easily from the above discussion. $\square$.

Corollary 2.1 *If there exists no external control, that is $\eta = 0$, then it is always optimal for L to take over.*

Proof: When $\eta = 0$ we see that $B = C = -\infty < A$. Then the corollary follows from the proposition. $\square$.

In the benchmark model without external control, it is always optimal for L to take over the firm. We now derive the implications to the takeover in the presence of external control.

Figure 3 displays the monitor region and the takeover region in the presence of the external control. In this figure, $Z_{max} = 8, \mathbb{E}[q] = 4, \alpha_0 = 1\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$. The takeover gain from L's initial share is $\alpha_0\mathbb{E}[Z] = \frac{1}{2}\alpha_0 Z_{max} = 0.04 > \frac{c}{2}$. According to Proposition 2.1, the critical monitor parameter $\gamma^* = 2\{(1-\theta)\mathbb{E}[q] - 1\} = 5.92$. In the region $\gamma \leq 5.92$, in this example, since $A \geq C$ for all possible choices of positive

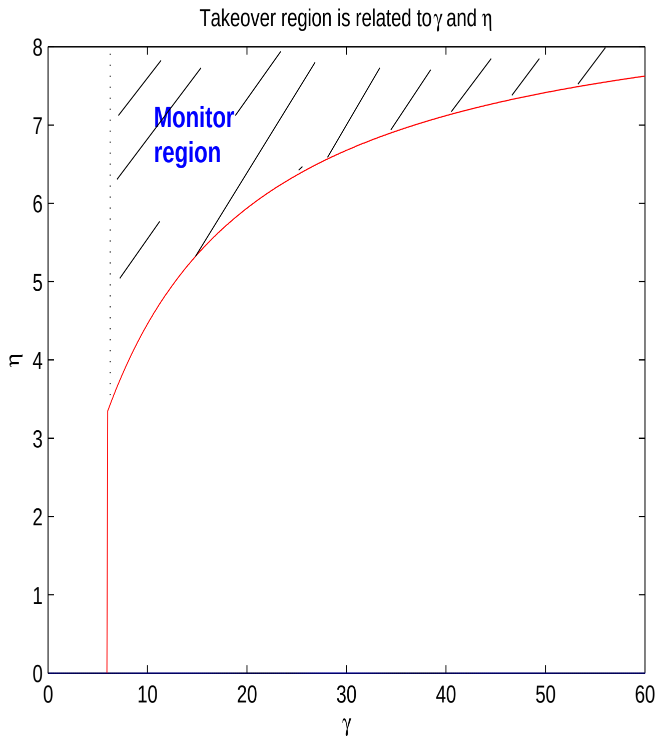

Figure 3: This figure displays the takeover region and the monitor region in the presence of external control. The parameters are $Z_{max} = 8, \mathbb{E}[q] = 4, \alpha_0 = 1\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$. There are two different situations for the monitor region γ: $\gamma \leq 5.92$ and $\gamma > 5.92$.

number η, L decides to take over. The interesting case is when the monitor cost is large, that is $\gamma > 5.92$. By Proposition 2.1 , L's takeover decision depends on γ and η. If η is bounded by the curve $\eta = \eta(\gamma)$ (as illustrated by the first situation of Proposition 2.1) where

$$\eta(\gamma) := \frac{(1-\theta)^2\mathbb{E}[q]}{\frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q]-1]^2 + \alpha_0 - \frac{A}{V_0}},$$

the takeover will take place. In particular, when $\eta = 0$ in the benchmark model, L always takes over the firm. However, when the external force is strong, i.e., η is above the curve $(\gamma, \eta(\gamma))$ displayed in Figure 3, it is not optimal for L to take over. Rather, because of the presence of the strong external control, L is willing to monitor the firm in this time period. Moreover, it is easy to see that the boundary $\eta(\gamma)$ is increasing with respect to the argument γ. This increasing property of $\eta(\gamma)$ implies that even γ is very large, L will monitor the firm in a strong external corporate control environment.

The next corollary explains how the internal monitor parameter γ affects L's takeover decision given the external parameter η. Intuitively, when γ is large, it is optimal for L to take over the firm instead of monitoring. On the other hand, when the monitoring cost parameter γ is small, L doesn't want to take over the firm. The intuition is verified in the following proposition.

Corollary 2.2 *Given an external parameter η, when γ is very large, L takes over the firm; when γ is very small, L prefers not to take over the firm; when γ is moderate, the decision of L is ambiguous.*

This corollary follows from Proposition 2.1 easily. When γ is *very large* in the sense that $\frac{1}{2}\gamma > (1-\theta)\mathbb{E}[q] - 1$ and

$$\frac{1}{2\gamma}\{(1-\theta)\mathbb{E}[q]-1\}^2 \le \frac{A}{V_0} + \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \alpha_0,$$

then by Proposition 2.1, the maximum expected utility A in the takeover region is greater than the maximum expected utility in the monitor region. Hence L takes over the firm. When the conditions hold, γ is large so that the internal control is not well enforced. The cost to monitor the manager is high and the effort L has to put on monitoring manager is high as well. L can then take over the firm for the purpose of reducing monitoring cost.

When γ is *very small* such that $\frac{1}{2}\gamma \leq (1-\theta)\mathbb{E}[q] - 1$ and

$$\frac{1}{8}\gamma < -\frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\mathbb{E}[q]}{2} - (\frac{1}{2} - \alpha_0) - \frac{A}{V_0},$$

then L doesn't take over the firm. In this case, γ is small and the firm's internal control is well functioning. It is easy for L to monitor M and so L can ensure the firm in the right track with very little effort. She then keeps M in control so that she may be able to exert her effort somewhere else. More importantly, L cannot impove firm efficiency much by reducing agency cost, so she is better off leaving the firm to M.

In the moderate level of γ, the situation is more complicated and its effect depends on the joint effects of all other parameters.[9] Figure 3 displays how the takeover decision is affected jointly by the external and the internal corporate control.

We now show how the external control parameter η affects the takeover decision for various levels of monitor cost parameter γ.

Proposition 2.2 *L will take over the firm when η is very small. When $\mathbb{E}[Z]$ is large enough and the cost structure c is bounded, then L will never take over the firm if the initial share $\alpha_0 \leq \frac{1}{6}$; L will take over the firm if the initial share $\alpha_0 > \frac{1}{6}$.*

[9]It is also possible that γ and η are negatively related. If so, the smaller η the greater γ. Consequently, then the probability of taking over increases more with decreasing of η. The interaction between η and γ is not investigated in this paper.

L will never take over the firm when the takeover cost c is very large and $\alpha_0 \leq \frac{1}{4}$.

Proof: See Appendix. □

The intuition of this proposition is clear. When η is very small, shareholders are not well protected by the external legal and social systems. While small shareholders cannot do much, large shareholder would prefer to control the firm. Moreover, if the initial holding is reasonable large, L wants to take over the firm to enjoy the improvement. On the other hand, when the expected improvement is large but the initial holding is very small, L prefers not to take over the firm. The transition cost c in this case is crucial. Because of the constraint $\alpha_0\mathbb{E}[Z] > \frac{c}{2}$, then $\mathbb{E}[Z]$ is very large if the cost structure, c, is also large when α_0 is fixed.

2.3 Equilibrium of Small Takeover Gain

In this section we derive the equilibrium model when L has a relatively small takeover gain from L's initial holding, namely, $\alpha_0\mathbb{E}[Z] < \frac{c}{2}$. As it turns out, the equilibrium and endogenous control decision are similar to those in the previous case for the large takeover gain region with some remarkable exceptions.

We first start with the optimal number of shares in the takeover region.

Lemma 4 *For a reasonable cost structure c, L takes over the firm by purchasing* 50% *of the firm's shares. For any cost structure c, L never takes the* interior *shares in the range* [50%, 100%].

Proof: See Appendix. □

In contrast to the large takeover gain, the optimal shares of L (α) doesn't depend on α_0. It is either 50%, or fully take over 100%. This surprising fact follows from the **convexity** of

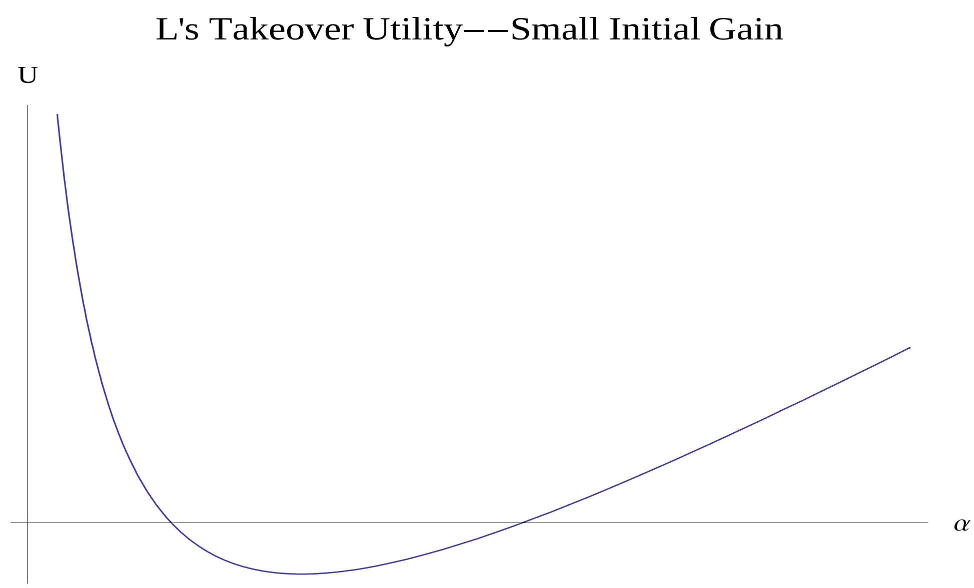

Figure 4: shows L's expected utility with respect to her ex post holding if she takes over in the case of $\alpha_0 E[Z] < \frac{c}{2}$.

the expected utility of the large shareholder (Figure 4). Since L's takeover utility function is convex, the maximum of this function is always at either 50% or 100%.

The Nash equilibrium can be given similarly. Actually, Prop 2.1 applies to this situation as well as long as we interpret $A = \max\{f(50\%), f(100\%)\}$ as the maximum expected utility in the monitor region by Lemma 4.

Proposition 2.3 *Assume $\alpha_0 \mathbb{E}[Z] < \frac{c}{2}$. Then L does not take over the firm when one of the following conditions holds:*

1. *The cost c is very large and $\mathbb{E}[Z]$ is bounded,*

2. *The expected improvement $\mathbb{E}[Z]$ is relatively large but $\alpha_0 \leq 25\%$.*

L takes over the firm if η is very small.

Proof: See Appendix. □

The intuition of this proposition is similar to Proposition 2.1. The expected utility in the monitor region is independent of the internal control parameters, η, γ, and/or θ. If η is very small, similar to the arguments in the large takeover gain region, A is large enough to compensate the monitor decision. Hence, L is willing to take over.

If either the cost structure c is very high, or the expected improvement $\mathbb{E}[Z]$ is relatively high but the initial share is not larger enough, then L prefers not to take over the firm. When c is high, by making the tender offer, L sends out the signal that the expected improvement is high. Small shareholders read the information and require a greater premium to tender. Given that the $\mathbb{E}[Z]$ can only be in a reasonable range and the initial holding is also bounded, L cannot recover from the transition cost so she has to forego the improvement opportunity. This also implies that L most likely takes over the firm due to the agency cost concern.

Similar to the large expected takeover gain case, the role of γ is more complicated then the role of η. The next proposition interprets the role of the internal control parameter ,γ, precisely, when α_0 is very small. The general case is similar to Proposition 2.1.[10]

Proposition 2.4 *Assume the initial share α_0 is very small. Then L takes over the firm if and only if one of the following conditions holds:*

1. *γ is large enough such that $\frac{1}{2}\gamma > (1-\theta)\mathbb{E}[q] - 1$ and*

$$\frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q]-1]^2 < \frac{\mathbb{E}[q-Z-1]}{2} + \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - c, \tag{9}$$

[10]The proof of this Proposition follows from the direct calculation of the limits of A, B and C when $\alpha_0 \downarrow 0$. We omit the details of this proposition and the general case, which are available from authors upon request.

2. $\frac{1}{2}\gamma \leq (1-\theta)\mathbb{E}[q] - 1$ *and*

$$\frac{1}{8}\gamma > -\frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\mathbb{E}[q]}{2} - \frac{\mathbb{E}[q-Z]}{2} + c. \tag{10}$$

According to this proposition, when γ is large enough, L takes over the firm. Under a reasonable range of parameters, the second condition barely holds, so L takes over the firm if and only if γ is large enough.

In either large or small initial holding gain cases, better external corporate governance (high η), and better internal corporate governance (low γ), reduce the possibility of change of control. In addition, appropriate compensation package designs are called to reduce or induce the possibility of change of control. If we view that an actual takeover causes nonproductive losses since the transition cost is wasted, then a better corporate governance practice, better firm transparency and a better disclosure environment improve firm efficiency. Takeover transition cost, on the other hand, reduces the economic efficiency since it reduces the monitoring value of market of corporate control. When c is large, L does not have much incentive to seek information to improve firm performance due to the low takeover probability.

Figure 5 displays the monitor region and the takeover region in the presence of the external control. In this figure, $Z_{max} = 8, \mathbb{E}[q] = 4, \alpha_0 = 0.5\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$. The takeover gain from L's initial share is $\alpha_0\mathbb{E}[Z] = \frac{1}{2}\alpha_0 Z_{max} = 0.02 < \frac{c}{2}$. Similar to Figure 3, the critical monitor parameter $\gamma^* = 2\{(1-\theta)\mathbb{E}[q] - 1\} = 5.92$. In the region $\gamma \leq 5.92$, since $A \geq C$ for all possible choices of positive number η, L decides to take over. The interesting case is when the monitor cost is large, that is when $\gamma > 5.92$. By Proposition 5 , L's takeover decision depends on γ and η. If η is bounded by a curve of γ (as illustrated by the first situation of Proposition 5), takeover will take place. In particular, when $\eta = 0$ in the benchmark model, L always takes over the firm. However, when the external force is

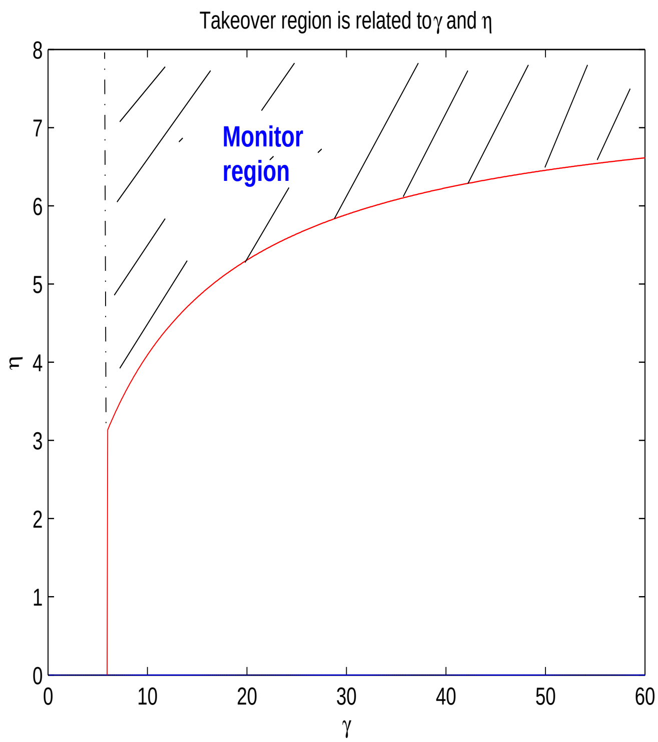

Figure 5: This figure displays the takeover region and the monitor region in the presence of external control. The parameters are $Z_{max} = 8, \mathbb{E}[q] = 4, \alpha_0 = 0.5\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$. There are two different situations for the monitor region γ: $\gamma \leq 5.92$ and $\gamma > 5.92$.

strong, i.e., η is above the curve displayed in Figure 3, it is not optimal for L to take over. Rather, because of the presence of the strong external control, L is willing to monitor the firm in this time period. The boundary of η in Figure 5 lies below the boundary of η in Figure 3.

In the case of the small initial holding, as we have explained, the remarkable feature is that the new level of holding α is independent of the initial α_0 . We will derive some important implications in a dynamic framework in Section 4.

3 Discussions

In this section, we present several implications from the sensitivity analysis.

The first corollary displays the property of α_1^* with respect to α_0. Its proof is straightforward and omitted.

Corollary 3.1 *If the new share α^* depends on α_0, then it is a concave function of α_0.*

The concavity suggests that there is an optimal level of initial holding for L if she takes over. In Goldman and Qian (2005), they find the same effect of the initial holding in the toehold content. In their model, as in Shleifer and Vishny (1986), L only attempt to acquire the binding percentage 50%, while in our model, L also decides the optimal ex post holding, α.

Corollary 3.2 *The tender premium is always negatively related to α_0. The higher L's initial holding α_0, the smaller the tender premium. In small takeover gain region, the tender premium is positively correlated to $\mathbb{E}[Z]$. In large takeover gain region, the role of $\mathbb{E}[Z]$ is ambiguous.*

Proof: See Appendix. □

For the case that the optimal holding for L is either 50% or 100%, it is straight forward from equation (2). If the optimal holding is determined by equation (6), we can substitute α^* back in and the result follows as shown in Figure 6. It is natural to expect that the tender premium increases with expected improvement and decreases with L's initial position. However, our model shows that, unlike what is shown in previous literature that the larger the initial position, the better bargaining power of L, large shareholder by nature can offer lower tender premium if she has a larger initial position.

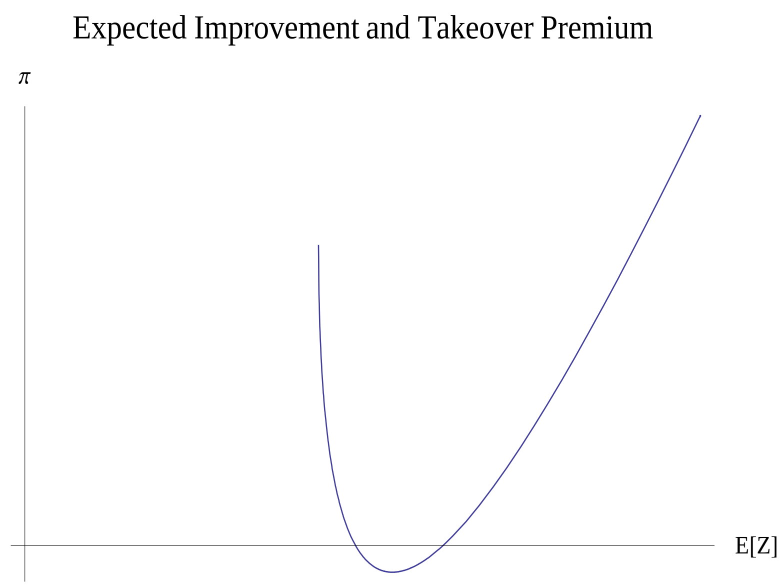

Figure 6: This figure shows the relationship between expected improvement if L takes over the firm and the takeover premium she has to pay to the small shareholders. It shows that the two can be either positive or negatively correlated.

Corollary 3.3 *If η is very small, then the optimal shares is determined by lemma 2; otherwise, the optimal shares is α_2^*. In either case the optimal shares doesn't depend on the size of η. However, β^* is negatively related to η.*

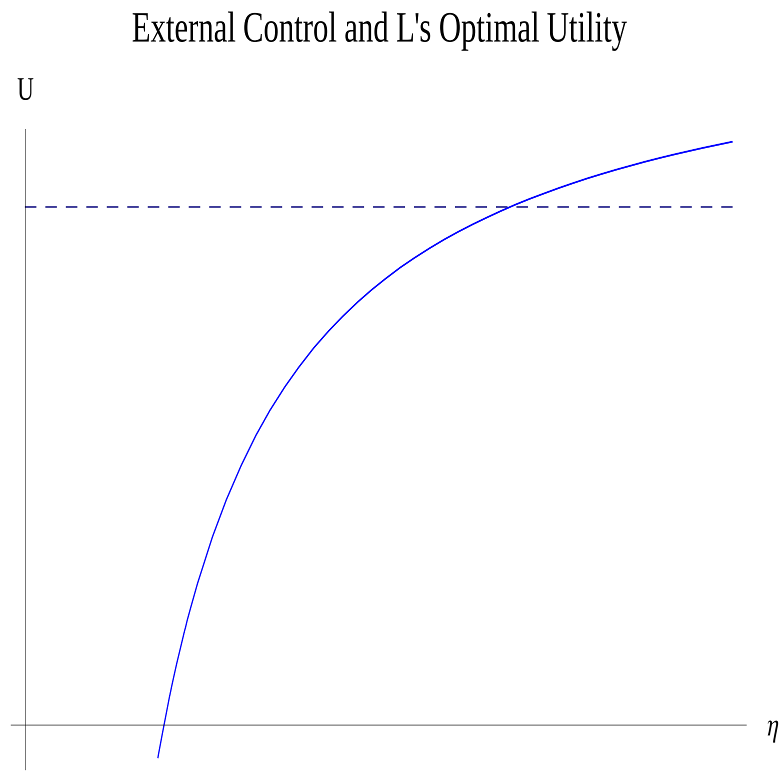

Figure 7: This figure shows how external corporate governance factor η effects L's expected utility. The dashed line is the optimal utility if L takes over the firm and another line is that if she does not take over. L takes over the firm when the dashed line is above the solid line and does not takeover the firm otherwise.

In the case that M retains, since M considers the legal and social cost, L mainly makes her decision based on her own cost of monitoring. Only if the social environment does not give adequate shareholder protection, L then decides to take over (Figure 7). It supports the fact that countries with poor shareholder protection are also associated with a high concentration of ownership. Shareholders need to have control of the firm so that they can protect their investment. In such countries, top manages are often the big shareholder themselves or their close relatives.

If η is large, then takeover action is not favorable to large shareholder. As mentioned in Shleifer and Vishny (1986), there are many firms with large shareholders in US but takeover is still an extreme phenomena.

Corollary 3.4 *If γ is very large, then the optimal shares is determined by Lemma 3; otherwise, the optimal shares is α_2^* which depends negatively on γ.*

In the situation where monitoring manager is difficult (Figure 8), shareholders choose to reduce their holdings. We see dispersed ownership in large firms where information is hard to collect.

Figure 9 show a general picture of how external control and internal control affect L's optimal utility. When L's utility increase with external control factor, η, it decreases with the internal control factor, γ.It is clear from Figure 9 that L is beneficial for better internal and external controls in general. It is interesting to compare the different role of the external control factor η and the internal control factor γ. The role of γ is not as straightforward as the effect of η. The different effects between γ and η follow from the dependence of α_2^* on the parameter γ.

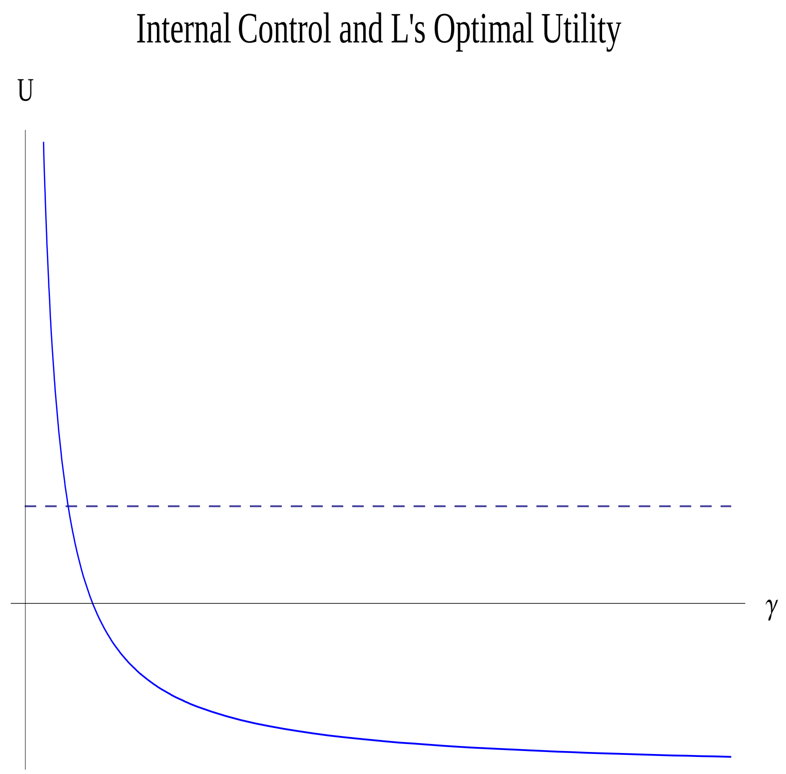

Figure 8: This figure shows how internal corporate governance factor γ effects L's expected utility. The dashed line is the optimal utility if L takes over the firm and another line is that if she does not take over. L takes over the firm when the dashed line is above the solid line and does not takeover the firm otherwise.

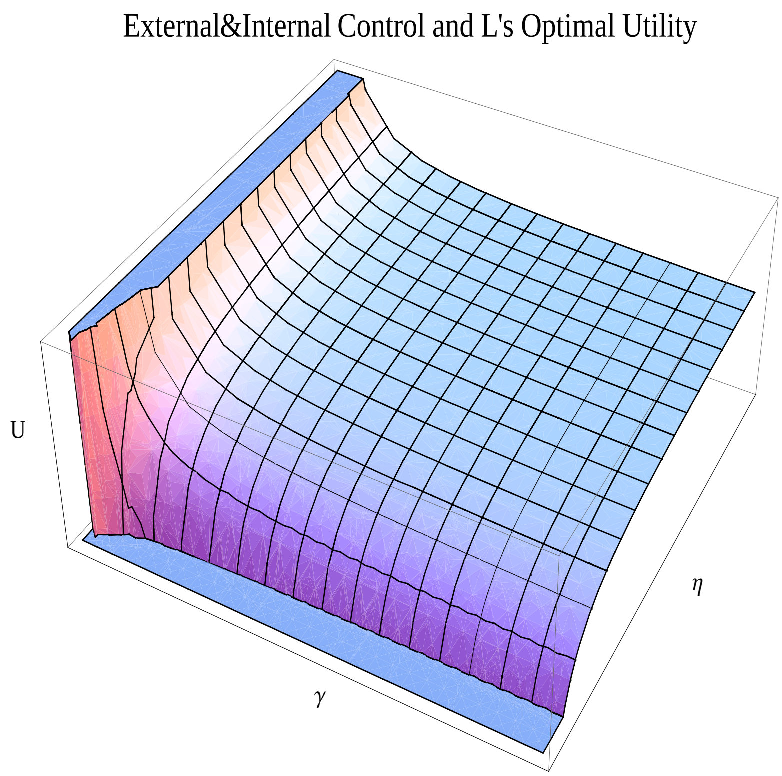

Figure 9: This figure shows how the internal γ and η control effects the maximum utility of L. In general, L's utility increases with decreasing of γ (less monitoring difficulty) and increasing of η (better shareholder protection)

4 Dynamic Extension

We have shown how the external and internal controls jointly affect the takeover decision, management stealing and the firm value endogenously in a single period model. The results of the previous section can be extended in a dynamic setting. Without loss of generality we also assume that $0.5Z_{max} = \mathbb{E}[Z] > c \geq \mathbb{E}[q_2] - 1$. We assume that those production rates q are i.i.d and improvement Z in each time period are i.i.d. Finally we assume that the compensation package is renewed in each time period.

We characterize the situation where L takes over the firm eventually in this dynamic framework. We start from a small initial holding α_0. Based on Proposition 2.1, it is possible that the new hoding α^* is less than 50%. So L does not takeover in the first time period. In this case, α^* either equals to $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma}$ (if it is smaller than 50%), or close to 50%.

Our next result states that, if $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} \geq \frac{1}{2}$, even though α^* is close but smaller than 50%, L will take over the firm in the subsequent time period.

Proposition 4.1 *If $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} \geq \frac{1}{2}$, then either L takes over the firm in this time period, or in the subsequent time period. Moreover, if L takes over the firm in the subsequent time period, L optimally takes 50% shares of the firm.*

Proof: See Appendix. □

We now move to the case that $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} < \frac{1}{2}$. In this case, as shown in the next result, the takeover decision depend on η and γ (the effect of two different corporate governance together).

Proposition 4.2 *Assume production rate q is i.i.d, improvement Z is i.i.d. and $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} < \frac{1}{2}$. Then L will take over the firm in the dynamic model when either γ is very large or η is* *relatively small comparing with γ (see Appendix formula (22)). Otherwise, L will stop in the second time period and always hold $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma}$ shares of the firm.*

Proof: See Appendix. □

The dynamic model demonstrates that even in the case without large shareholder, market of corporate control can be enforced by fostering a large shareholder. Raider can accumulate shares up to the point that takeover is favorable to her. It can also be the case that the business reaches the long-term dynamic equilibrium where an unilateral move is not optimal for any player. It is possible that the change in exogenous factors change will shift the dynamic of the game. As we mention above, γ and η are likely to be negatively related.

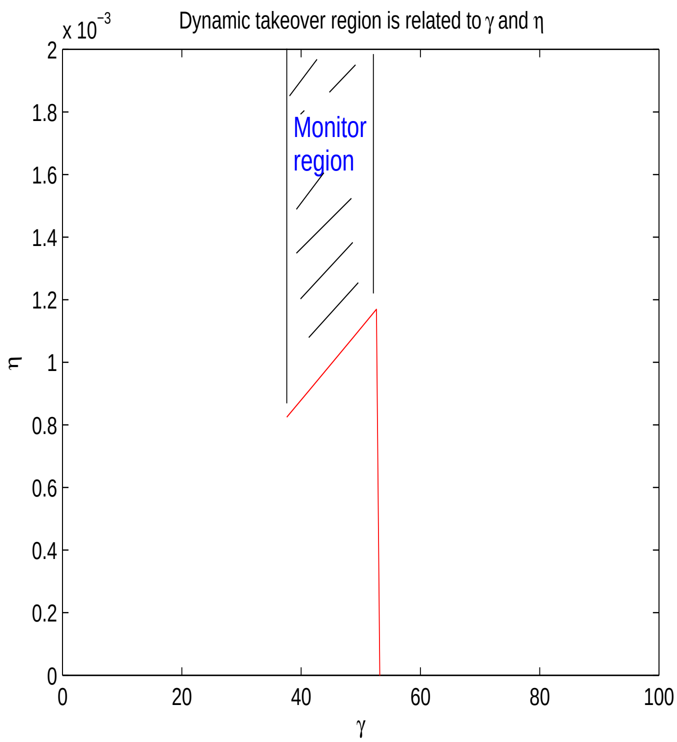

Figure 10: This figure displays the takeover region and the monitor region in the presence of external control. The parameters are $Z_{max} = 8, \mathbb{E}[q] = 18, \alpha_0 = 1\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$.

Figure 10 displays the takeover region and the monitor region in the dynamic setting. The parameters are $Z_{max} = 8, \alpha_0 = 1\%, \theta = 1\%, V_0 = 1$ and the cost $c = 0.05$. If the expected production $\mathbb{E}[q]$ is relatively small, such as $\mathbb{E}[q] = 4$. Based on Proposition 2.1, we see that L always take over the firm. The only difference in the dynamic setting and the static setting is when $\mathbb{E}[q]$ is large, for instance $\mathbb{E}[q] = 18$ as in Figure 10. When γ is large, then L always takeover the firm regardless of the external control. This is in contrast to the static setting in which L takes over the firm only when η is not strong enough. However, the monitor region is not empty even in the dynamic setting. When the γ remains in a reasonable range, we see from Figure 10 that L's optimal decision is to monitor when the external control is strong.

5 Managerial Defense

In the previous sections we assume that the manager puts up no defense when the large shareholder make the takeover announcement. L predicts the successful probability to be either one or zero. In the real world, management has the options to defend the target firm with various anti-takeover actions.

In this section we show that even in the presence of managerial defenses, our main results of the previous sections hold. Following Hirshleifer and Titman (1990), we consider three categories of defensive measures. These defensive measures are assume to be exogenously given. L anticipates such anti-takeover actions. So she takes these action into consideration when L makes the takeover decision.

5.1 Contingent Defense

In the first case, the defense strategy is in some form of poison pills which impose costs on the bidder, and thus redistributes wealth from L to S. For this reason, let the excess return to S be

$$Z + a,$$

where $0 \leq a \leq 1$ represents the extra amount shifted to S in the successful takeover.

By using the same procedure as in Proposition 3.1, we can easily obtain

$$\pi_a(\alpha) = \frac{c + \alpha Z_{max} + a(1+\alpha)}{\alpha + \alpha_0} \tag{11}$$

and

$$\frac{\partial \pi_a(\alpha)}{\partial \alpha} = \frac{\alpha_0 Z_{max} - c - a(1-\alpha_0)}{(\alpha + \alpha_0)^2} \tag{12}$$

By comparing the takeover premium with equation (2), this kind of contingent defense is in essence to impose a higher takeover cost, from c to $c + a(1-\alpha_0)$. Therefore, previous results can be applied in this case. For instance, equation (3) is replaced by

$$\alpha^* = -\alpha_0 + \sqrt{\frac{2\alpha_0 \left[\alpha_0 Z_{amx} - C\right]}{\mathbb{E}[Z] - (q-1)}} \tag{13}$$

where

$$C = c + a\,(1 - \alpha_0)$$

This strategy essentially introduces higher barriers of takeover. If raider anticipates such action ex anti, but still decides to go ahead with the tender offer, then minority shareholders are beneficial from such action. However, if the raider decides not to acquire the firm, the manager can then steal a larger proportion of the firm and remains in control. Even the raider eventually may take over the firm, the overall payoff to the small shareholders can be much less. Raider in this case will also get less wealth than in the non-managerial defense situation.

If we take a and α_0 as exogenously given, all the results from the previous sections follow. In a dynamic setting, α_0 changes period to period and is endogenous. If L accumulates shares period by period, the strategy will delay the takeover. Small shareholders in the final takeover period will benefit from such anti-takeover defense strategy.

It is also interesting to note that if $a < 0$, then L has the dilution right as described in Grossman and Hart (1980). In this case, to maximize her return, L is more likely to acquire less to obtain control.

5.2 Reduction in Firm Improvement

Manager can use the strategy of selling the units from which the most improvements can be made. We can model this by examining the effect of the possible improvements, reduced from Z to $\tilde{Z}$. Due to the defense, $\tilde{Z}$ is still distributed uniformly, however $\tilde{Z}_{max} = Z_{max} - \delta$, where δ is a positive number.

Our main results in the previous sections apply for given δ. If the strategy not only reduces the improvement but also is large enough to reduce the improvement from the large gain region (Section 3.2) to small gain region (Section 3.3), then it is clear that the decreasing $\mathbb{E}[Z]$ takes away some flexibility of L. For certain cases, L is more likely to take 100% of shares. This is clear that in this region, the strategy hurts minority shareholders since the tender premium is reduced along with the gain for tendering shareholders. However, even though the strategy takes away some of L's flexibility, it does not necessarily lead to a reduction of L's utility. In addition to paying less tender premium, L can adjust her holding decision to reflect the reduction of improvement.

If manager can reduce the improvement to the small gain region (Section 3.3), the results in section 3.3 apply. When the jump happens, L's utility also decreases. Since the improvement opportunity is reduced, the tender premium is also reduced if L still makes the tender offer. Even though the manager's compensation is affected negatively, he is still better off given that he is more likely to keep his job.

5.3 Impose Cost to Raider before Takeover

In this model, whenever a tender offer is made, it is going to be successful. Therefore, there are only two possible situations, tender offer is not made or the firm is taking over by the large shareholder. Since L expects manager will impose cost, she makes her take-over decision as if the cost only applies in successful take-over. L will simply not make an offer if she find that takeover is suboptimal. Therefore, she does not have to bear such cost imposed by the managerial defense. Such cost equivalent to an increase in the takeover transaction cost c.

L's utility decreases if manager impose a cost on the transaction. S is not worse off in this case. With a greater transaction cost, L has to pay more to S so that S will tender. Therefore, such anti-takeover strategy leads to losses for L if L takes over the firm. On the other hand, by imposing a high enough cost to L, this cost can prevent takeovers. Then no one is able to realize the possible improvement. Both S and L are not able to improve their utility.

6 Conclusion

By incorporating market of corporate control, monitoring and external control, we are able to demonstrate the inter play of all relevant factors. While the evidence for improvement hypothesis is strong as shown in previous studies and this paper, we also show that the inefficiency hypothesis is rather complicated. We show that not just pre-takeover performance, but the matrix of all factors is the driving force of takeover. Under current legal and market system, it is difficult to justify the inefficiency hypothesis. But we can clearly see that market of corporate control, monitoring and legal and social environment are important disciplinary factors of corporate governance.

Our model has assumed that the large shareholder have predicated and taken into account the manager and the minority shareholder's decision. In real life, manager might employ managerial defense and small shareholders could against the takeover decision. Thus there is likely to be asymmetric information about the takeover, stealing and the improvement, as in Hirshleifer and Titman (1990), Goldman and Qian (2005). Moreover, this model assumes there is one large shareholder who is also a possible raider. In the real world, there exists a competitive market among raiders and raiders have different role as the large shareholders[11] It would be interesting to incorporate the successful takeover probability and the competitive raider market into our framework.

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Appendix

Proof of Lemma 2 L's expected utility is

$$\begin{aligned} f(\alpha) := & \quad \alpha\mathbb{E}[V_2] - (\alpha-\alpha_0)(V_1+V_1\pi(\alpha)) - cV_1 \\ = & \quad \alpha_0 V_1 + \alpha(\mathbb{E}[V_2]-V_1) - (\alpha-\alpha_0)\pi(\alpha)V_1 - cV_1. \end{aligned}$$

By straightforward calculation, yield

$$\begin{aligned} f'(\alpha) &= V_1\left\{\mathbb{E}[q_2] + \frac{1}{2}Z_{max} - 1\right\} - \frac{Z_{max}\alpha^2 + 2\alpha_0 Z_{max}\alpha + 2\alpha_0 c - \alpha_0^2 Z_{max}}{(\alpha+\alpha_0)^2}V_1 \\ &= \frac{V_1}{(\alpha+\alpha_0)^2}\left\{(\alpha+\alpha_0)^2(\mathbb{E}[q]-\mathbb{E}[Z]-1) + 2\alpha_0(\alpha_0 Z_{max} - c)\right\}, \end{aligned}$$

and

$$f''(\alpha) = -\frac{4\alpha_0(Z_{max}\alpha_0 - c)}{(\alpha+\alpha_0)^3}V_1. \tag{14}$$

By assumption, $Z_{max}\alpha_0 > c$, the function $f(\alpha)$ is concave with respect to the percentage α.

Case (1). Assume that $\mathbb{E}[q_2] < \mathbb{E}[Z]+1$.Then by the first order condition, the maximum point of the function $f(\alpha)$ is

$$\alpha_1^* = \sqrt{\frac{s}{\mathbb{E}[Z]+1-\mathbb{E}[q_2]}} - \alpha_0. \tag{15}$$

Since $\alpha^* \in [50\%, 100\%]$, then

$$\alpha^* = \min\{\max\{50\%, \alpha_1^*\}, 100\%\}. \tag{16}$$

Case (2). Assume that $\mathbb{E}[q_2] \geq \mathbb{E}[Z]+1$. Then $f'(\alpha) > 0$. Hence the optimal $\alpha^* = 100\%$. The proof of this proposition is completed. ∎

Proof of Lemma 3. By Lemma 2, $\alpha^* > 50\%$ if and only if $\alpha_1^* > 50\%$ and equivalently,

$$u(\frac{1}{2} + \alpha_0)^2 < 2\alpha_0(\alpha_0 Z_{max} - c).$$

It is equivalent to

$$\alpha_0^2(4\mathbb{E}[Z] - u) - \alpha_0(u + 2c) - \frac{1}{4}u > 0.$$

Since $\mathbb{E}[Z] > \mathbb{E}[q] - 1 \geq 0$ by assumption, then there exists one positive and one negative root of the quadratic equation $(4\mathbb{E}[Z] - u)x^2 - (u + 2c)x - \frac{1}{4}u = 0$. Therefore, the last inequality holds if and only if the initial α_0 satisfies

$$\alpha_0 > \frac{u + 2c + 2\sqrt{u(c + \mathbb{E}[Z]) + c^2}}{2(4\mathbb{E}[Z] - u)}. \tag{17}$$

Note that, the right side of the above inequality is a increasing function of the variable u and takes limit $\frac{c}{Z_{max}}$ when $u \downarrow 0$. Therefore, when u is closes to zero, or when α_0 is large, the above inequality holds. Hence $\alpha_1^* > 50\%$. On the other hand, if u is relatively large, or α_0 is small, we see $\alpha_1^* \leq 50\%$.

By the same argument, we show that $\alpha_1^* < 1$ if and only if

$$\frac{u + c + 2\sqrt{2u(c + \mathbb{E}[Z]) + c^2}}{4\mathbb{E}[Z] - u} < \alpha_0.$$

∎

Proof of Proposition 2.2.

In the monitor region, it is straightforward to see that

$$B = V_1 \left\{ \frac{1}{2\gamma}[(1-\theta)\mathbb{E}[q]-1]^2 - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] + \alpha_0 \right\},$$

and

$$C = V_1 \left\{ -\frac{1}{8}\gamma - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] - \frac{(1-\theta)\theta\mathbb{E}[q]}{2} - (\frac{1}{2} - \alpha_0) \right\}.$$

Since

$$\lim_{\eta\to 0} B = \lim_{\eta\to 0} C = -\infty,$$

then $A > \max\{B, C\}$ when η is very small. Hence L will take over the firm.

On the other hand, since

$$\lim_{u\to\infty} \frac{u + 2c + 2\sqrt{u(c+\mathbb{E}[Z]) + c^2}}{2(4\mathbb{E}[Z] - u)} = 50\%, \tag{18}$$

then by Lemma 3, the optimal share in the taking over region is $\alpha^* = 50\%$. In this case, it is easy to see that

$$\lim_{u\to\infty} A = \begin{cases} -\infty, & \text{if } \alpha_0 < \frac{1}{6}; \\ \infty, & \text{if } \alpha_0 > \frac{1}{6} \end{cases} \tag{19}$$

Assume $\alpha_0 < \frac{1}{6}$. Hence $A < B$ and $A < C$ if the expected increment is very large. Then L doesn't take over the firm. By the same derivation, L takes over the firm by the same reason when $\alpha_0 > \frac{1}{6}$.

When the cost structure c is very large, by assumption $\alpha_0\mathbb{E}[Z] > \frac{c}{2}$, the expected increment $\mathbb{E}[Z]$ must be large too. Hence

$$\lim_{c\to\infty,\alpha_0\mathbb{E}[Z]>\frac{c}{2}} \frac{u+2c+2\sqrt{u(c+\mathbb{E}[Z])+c^2}}{2(4\mathbb{E}[Z]-u)} \geq 50\%, \tag{20}$$

then by Lemma 3, the optimal share in the takeover region is $\alpha^* = 50\%$. Therefore, the maximum expected in the takeover region is

$$\begin{aligned} A &= \alpha_0 + \frac{\mathbb{E}[q]-1}{2} - \frac{1-2\alpha_0}{1+2\alpha_0}\left\{c + \frac{1-6\alpha_0}{2(1-2\alpha_0)}\mathbb{E}[Z]\right\} \\ &\leq \alpha_0 + \frac{\mathbb{E}[q]-1}{2} - \frac{c}{4\alpha_0(1+2\alpha_0)}[-8\alpha_0^2 - 2\alpha_0 + 1] \end{aligned}$$

Hence for any $\alpha_0 < \frac{1}{4}$, since $8\alpha_0^2 + 2\alpha_0 - 1 < 0$, we have

$$\lim_{c\to\infty} A = -\infty.$$

Hence L doesn't take over the firm. ∎

Proof of Lemma 4.

When $\alpha_0 Z_{max} < c$, by formula (14), the expected utility $f(\alpha)$ is a **convex** function of the percentage in the takeover situation. Hence, in the general cost structure of c, $f(\alpha)$ never takes the interior optimal over the available region $[50\%, 100\%]$. We then show that $\alpha^* = 50\%$ for a reasonable cost structure of c.

First, assume that $\mathbb{E}[q]-1 \leq \mathbb{E}[Z]$. Then by the same proof as in Lemma 2, the expected utility $f(\alpha)$ is decreasing with respect to the percentage. Hence $\alpha^* = 50\%$.

Second, assume that $\mathbb{E}[q] - 1 > \mathbb{E}[Z]$. Then $\alpha^* = 50\%$ if either $\alpha_1^* \leq 50\%$, or $\alpha_1 > 50\%, f(50\%) \geq f(100\%)$. On one hand, by the proof of lemma 3, $\alpha_1^* \leq 50\%$ if and only if

$$c \geq \alpha_0 \frac{\mathbb{E}[Z]}{2} - \frac{(2\alpha_0 + 1)^2}{8\alpha_0} u.$$

On the other hand, $f(50\%) \geq f(100\%)$ if and only if the following inequality holds:

$$\frac{2(1-\alpha_0)(Z_{max} + c)}{1 + \alpha_0} - \frac{(1 - 2\alpha_0)(Z_{max} + 2c)}{1 + 2\alpha_0} \geq \mathbb{E}[q + Z] - 1,$$

or equivalently,

$$c < 2\alpha_0 \mathbb{E}[Z] - \frac{(1+\alpha_0)(1+2\alpha_0)}{4\alpha langetal1989JFElangetal1989JFE_0} u. \tag{21}$$

Hence, for a reasonable cost structure c, the optimal shares in the taking over region is always 50%. ∎

Proof of Proposition 2.3.

The proof is similar to the proof of Proposition 2.1 when the η is very small. By Lemma 4, $A = f(50\%)$. It is easy to see that $\lim_{c \to \infty} A = -\infty$ while $\mathbb{E}[Z]$ is bounded. On the other hand, since $\alpha_0 \mathbb{E}[Z] \leq \frac{c}{2}$, we have

$$\begin{aligned} A &= \alpha_0 + \frac{\mathbb{E}[q] - 1}{2} - \frac{1 - 2\alpha_0}{1 + 2\alpha_0} \left\{ c + \frac{1 - 6\alpha_0}{2(1 - 2\alpha_0)} \mathbb{E}[Z] \right\} \\ &\leq \alpha_0 + \frac{\mathbb{E}[q] - 1}{2} + \frac{1 - 2\alpha_0}{2(1 + 2\alpha_0)} (8\alpha_0^2 + 2\alpha_0 - 1)\mathbb{E}[Z]. \end{aligned}$$

Hence $\lim_{\mathbb{E}[Z] \to \infty, \alpha_0 \mathbb{E}[Z] \leq \frac{c}{2}} A = -\infty$ when $\alpha_0 < 25\%$. The proof of this proposition is finished. ∎

Proof of Proposition 4.1.

By prop 2.1, after the first time period, the optimal share $\alpha^* = 50\%, 100\%, \alpha_1^*, \alpha_2^*$, or a number which is close to 50% as much as possible. Under assumption, in all cases except for the last one, L has taking over the firm. Hence it suffices to assume that L takes a number of shares which is close to 50% and what happens in the next time period.

We follow the same derivation in Prop 2.1 in the next time period with a new initial share $\alpha_0 \to 50\%$ and we will prove that L would take at least 50% in this case.

In fact, since $c \geq \mathbb{E}[q_2] - 1$, we have

$$\lim_{\alpha_0 \to 50\%} \alpha_1^* = \sqrt{\frac{0.5Z_{max} - c}{\mathbb{E}[Z] + 1 - \mathbb{E}[q]}} - 0.5 \leq 50\%.$$

Then by Lemma 3, L takes exactly 50% of the firm in the takeover region and $A = f(50\%)$. By using the assumption that $\alpha_2^* \geq \frac{1}{2}$, it suffices to compare A and C when $\alpha_0 \to 50\%$. It is straightforward to check that

$$\lim_{\alpha_0 \to 50\%} A = \frac{1}{2}\mathbb{E}[q_2 + Z]V_1.$$

and

$$\lim_{\alpha_0 \to 50\%} C = V_1 \left\{ -\frac{1}{8}\gamma + \frac{1-\theta}{2}\mathbb{E}[q] - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q] \right\}.$$

Hence $\lim_{\alpha_0 \to 50\%} A > \lim_{\alpha_0 \to 50\%} C$. Therefore, by Proposition 2.1, L takes over the firm. The proof of this proposition is finished. ∎

Proof of Proposition 4.2.

It suffices to assume that $\alpha_0 = \frac{(1-\theta)\mathbb{E}[q]-1}{\gamma}$ in the subsequent time period. We use $f(\alpha, \alpha_0)$, $g(\alpha, \alpha_0)$ denote the utility $f(\alpha)$ and $g(\alpha)$, respectively, to highlight the initial input α_0. $A = \max_{\alpha \geq 50\%} f(\alpha, \alpha_0)$ is determined by lemma 2 and $V = g(\alpha_0, \alpha_0)$. Since $\frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} < 0.5$

by assumption, it suffices to compare A and B. First note that A doesn't depend on η and in this case (with the choice of α_0) (by ignoring the firm value)

$$B = \frac{[(1-\theta)\mathbb{E}[q]-1]^2}{2\gamma} + \frac{(1-\theta)\mathbb{E}[q]-1}{\gamma} - \frac{1}{\eta}(1-\theta)^2\mathbb{E}[q].$$

Moreover,

$$\lim_{\eta\to\infty} B = \frac{[(1-\theta)\mathbb{E}[q]-1]^2}{2\gamma} + \frac{(1-\theta)\mathbb{E}[q]-1}{\gamma}.$$

If γ is very large in the sense that $A \geq \lim_{\eta\to\infty} B$, then L takes over the firm. If $A < \lim_{\eta\to\infty} B$, since $\lim_{\eta\to 0} B = -\infty$, when η is very small in the sense that

$$\eta \leq \frac{(1-\theta)^2\mathbb{E}[q]}{\lim_{\eta\to\infty} B - A} \tag{22}$$

then $B < A$. Therefore L takes over the firm. Otherwise, A is less than $B$and L doesn't take over the firm and will keep the same number shares α_0. Hence we have proved the following proposition. ∎

Proof of Corollary 3.2

For $\alpha^* = k$, where $k = 0.5$ or 1 :

$$\begin{aligned} \pi &= \frac{\alpha Z_{max} + c}{\alpha + \alpha_0} \\ &= \frac{k Z_{max} + c}{k + \alpha_0} \end{aligned}$$

So, π is increasing with $Z_{\max} = 2\mathbb{E}[Z]$ but decreasing with α_0.

In large takeover gain region, if $\alpha^* = \sqrt{\frac{2\alpha_0(\alpha_0 Z_{max}-c)}{\mathbb{E}[Z]+1-\mathbb{E}[q]}} - \alpha_0$:

$$\begin{aligned}
\pi &= \frac{\alpha Z_{max} + c}{\alpha + \alpha_0} \\
&= \frac{\left(\sqrt{\frac{2\alpha_0(\alpha_0 Z_{max}-c)}{\mathbb{E}[Z]+1-\mathbb{E}[q]}} - \alpha_0\right) Z_{max} + c}{\sqrt{\frac{2\alpha_0(\alpha_0 Z_{max}-c)}{\mathbb{E}[Z]+1-\mathbb{E}[q]}}} \\
&= Z_{max} - \frac{\alpha_0 Z_{max} - c}{\sqrt{\frac{2\alpha_0(\alpha_0 Z_{max}-c)}{0.5Z_{max}+1-q}}} \\
&= Z_{max} - \sqrt{\frac{(\alpha_0 Z_{max} - c)(0.5Z_{max} + 1 - q)}{2\alpha_0}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi}{\partial \alpha_0} &= \frac{\partial\left(Z_{max} - \sqrt{\frac{(\alpha_0 Z_{max}-c)(0.5Z_{max}+1-q)}{2\alpha_0}}\right)}{\partial \alpha_0} \\
&= \frac{-\sqrt{2}c\,(0.5Z_{amx} - q + 1)}{2\alpha_0^2\sqrt{\frac{1}{\alpha_0}\,(\alpha_0 Z_{amx} - c)\,(0.5Z_{amx} - q + 1)}}
\end{aligned}$$

Apply inequality (7) and Lemma 3, $\frac{\partial \pi}{\partial \alpha_0} < 0$.

$$\begin{aligned}
\frac{\partial \pi}{\partial \alpha_0} &= \frac{\partial\left(Z_{max} - \sqrt{\frac{(\alpha_0 Z_{max}-c)(0.5Z_{max}+1-q)}{2\alpha_0}}\right)}{\partial Z_{max}} \\
&= 1 - \frac{\alpha_0\,(1 - q + 0.5Z_{max}) + 0.5(-c + \alpha_0 Z_{max})}{2\sqrt{2}\sqrt{\alpha_0(1 - q + 0.5Z_{max})(\alpha_0 Z_{max} - c)}}
\end{aligned}$$

So, π is decreasing with α_0 but the relation between Z_{max} and π is not definite if inequality (7) is satisfied. ∎